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Slow quenches in topological insulators

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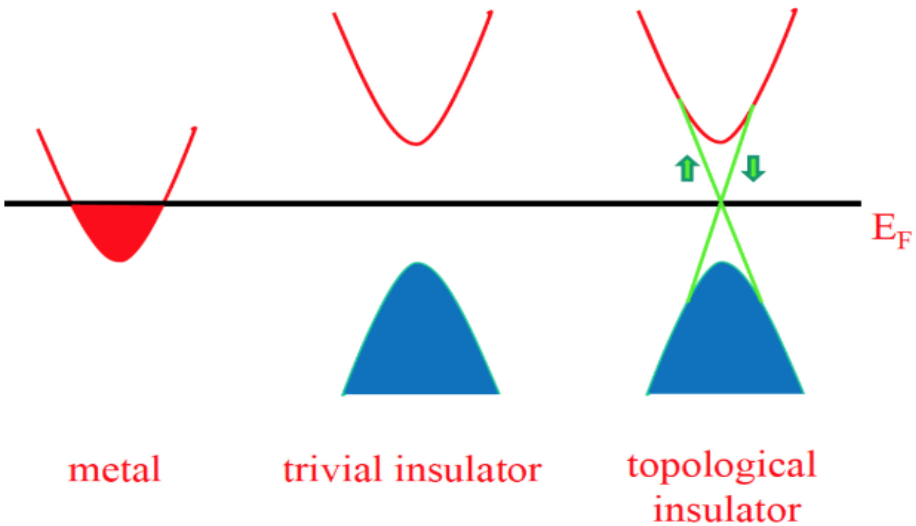
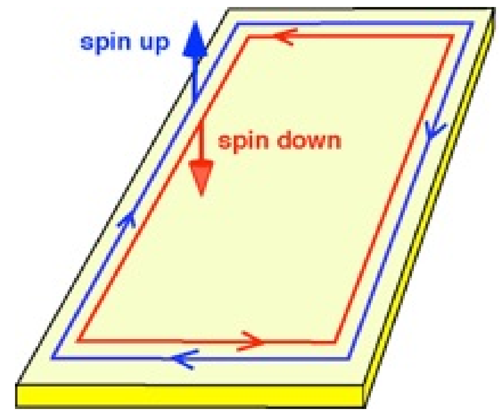
- Introduction
- Quenches in bulk systems
- Quenches in systems with edges
- Kibble-Zurek mechanism
- Quenches in disordered systems
- Conclusion

What are topological insulators?

- Non-interacting systems -> band theory

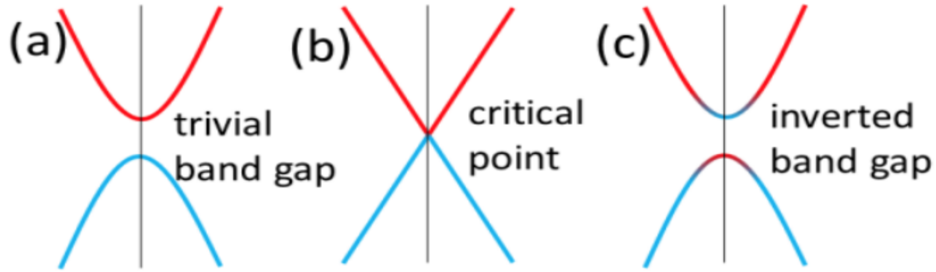




- Bulk: insulator, Fermi energy in the band gap
- Edge: conducting states inside the gap. Topologically protected a.k.a. avoid dissipation.
- Topological invariant: integer non-local order parameter, property of the bulk
- Bulk-boundary correspondence: the number of edge states is related to the topological invariant

