Molecular dynamics simulation of entanglement growth in generalized hydrodynamics

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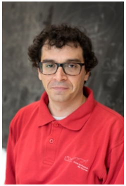
SISSA

Based on 1905.03206
Joint work with *Vincenzo Alba*

Collaborators and Funding



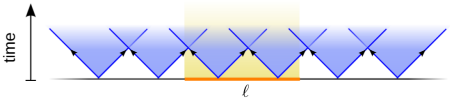
Vincenzo Alba
(Univ. Amsterdam / D-ITP)

Subject

Quasiparticle picture of entanglement evolution

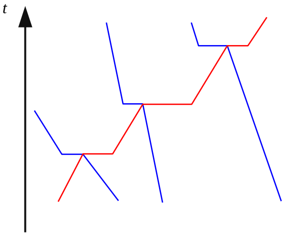
Calabrese, Cardy (JStat 2005)

Alba, Calabrese (PNAS 2017)



Soliton gas picture of Generalized Hydrodynamics (GHD)

Yoshimura, Doyon, Caux (PRL 2018)



1. Entanglement evolution

Quantum quench

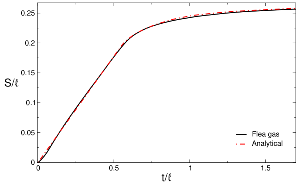
$$\rho(t=0) := |\Psi_0\rangle\langle\Psi_0|, \qquad \rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

Von Neumann entanglement entropy

$$S_A(t) = -\mathrm{Tr}\,\rho_A(t)\ln\rho_A(t), \quad \rho_A(t) = \mathrm{Tr}_\mathrm{B}\,\rho(t)$$

Typical behaviour of $S_A(t)$

$$S_A(t) \sim t \quad (v_\mathrm{M} t \ll \ell),$$
$$S_A(t)/\ell \sim S_\mathrm{th} \quad (t \to \infty)$$



Exact analytical results on the lattice:

- XY chain (free fermionic) Fagotti, Calabrese (PRA 2008)
- Kicked Ising chain (chaotic) Bertini, Kos, Prosen (PRX 2019)

Effective description:

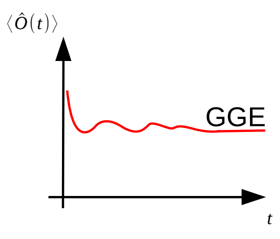
- Minimal membrane picture (non-integrable): Nahum, Ruhman, Vija, Haah (PRX 2017)
- Quasiparticle picture (integrable): Calabrese, Cardy (JStat 2005)

2. Quenches in integrable quantum systems

Homogeneous systems: GGE

- Infinite number of (quasi)local conserved charges: $[\hat{Q}_i, \hat{Q}_j] = 0$.
- Expectation values of *local* operators in the steady state are described by a *Generalized Gibbs Ensemble*

$$\hat{\rho}_{\mathrm{GGE}} = \frac{1}{Z} e^{-\sum_j \beta_j \hat{Q}_j}$$



Rigol, Dunjko, Yurovski, Olshanii (2007)

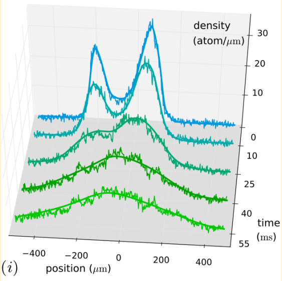
Inhomogeneous systems: GHD

- GHD: hydrodynamics with infinite number of continuity equations

$$\partial_t \hat{q}_i(x,t) + \partial_x \hat{j}_i(x,t) = 0$$

Bertini, Collura, De Nardis, Fagotti (2016)
Castro-Alvaredo, Doyon, Yoshimura (2016)

- Recently confirmed in ultracold atomic experiment



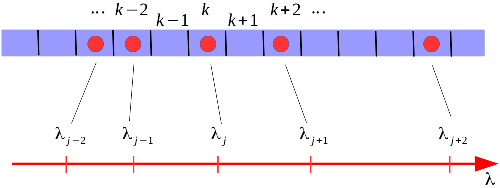
Schemmer, Bouchoule, Doyon, Dubail (2019)

