Spectral statistics and many-body localization

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Many-body localization (MBL) - what is it about?

1. Occurring in **INTERACTING** quantum systems with **DISORDER**

2. An **IDEAL INSULATOR** $\rightarrow$ at **ANY** temperature

3. Explains the **FAILURE** of some systems to **THERMALIZE**

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The outline of the presentation

1. The properties of MBL systems
2. Introduction of the physical model
3. Numerical analysis of the **spectral statistics**
   - A brief introduction to spectral statistics
   - **Spectral form factor** (**SFF**)
4. Our recent results and conclusion

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4. **Quantum chaos challenges many-body localization**

Jan Šuntajs,[1] Janez Bonča,[2,1] Tomaž Prosen,[2] and Lev Vidmar[1]
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MBL - a quick recap

- **Closed** quantum systems

Nandkishore, Huse, 2015

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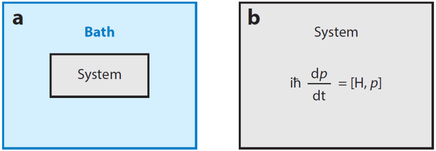

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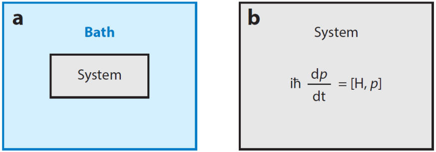

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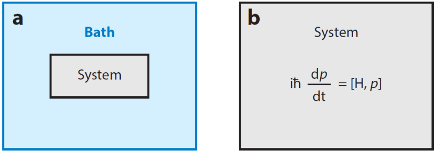

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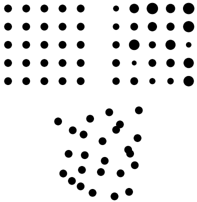

The properties of MBL systems

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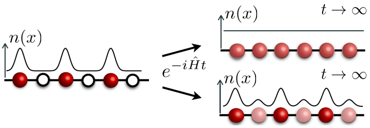

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  - **Area law scaling for all eigenstates**
  - Logarithmic growth in time

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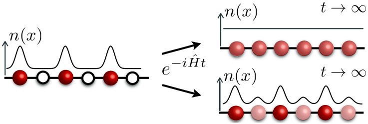

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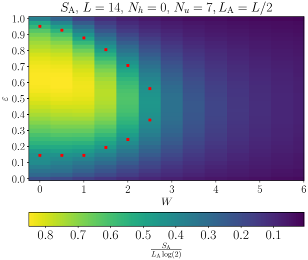

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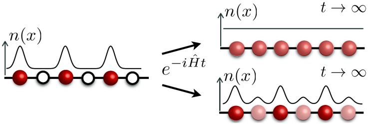

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PHYSICAL REVIEW B **77**, 064426 (2008)

Many-body localization in the Heisenberg *XXZ* magnet in a random field

Marko Žnidarič,[1] Tomaž Prosen,[1] and Peter Prelovšek[1,2]
[1]*Department of Physics, FMF, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia*
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(Received 31 August 2007; revised manuscript received 8 November 2007; published 25 February 2008)

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  - The subject of our numerical analysis

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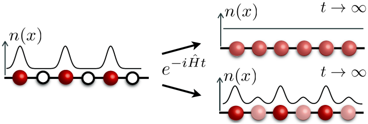

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The model

The hamiltonian - **paradigmatic ‘quantum chaotic’/ergodic**:

Heisenberg-like 1D

$$\hat{H} = \sum_{j \in \{1,2\}} J_j \sum_{\ell}^{L} \left(\hat{s}_\ell^x \hat{s}_{\ell+j}^x + \hat{s}_\ell^y \hat{s}_{\ell+j}^y + \Delta_j \hat{s}_\ell^z \hat{s}_{\ell+j}^z \right) + \sum_{\ell=1}^{L} w_\ell \hat{s}_\ell^z$$

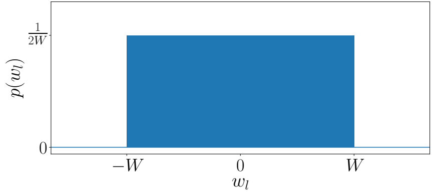
- l - site, L - chain length
- w_ℓ: randomly disordered potential

The model

$$J_1 = J_2 = 1$$

$$\Delta_1 = \Delta_2 = 0.55$$

Disorder probability distribution:



W - the disorder strength parameter

Our (numerical) analysis of the MBL systems

- We perform **full** or **partial** diagonalization of the Hamiltonians
  Spectrum: $\{E_1 \leq E_2 \leq \cdots \leq E_{\mathcal{D}}\}$

- **Partial diagonalization:** ≈ 500 eigenstates from the middle of the spectra

- Maximum Hilbert space dimensions:

$$\mathcal{D} = 48620 \quad \text{(full)}$$
$$\mathcal{D} = 184756 \quad \text{(partial)}$$

- Between $10^2 - 10^3$ **different** disorder realizations for **each model parameter**

Why do we study energy spectra?