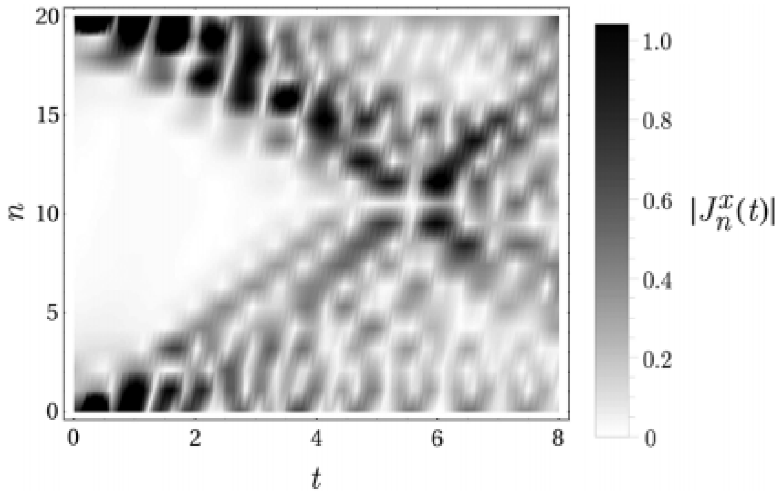
Time dependence of topological insulators

- Quenches between different topological regimes:



$$E_{\text{gap}} > 0 \longrightarrow E_{\text{gap}} = 0 \longrightarrow E_{\text{gap}} < 0$$

- Bulk Hall conductivity approaches the new ground-state value Hu, Zoller, Budich, PRL 117 (2016)
- Edge states relax towards new ground-state values Caio, Cooper, Bhaseen, PRL 115 (2015)



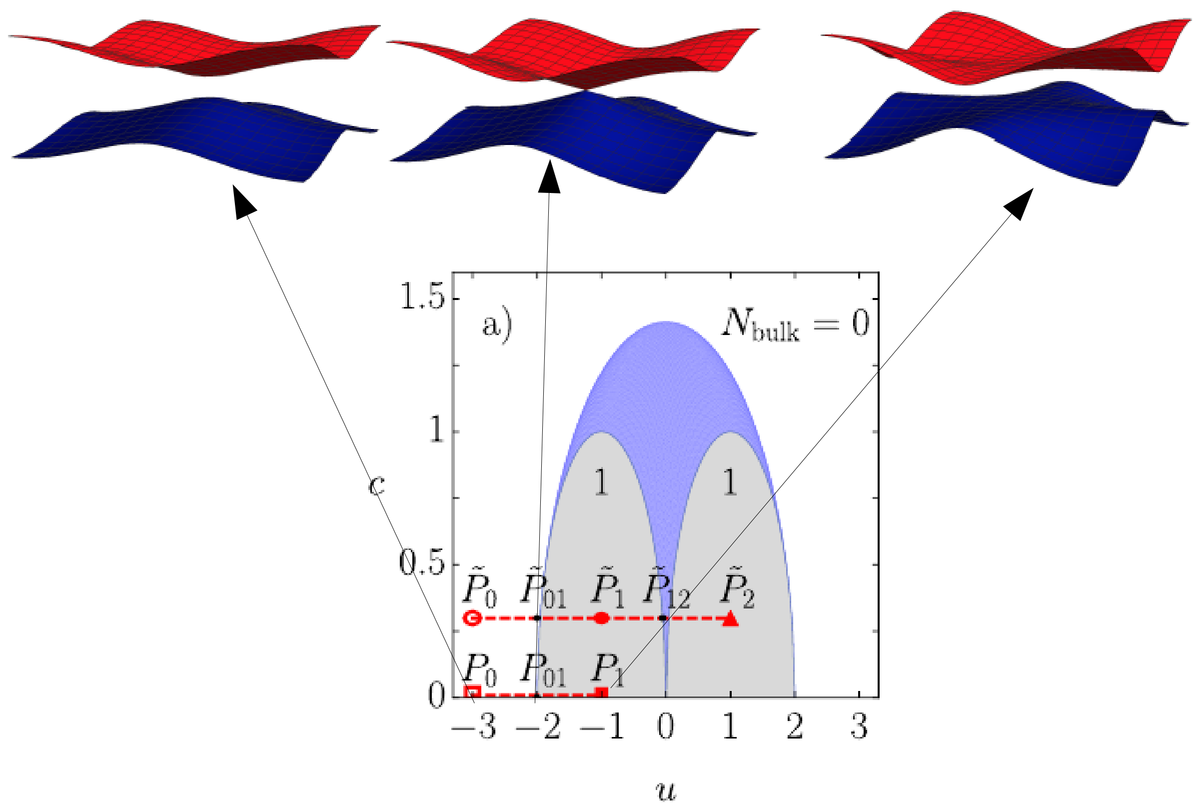
Systems with time-reversal symmetry – BHZ model