3. Thermodynamic limit of Bethe ansatz solvable systems

Energy eigenstates

Energy eigenstates are enumerated by sets of (half)integer *quantum numbers,* which correspond to a set of *rapidities*

$$|\{I_j\}_{j=1}^N\rangle \to |\{\lambda_j\}_{j=1}^N\rangle$$



Densities

In the thermodynamic limit, eigenstates are characterized by the density of states, particles and holes in rapidity space:

$$\rho_{\mathrm{t},n,\lambda} = \rho_{n,\lambda} + \rho_{\mathrm{h},n,\lambda}$$

Bethe–Gaudin–Takahashi equations

$$\rho_{\mathrm{t},n,\lambda} = a_n(\lambda) - \sum_m \int d\mu T_{nm}(\lambda-\mu)\rho_{m,\mu}$$

Expectation values of conserved charges

$$\langle \hat{q}_j \rangle = \sum_n \int d\lambda \rho_{n,\lambda} q_{j,n}(\lambda)$$

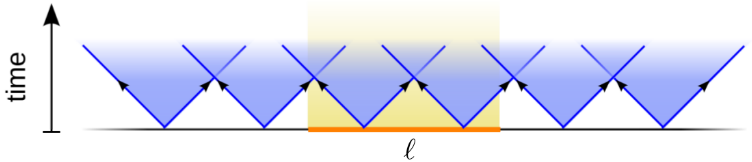
Yang–Yang entropy (∼ ln # of eigenstates)

$$s_{\mathrm{YY}} = \sum_n \int d\lambda \rho_{\mathrm{t},\lambda} \ln \rho_{\mathrm{t},\lambda} - \rho_\lambda \ln \rho_\lambda - \rho_{\mathrm{h},\lambda} \ln \rho_{\mathrm{h},\lambda}$$

Review: M. Takahashi (Cambridge University Press, 1999)

4. The quasiparticle picture of entanglement evolution

Calabrese, Cardy (JStat 2005)
Generic integrable systems: Alba, Calabrese (PNAS 2017)

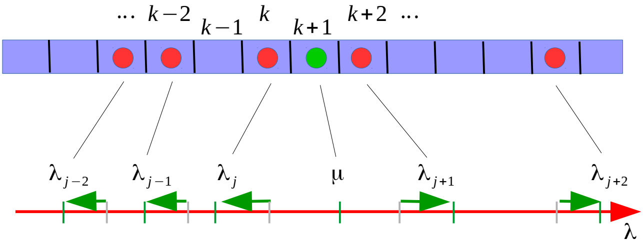


- Valid at large space-time scales
- Each segment $[x, x + \Delta x]$ is a source of quasiparticles
- In the quenches considered here, quasiparticles are emitted in pairs with rapidity $\pm\lambda$
  - Different configurations are possible Bertini, Tartaglia, Calabrese (JStat 2018)
- Quasiparticles move linearly with the effective velocity $v_{n,\lambda}$
- A pair contributes to the entanglement iff one of them is in A and the other is outside
- Each shared pair contributes to the entanglement $s_{n,\lambda}$, the Yang–Yang entropy density of the GGE
- $S_A(t)$ is obtained by counting shared pairs and integrating over all modes

$$S_A(t) \sim \left\{ 2t \sum_n \int_{2|v_{n,\lambda}|t<\ell} d\lambda |v_{n,\lambda}| s_{n,\lambda} + \ell \sum_n \int_{2|v_{n,\lambda}|t>\ell} d\lambda s_{n,\lambda} \right\}$$

5. The quasiparticle velocities

Bonnes, Essler, Lauchli (PRL 2014)



- When a quasiparticle is added, the rapidities of other quasiparticles are shifted
- This results in a *dressing* of charges

$$q_{j,n}^{\mathrm{dr}}(\mu) = q_{j,n}(\mu) + \sum_{k=1}^{N} \left[q_{j,n}(\tilde{\lambda}_k) - q_{k,n}(\lambda_k) \right]$$

- The effective velocities of quasiparticles are

$$v_{n,\lambda} = \frac{e_n^{\mathrm{dr}\prime}(\lambda)}{p_n^{\mathrm{dr}\prime}(\lambda)}$$