- Quantum chaos conjecture (Bohigas, Giannoni, Schmidt, 1984):

Quantum systems

Spectral properties match the predictions of the **random matrix theory (RMT)**.

Corresponding classical systems

The dynamics are completely **chaotic**.

- **What about systems without a classical analogue?**

Quantum chaos and energy spectra

- **Many-body quantum chaos**

GENERIC systems $\rightarrow$ **RMT-like** spectral statistics

Montambaux *et. al.* (1993), Prosen (1999), Santos and Rigol (2008)

RMT statistics $\rightarrow$ hallmarks of **ergodicity** and **thermalization** in an isolated quantum system.

D'Alessio, Kafri, Polkovnikov, Rigol (2016)

Statistical properties of the energy spectra

- We analyse the **statistical properties** of the energy spectra
- We rely on the findings of the **RMT:**
  - **Ergodic** systems: spectral statistic match the **Gaussian orthogonal ensemble** (**GOE**)
  - **MBL** systems: nearest levels distributed in accordance with the **Poisson** distribution
- We compare our **RESULTS** with the **above cases.**

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The mean ratio of the level spacings

- The spacings between the **nearest** energy levels:

$$\delta_n = E_{n+1} - E_n \geq 0$$

- We define the **level spacing ratio**:

$$0 \leq \tilde{r}_n = \min\{\delta_n, \delta_{n-1}\}/\max\{\delta_n, \delta_{n-1}\} \leq 1$$

- **KEYNOTE:** the limiting values of $\langle \tilde{r} \rangle$ are well known:

Ergodic: $\langle \tilde{r} \rangle_{\mathrm{GOE}} = 0.5307$

MBL: $\langle \tilde{r} \rangle_{\mathrm{P}} = 2\ln 2 - 1 \approx 0.3863$

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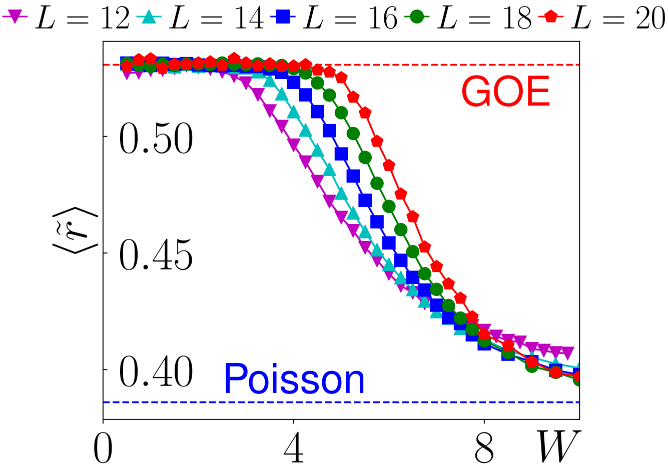

Pros and cons of $\langle \tilde{r} \rangle$

Mean level spacings ratio:

- a commonly used indicator of a given system's ergodicity
- + straightforward implementation
- - only considers correlation between the **nearest** energy levels

We would like to consider correlations between all the levels

- This is why we implement the **spectral form factor (SFF)**
- the implementation is more demanding

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The spectral form factor (SFF)

Definition:

$$K(\tau) \coloneqq \left\langle \frac{1}{D} \sum_{i,j}^{D} \mathrm{e}^{-i(\varepsilon_i - \varepsilon_j)\tau} \right\rangle; \qquad K(0) = D, \qquad K(\tau \to \tau_{\mathrm{H}}) = 1$$

D - Hilbert space dimension $\tau \to$ an external parameter

$\langle ... \rangle$ over disorder realizations

Heisenberg time $\tau_{\mathrm{H}} \propto$ inverse *mean level spacing*
(largest **sensible** timescale of a system)