- Describes low energy physics of quantum wells (2D)

$$\hat{H}(\mathbf{k}) = \hat{s}_0 \otimes \left[(u + \cos k_x + \cos k_y)\hat{\sigma}_z + \sin k_y \hat{\sigma}_y\right] + \hat{s}_z \otimes \sin k_x \hat{\sigma}_x + c\,\hat{s}_x \otimes \hat{\sigma}_y$$

spin — Staggered orbital binding energy — orbital — Spin coupling

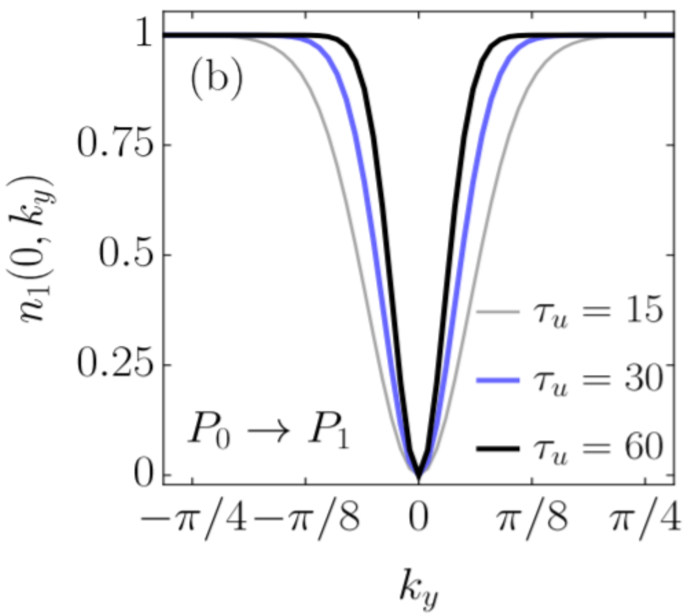
- $\mathbb{Z}_2$ Topological invariant: 0 or 1
- 4 energy bands:

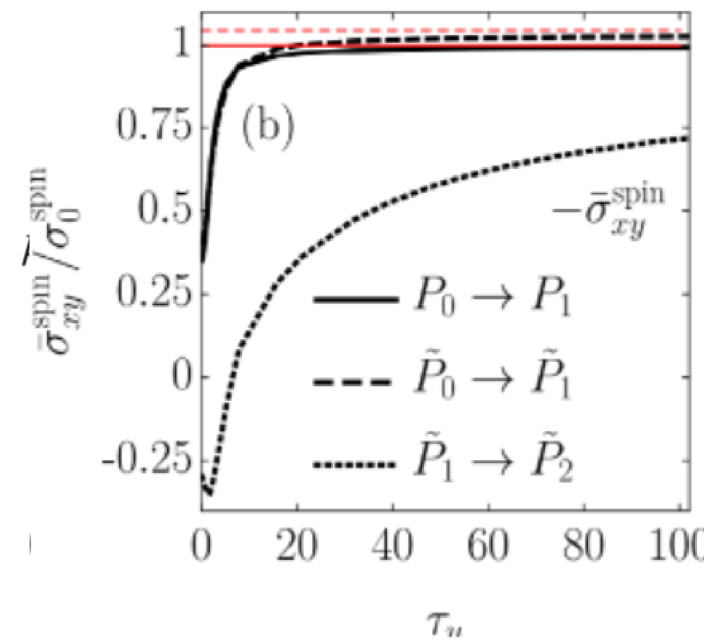


$$u(t) = u_0 + (u_1 - u_0)\sin^2(\tfrac{\pi}{2}t/\tau_u)$$

BHZ model after a quench

- Gap closes, electrons are excited near closing, Landau-Zener dynamics





$$n_{\rm exc} = \exp[-\frac{\pi(|\mathbf{k}-\mathbf{k}_c|-c)^2}{v_u}]$$

$$v_u = |\frac{\mathrm{d}u}{\mathrm{d}t}\Big|_{t=\frac{\tau_u}{2}}| = \frac{\pi|u_1-u_0|}{2\tau_u}$$