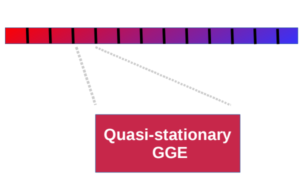
- In the TDL,

$$v_{n,\lambda} = v_{n,\lambda}^{\mathrm{bare}} + \sum_m \int d\mu \frac{T_{nm}(\lambda-\mu)}{a_n(\lambda)} \rho_m(\mu)(v_{n,\lambda} - v_{m,\mu})$$

6. Generalized hydrodynamics (at ballistic scale)

Infinite number of conservation laws + local quasi-stationarity

$$\partial_t \hat{q}_i(x,t) + \partial_x \hat{j}_i(x,t) = 0$$



Continuity equations for modes

$$\partial_t \rho_{n,\lambda}(x,t) + \partial_x (v_{n,\lambda}(x,t)\rho_{n,\lambda}(x,t)) = 0$$

Castro-Alvaredo, Doyon, Yoshimura (PRX 2016)
Bertini, Collura, De Nardis, Fagotti (PRL 2016)

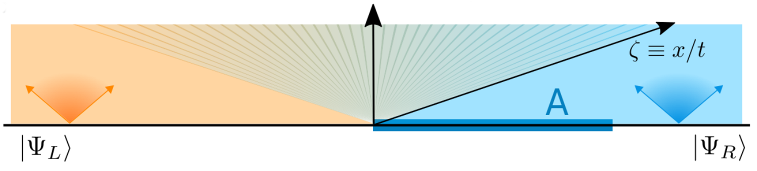
7. An inhomogeneous setting

XXZ Heisenberg spin chain

$$H = \sum_{j=1}^{L} \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right]$$

Bipartite quantum quench - extension of quasiparticle picture

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle, \qquad |\Psi_0\rangle = |\Psi_{0,\mathrm{L}}\rangle \otimes |\Psi_{0,\mathrm{R}}\rangle$$

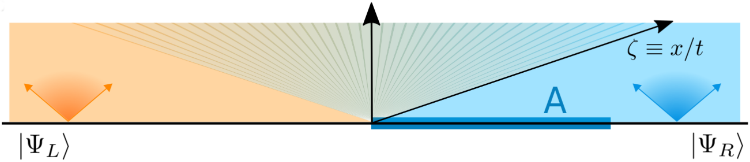


Example of an initial state

$$|\Psi_L\rangle = |\text{Néel}\rangle \equiv \left(\frac{1+\mathcal{T}}{\sqrt{2}}\right) (|\uparrow\downarrow\rangle)^{\otimes L/2}$$

$$|\Psi_R\rangle = |\text{dimer}\rangle \equiv \left(\frac{1+\mathcal{T}}{\sqrt{2}}\right) \left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right)^{\otimes L/2}$$

8. Analytical vs. numerical approach



The effective velocities

$$v_{n,\lambda}(\zeta) = v_{n,\lambda}^{\text{bare}}(\zeta) + \sum_m \int d\mu \frac{T_{nm}(\lambda - \mu)}{a_n(\lambda)} \rho_{m,\mu}(\zeta)(v_{n,\lambda}(\zeta) - v_{m,\mu}(\zeta))$$

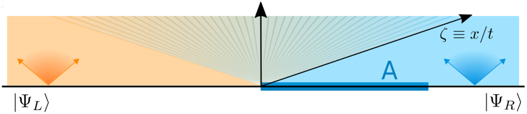
Possibilities for following quasiparticles & computing $S_A(t)$

- Analytically, by solving — V. Alba, B. Bertini, M. Fagotti (1903.00467)

$$\frac{d}{dt} X_{n,\lambda}(x,t) = v_{n,\lambda}(X_{n,\lambda}(t,x),t)$$

- Numerically, using the *flea gas* picture of GHD — MM, V. Alba (1905.03206)

9. The flea gas picture of GHD



The flea gas algorithm for simulating GHD: LL: Yoshimura, Doyon, Caux (PRL 2018)