$\{\varepsilon_i\} \to$ energy levels after *spectral unfolding*

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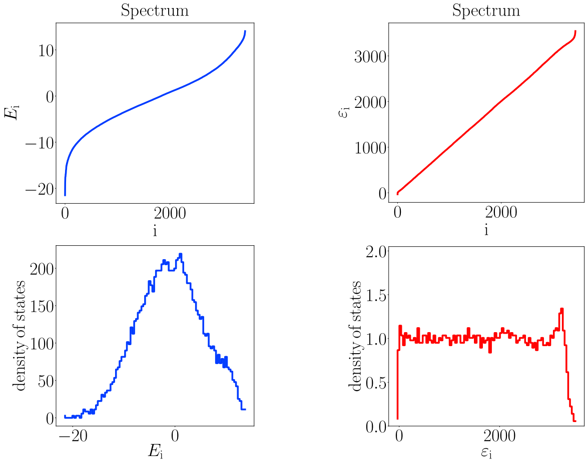
The spectral form factor (SFF)

- A quick introduction to unfolding



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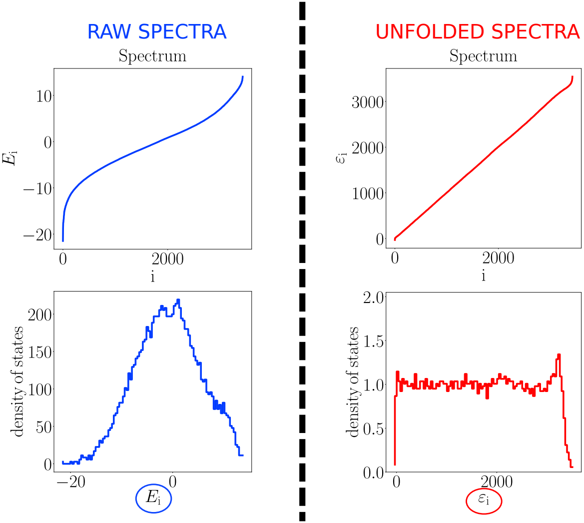
- **Unfolding** $\rightarrow$ mean level spacing $= 1$



The spectral form factor (SFF)

SFF - KEYNOTES:

- we again expect different behaviour for **ergodic** and **uncorrelated** spectra
- we investigated the behaviour of the **Thouless time** τ_{Th}
- **Thouless time** $\tau_{\mathrm{Th}} \to$ the onset of **UNIVERSAL DYNAMICS**

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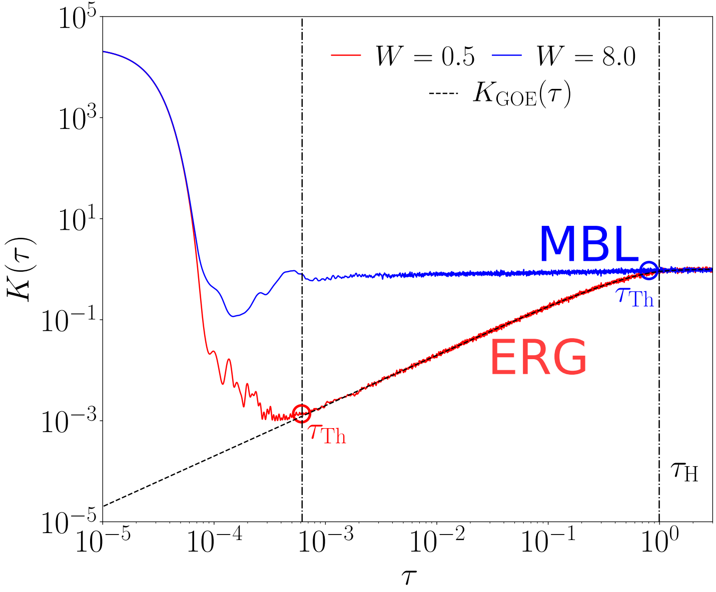

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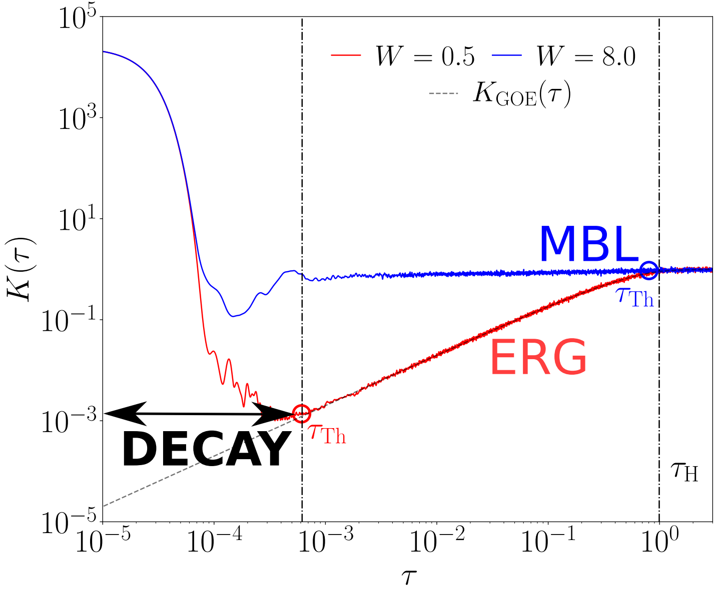