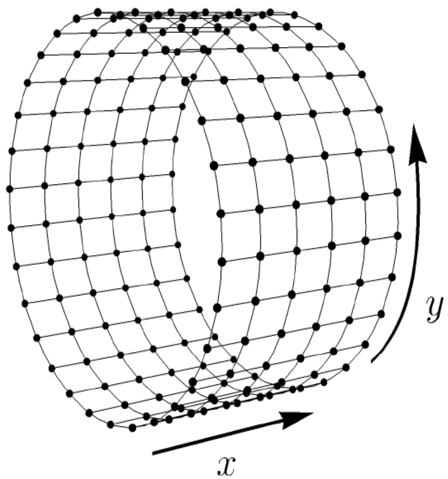
$$\delta\sigma_{xy}^{\rm spin} \propto n_{\rm tot} \propto \begin{cases} 1/\tau_u, \; [\hat{H},\hat{s}_z]=0 \\ 1/\sqrt{\tau_u}, [\hat{H},\hat{s}_z] \neq 0 \end{cases}$$

Chern ribbon – QWZ model

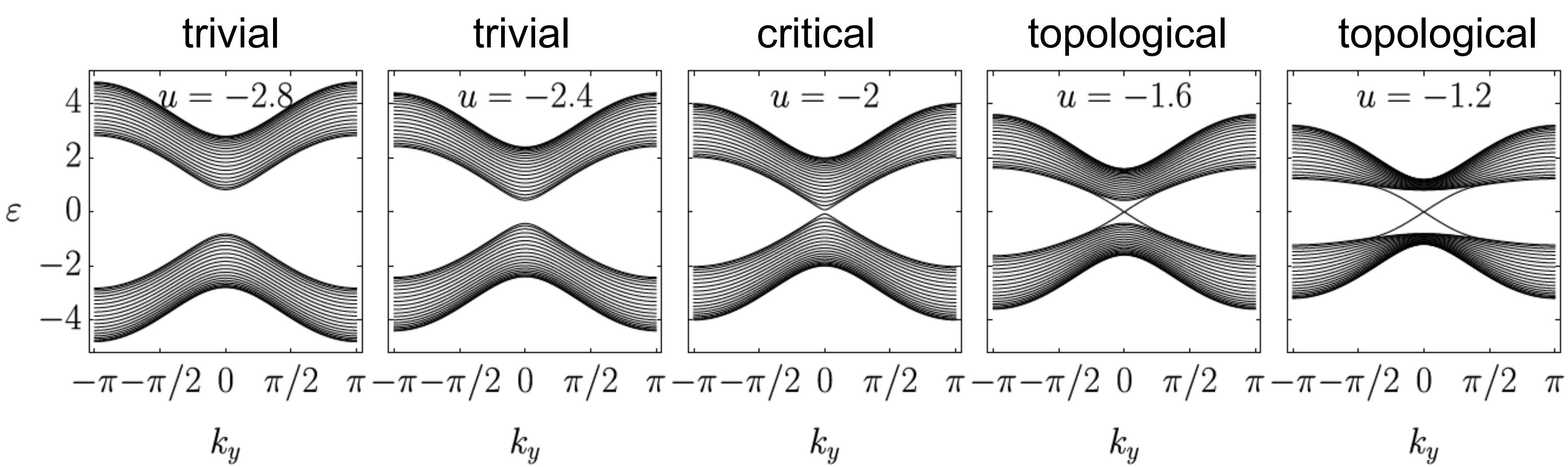
$$\hat{H}(k_y) = \sum_{x=1}^{N_x-1} \left(|x+1\rangle\langle x| \otimes \frac{\hat{\sigma}_z + i\hat{\sigma}_x}{2} + h.c. \right) + \sum_{x=1}^{N_x} |x\rangle\langle x| \otimes \left((\cos k_y + u)\hat{\sigma}_z + \sin k_y \hat{\sigma}_y \right)$$