1. Generate *randomly* a configuration of quasiparticles according to the initial distributions $\rho_{n,\lambda}(\pm\infty)$
2. Move the particles linearly with their bare velocities $v_{n,\lambda}^{\mathrm{bare}}$
3. When two particles (n,λ) (on the left) and (m,μ) (on the right) meet, make them jump with

$$+\frac{T_{nm}(\lambda-\mu)}{a_n(\lambda)} \quad \text{for } (n,\lambda)$$

$$-\frac{T_{mn}(\mu-\lambda)}{a_m(\mu)} \quad \text{for } (m,\mu)$$

4. After the simulation time T has elapsed, compute profiles of charges / entropy in the configuration and store it
5. Repeat the above many times ($\sim 10^2 - 10^5$) and take average of quantities over realizations

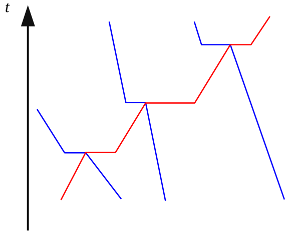
10. The velocities in the flea gas (heuristic argument)

- In a time Δt, the number of times a particle (n,λ) meets particles (m,μ) is (on average)

$$\rho_{m,\mu}|v_{n,\lambda}(\zeta)-v_{m,\mu}(\zeta)|\Delta t$$

- At each scattering, the particle (n,λ) jumps

$$\mathrm{sgn}\,(v_{n,\lambda}(\zeta)-v_{m,\mu}(\zeta))\cdot\frac{T_{nm}(\lambda-\mu)}{a_n(\lambda)}$$

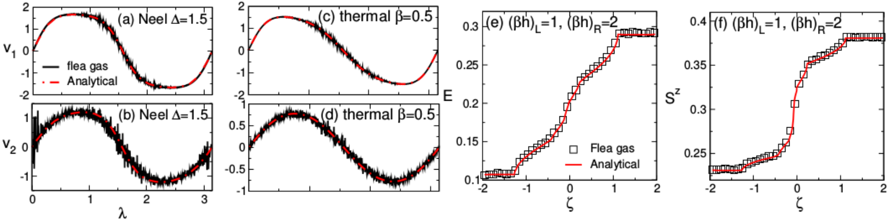


Effective velocities of flea gas particles

$$v_{n,\lambda}(\zeta)=v^{\mathrm{bare}}_{n,\lambda}(\zeta)+\sum_m\int d\mu\,\frac{T_{nm}(\lambda-\mu)}{a_n(\lambda)}\rho_{m,\mu}(\zeta)(v_{n,\lambda}(\zeta)-v_{m,\mu}(\zeta))$$

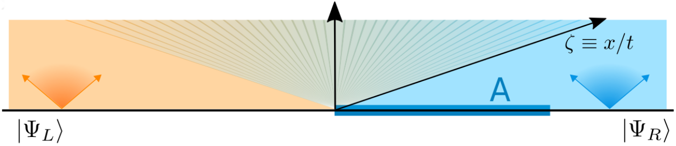
This is the same equation as the effective velocity equation in GHD.

11. Testing effective velocities in the XXZ flea gas



Rightmost panels: analytical result from
Piroli, De Nardis, Collura, Bertini, Fagotti (2017)