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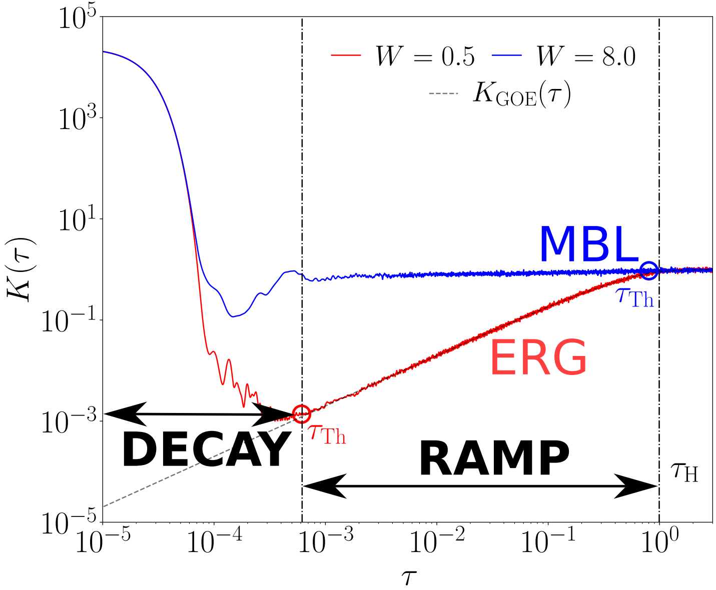

The spectral form factor (SFF)

We see universal behaviour after some transient time τ_{Th}

$K(\tau)$ for **uncorrelated** spectra

$$K_{\mathrm{P}}(\tau) = 1$$

$K(\tau)$ in ergodic systems

$$K_{\mathrm{GOE}}(\tau) = 2 - \tau \log\left(\frac{2\tau + 1}{2\tau - 1}\right)$$

SFF - explaining the Thouless time τ_{Th}

- **Thouless time** τ_{Th}: determines the energy scale at which the **spectral correlations** are universally determined by the **GOE** predictions (e.g., when the **RAMP** appears)

- large(r) $\tau_{\mathrm{Th}} \rightarrow$ small(er) **spectral correlation length** E_{Th}

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SFF - questions

- **We set out to find out different scalings**

- How does τ_{Th} scale with the system size L?

- How does τ_{Th} scale with disorder strength parameter W

We noticed some surprising results along the way

Quantum chaos challenges many-body localization

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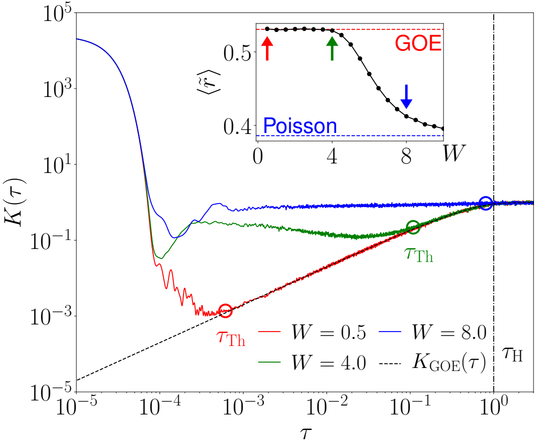
SFF - results

We first checked for consistency of the $K(\tau)$ and $\langle \tilde{r} \rangle$ results



$L = 18$

SFF - results

- We then numerically extracted τ_{Th} values and performed a scaling analysis w.r.t. both L and W

- In the subsequent scaling analysis, we introduce the **PHYSICAL THOULESS TIME** t_{Th}, rescaling τ_{Th} by the mean level spacing $\overline{\delta E}$ of the **RAW** spectra:

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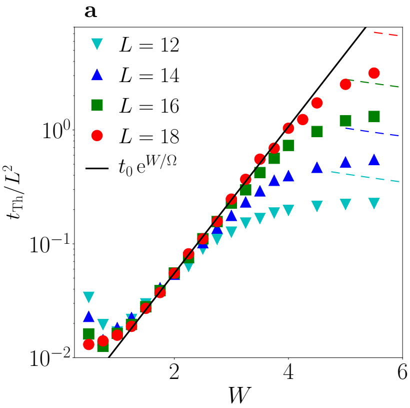
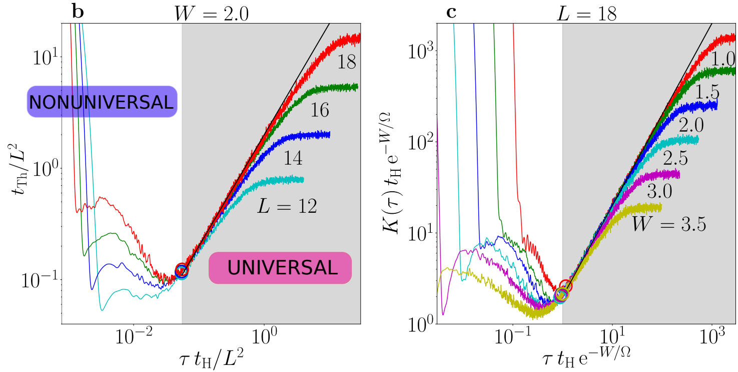
The results of the scaling analysis



We obtain the following double scaling:

$$t_{\mathrm{Th}} = t_0 \mathrm{e}^{W/\Omega} L^2$$

Verification of the scaling: data collapse



SFF - results

... Is there a MBL transition at all?

- a **SUPPOSED** MBL transition occurs for some **CRITICAL** disorder W^* when the energy spectrum becomes uncorrelated:

$$t_{\mathrm{Th}}(W^*) = t_{\mathrm{H}}$$

- t_{H} scaling is given by:

$$t_{\mathrm{H}} \propto \exp\left(L \ln 2\right)$$

- combining these results gives us

$$W^* \approx \Omega \ln(2)\, L \propto L$$

- **IMPLIES ABSENCE OF MBL IN THE THERMODYNAMIC LIMIT!**

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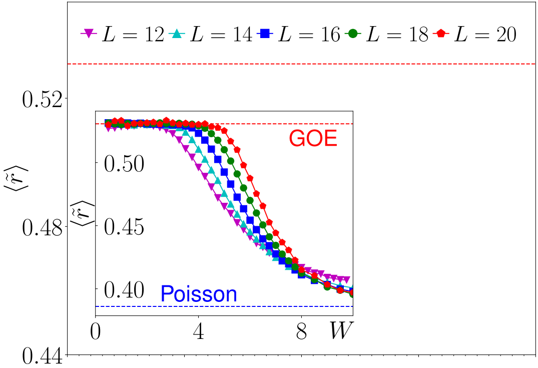
Verification of our results

- We tested our conclusions against more commonly used statistics
- However, we interpreted our results **DIFFERENTLY**



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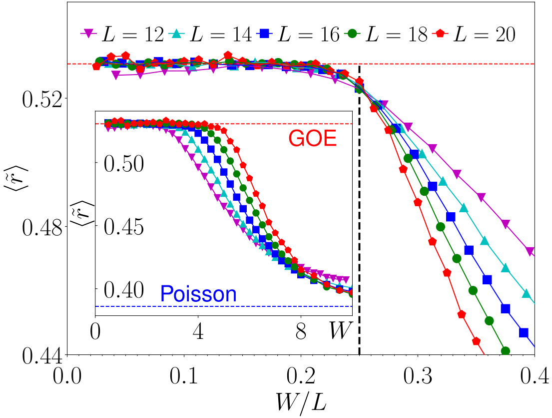

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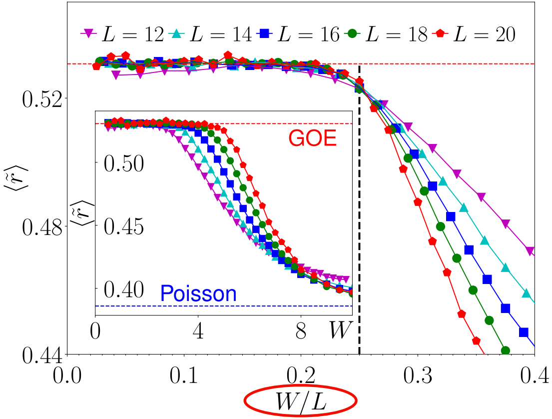

Verification of our results

- **BOTTOM LINE:** results for different spectral statistics seem consistent

Conclusions and further work

- Our results for the **PARADIGMATIC** class of models expected to give **MBL** show **NO INDICATIONS** of the MBL transition

- The emergence of **QUANTUM CHAOS** for **ANY** disorder strength in the TD limit

Conclusions and further work

- We need to test our assumptions on other models in which MBL is predicted
- We need to examine and better understand the relationship between **our results** and the **transport properties**

Conclusions and further work

Thank you for your attention!