orbital

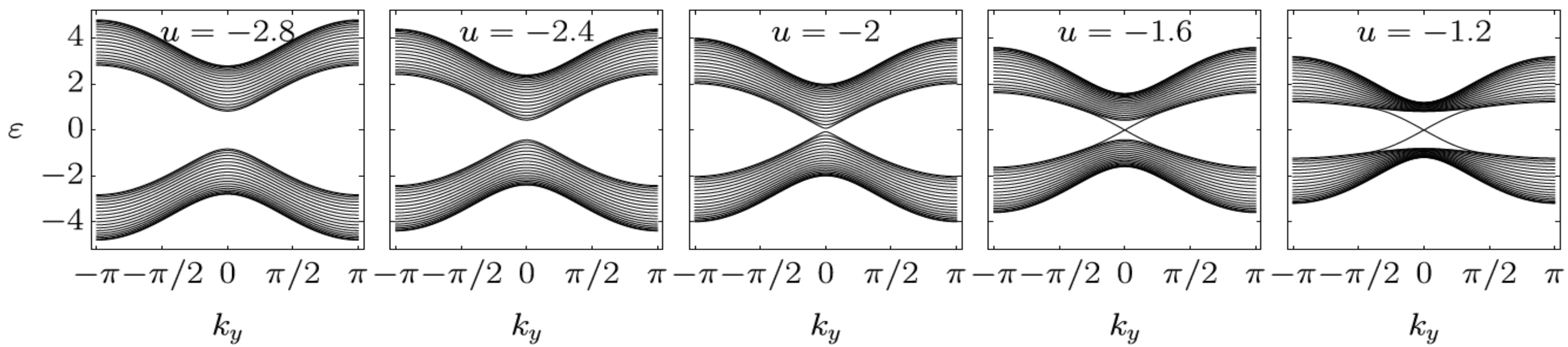
Staggered orbital binding energy



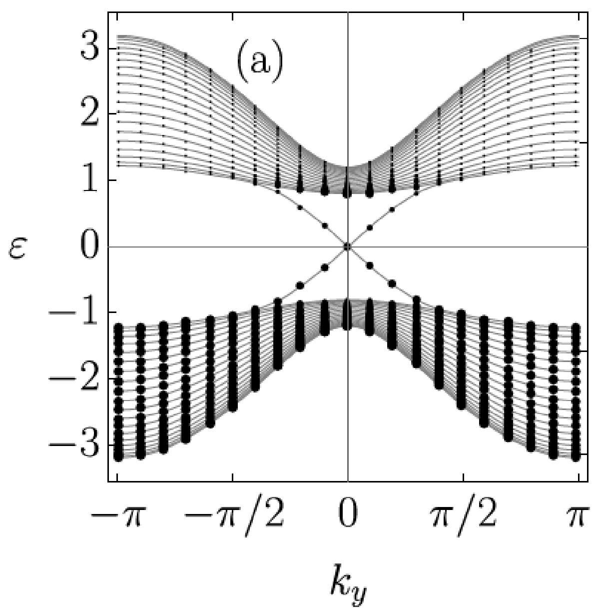
$$C = \begin{cases} 1, & 0 < u < 2, \\ -1, & -2 < u < 0, \\ 0, & |u| > 2. \end{cases}$$