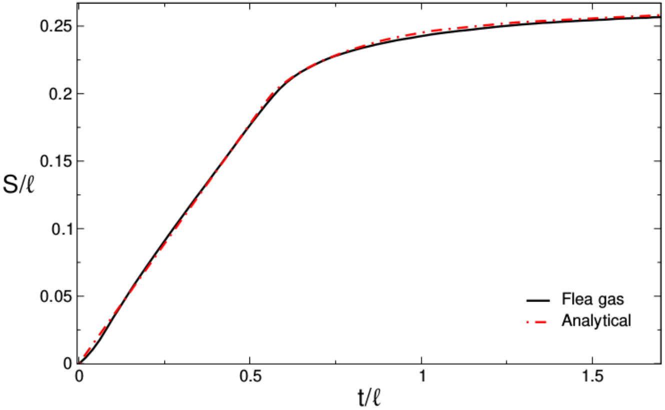
12. Computing entanglement entropy



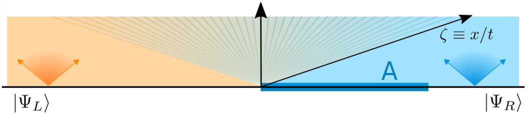
1. Prepare the initial state with particle pairs with rapidity $\pm\lambda_i$
2. For each pair, compute the Yang-Yang entropy contribution $s(\lambda_i)$
3. Evolve the flea gas in time
4. Find the "shared pairs" and sum their contribution $\sum_{\text{shared pairs}} s(\lambda_i)$
5. Repeat many times and compute the average

$$S_A(t) = \left\langle \sum_{\text{shared pairs}} s(\lambda_i) \right\rangle$$

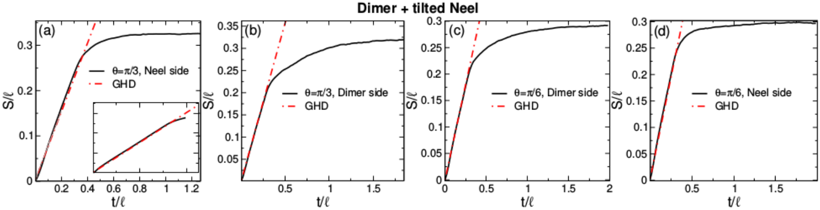
Homogeneous global quench from the Néel state



14. Test of the flea gas picture against analytical results II.



Bipartite quench from (tilted Néel $\otimes$ dimer)

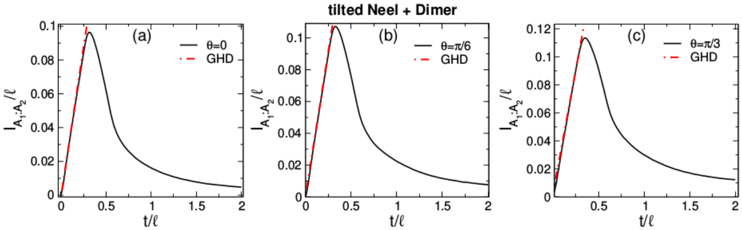


Initial rate: $S' = \sum_n \int d\lambda \left[\mathrm{sgn}(v_{n,\lambda}(0)) s_{n,\lambda}(0) + |v_{n,\lambda}(\sigma\infty)| s_{n,\lambda}(\sigma\infty)\right]$

Analytical: Alba, Bertini, Fagotti (1905.03206)

15. Mutual information

$$I_{A_1:A_2} = S_{A_1} + S_{A_2} - S_{A_1 \cup A_2}$$



15. Conclusions and outlook

Conclusions

- In integrable models, the quasiparticle picture of entanglement evolution can be matched with the flea gas picture of generalized hydrodynamics.
- We have tested the flea gas algorithm in the XXZ model
- We computed the full time evolution of the entanglement and the mutual information in bipartite quenches

Outlook: future research directions

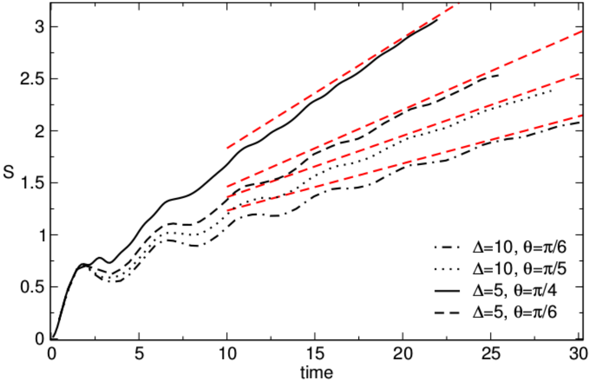
- Rigorous proof that the flea gas algorithm is equivalent to GHD
- Robust DMRG check of the quasiparticle picture
- More complicated setups
- Operator entanglement, diffusion and more

Reference: 1905.03206
Joint work with *Vincenzo Alba*

Thank you for your attention!

DMRG test

Bipartite quench from Neel + tilted Ferromagnetic state.



Alba, Bertini, Fagotti (1905.03206)