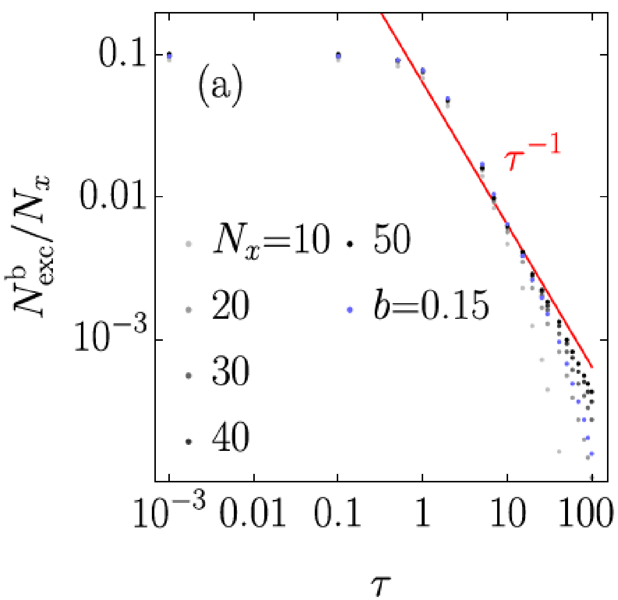


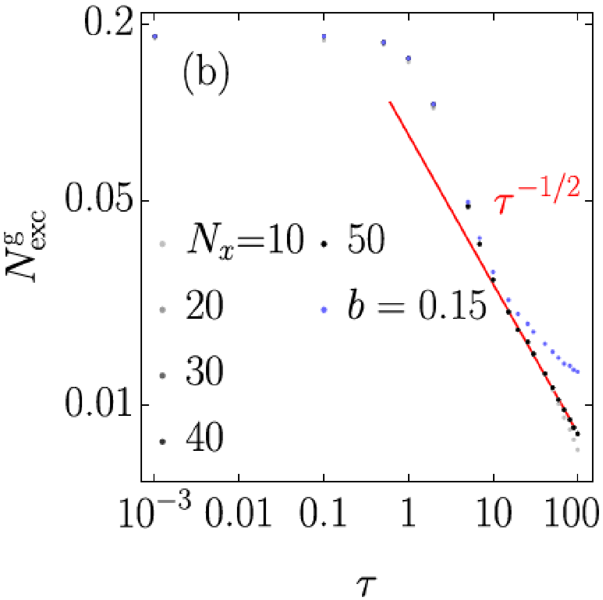
Chern ribbon - excitations



$$u(t) = u_0 + (u_1 - u_0)\sin^2(\tfrac{\pi}{2}t/\tau_u)$$



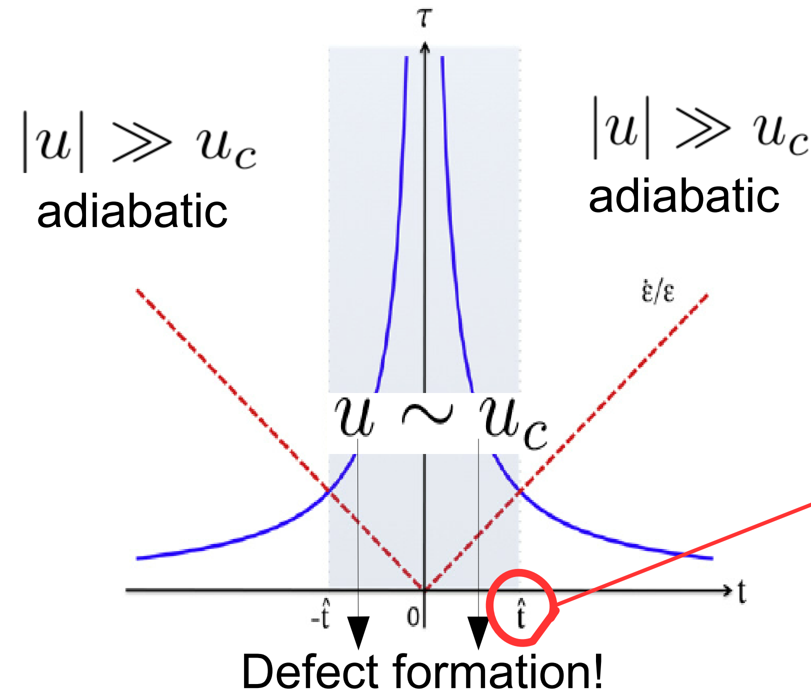




Kibble-Zurek mechanism (KZM)

- Systems driven through a continuous phase transition: $u = u_c(1 - t/\tau_u)$

$$\tau(u) \propto |u - u_c|^{-z\nu} \qquad \xi(u) \propto |u - u_c|^{-\nu}$$



Freeze-out time:

$$\hat{t} = \tau(\hat{t})$$

$$\hat{t} \propto \tau_u^{\frac{z\nu}{1+z\nu}}$$

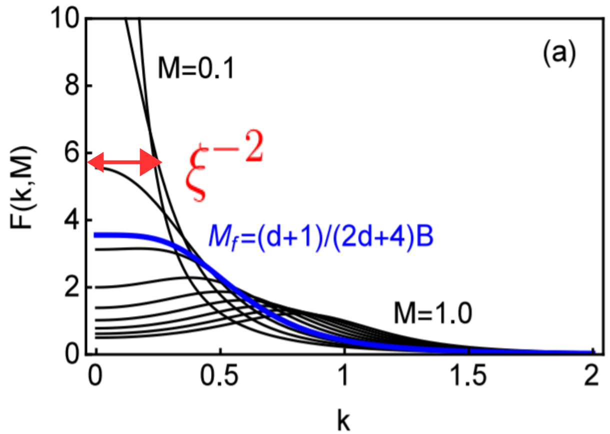
- Average defect size: $\xi(\hat{t}) \propto \tau_u^{\frac{\nu}{1+z\nu}}$

- Density of defects: $n = \xi(\hat{t})^{-d} \propto \tau_u^{-\frac{d\nu}{1+z\nu}}$

KZM – critical exponents of our models

- $u(t)$ is the control parameter
- Eq. relaxation time: $\tau(u) = E_{\text{gap}}(u)^{-1}$
- Eq. correlation length:

$$\mathcal{C} = \int d^d\mathbf{k}\ F(\mathbf{k},\Gamma)$$



W. Chen, J. Phys.: Condens. Matter 28, (2016)

	BHZ, $[\hat{H},\hat{s}_z]=0$	BHZ, $[\hat{H},\hat{s}_z]\neq 0$	QWZ, in-gap
ν	1	1/2	1
z	1	2	1
$n \propto \tau^{\frac{-d\nu}{1+z\nu}}$	τ^{-1}	$\tau^{-1/2}$	$\tau^{-1/2}$

- KZM holds!

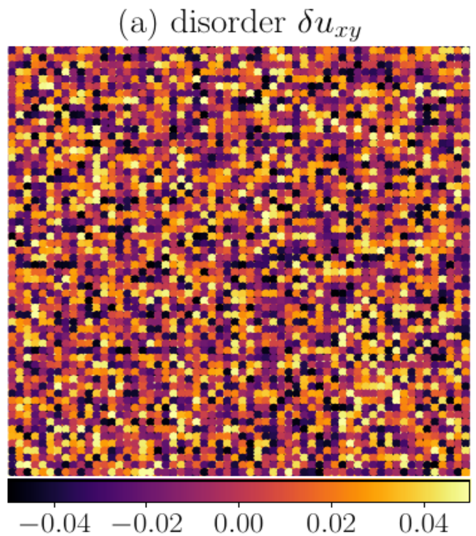
Quenches in disordered systems

- Do quenches create defect domains in real space?
- What would the defects be?

$$\hat{H} = \sum_{x,y} (|x,y\rangle\langle x,y| \otimes u_{xy}\hat{\sigma}_z + |x+1,y\rangle\langle x,y| \otimes \frac{\hat{\sigma}_z + i\hat{\sigma}_x}{2} + |x,y+1\rangle\langle x,y| \otimes \frac{\hat{\sigma}_z + i\hat{\sigma}_y}{2} + \text{h.c.})$$

- Disorder breaks translation invariance $u_{xy}(t) = \bar{u}(t) + \delta u_{xy}$
- Local Chern marker: $c(\mathbf{r}) = -2\pi i \sum_{\alpha} \langle \mathbf{r}, \alpha | \hat{P}[-i[\hat{X}, \hat{P}], -i[\hat{Y}, \hat{P}]] | \mathbf{r}, \alpha \rangle$

$$\hat{P} = \sum_{n \in \text{occup}} |\psi_n\rangle\langle\psi_n|$$



Conclusion

- Quenches produce excitations
- Power-law scaling of the number of excitations
- Transport properties approach new ground-state one values
- Kibble-Zurek mechanism connects the power law scaling of defects with the equilibrium critical exponents
- Outlook:
  - quenches in disordered systems
  - Are defects formed in real space?
  - What are the defects?

Thank you for your attention!

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