Anomaly non-renormalization in Weyl semimetals (and related problems)

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Introduction

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- I will focus mainly on the analogue of the chiral anomaly in interacting lattice Weyl semimetals ; later I will briefly mention related results on 1d interacting fermions close to the quantum critical point or with quasi-periodic disorder.

Weyl semimetals

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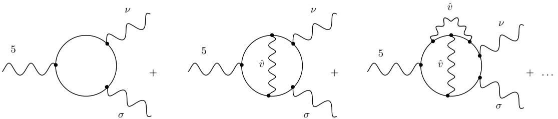
- The goal seemed attainable in Weyl semimetals (e.g. Burkov et al (2011)) and indeed observation have been reported (see e.g. Ong et al (2015))
- The anomaly is expressed by the celebrated triangle graph and in principle by a series of radiative corrections



Anomaly and Adler-Bardeen non-renormalziation

- One of the main properties of the anomaly in QFT is the non-renormalization (Adler-Bardeen 1969): all the radiative interaction corrections cancel out and the anomaly is exactly determined by its lowest order contribution in perturbation theory (triangle graph).

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- The AB result is based on a delicate cancellation between Feynman graphs, plagued by ultraviolet divergences. Cancellations order by order rely heavily on Lorentz invariance and chiral classical symmetry. Simplified derivations (Fujkawa, Atiah-Singer) works in semicalssical case.

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- Is the anomaly non.renormalization valid with a finite lattice and without Lorentz invariance, that is in Weyl semimetals? In other words, the lattice corrections do exactly cancel, or produce a small but finite contribution?

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- Also interesting from a QFT perspective; violation of Lorentz invariance have implications or not on the anomaly cancellation (Coleman Glashow 1999)
- In the case of weak short range interactions non-renormalization can be established non-perturbatively (series uniformlly convergent summing up to zero). The proof do not uses cancellations but a completely different and more robust mechanism.

Interacting lattice Weyl semimetal

- We focus on the simple situation of a minimal number of Weyl nodes, i.e. two with opposite chirality, assuming broken time reversal symmetry. It is not restrictive to consider a simple model (Delplace Carpentier EPL 2010)

$$h_0(\vec{k}) = \begin{pmatrix} t_\perp \cos k_3 - \mu + t' + \alpha(\vec{k}) & t(\sin k_+ + i \sin k_-) \\ t(\sin k_+ - i \sin k_-) & -t_\perp \cos k_3 + \mu - t' - \alpha(\vec{k}) \end{pmatrix}$$

with $\alpha(k) = -t'(\cos k_+ \cos k_- - 1)$, where t, t' are planar hoppings, $t_\perp$ is the perpendicular hopping and μ describes the difference of densities. If $|\mu - t'| \leq t_\perp$ there are 2 Fermi points $(0, 0, \pm p_F)$ with $t_\perp \cos p_F = \mu - t$.

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- We add an interaction so that the interating model is

$$H = \int d\vec{k} \psi^+_{\vec{k}} h_0(\vec{k}) \psi^-_{\vec{k}} + U \int d\vec{p} \hat{v}(\vec{p}) \rho_{\vec{p}} \rho_{-\vec{p}} + \nu N_3$$

  where $\rho_{\vec{p}} = \int \frac{d\vec{k}}{(2\pi)^3} \hat{\psi}^+_{\vec{k}+\vec{p}} \hat{\psi}^-_{\vec{k}}$ is the local density, $\hat{v}(\vec{p})$ is a short range potential, and $\psi^\pm = (a^\pm, b^\pm)$; $N_3 = N_A - N_B$ is the staggered fermion number. ν is a counterterm to fix the Weyl points.

Emerging description

- In absence of interaction the 2-point function $g(\mathbf{x})$ close to the Weyl point have the form, if $\mathbf{k} = \mathbf{k}' \pm \mathbf{p}_F$

$$\frac{\chi(\mathbf{k}')}{Z_0} \begin{pmatrix} -ik_0 \pm v_3^0 k_3' & v_+(k_+ - ik_-) \\ v_+^0(k_+ + ik_-) & -ik_0 \mp v_3^0 k_3' \end{pmatrix}^{-1} (1 + R(\mathbf{k}))$$

with $Z_0 = 1$, $v_3^0 = t_\perp \sin p_F$, $v_\pm^0 = t$.

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- The model admits an effective description in term of massless Dirac particles with a local interaction. This QFT model is invariant under a global $\psi^\pm \to e^{\pm i\alpha}\psi^\pm$ and chiral $\psi^\pm \to e^{\pm i\gamma_5 \alpha}\psi^\pm$; the second symmetry is not true in the lattice model.

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- In contrast to graphene, the velocity can be small and the Weyl points arbitrarily close. They are in general renormalized (their location is obtaining from ν by inversion).

Background e.m. field

- Let us now couple the system to an external e.m. field A_μ, $\mu = 0, 1, 2, 3$. We denote by $\langle\cdot\rangle_{\mathbf{A}}$ the interacting Gibbs state of the system in the presence of the external field and $\langle\cdot\rangle = \langle\cdot\rangle_{\mathbf{0}}$. The coupling is defined via the Peierls substitution.
- The axial density is chosen, following NN, as the flow between Weyl points.

$$\hat{\rho}^5_{\vec{p}} = \int \frac{d\vec{k}}{(2\pi)^3} \frac{\sin k_3}{\bar{Z}} \hat{\psi}^+_{\vec{k}+\vec{p}} \hat{\psi}^-_{\vec{k}}$$

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- $\tilde{Z}$ has to be chosen so that $\hat{\rho}^5_{\vec{p}}$ is proportional to $\pm$(total density)

$$\langle \hat{\rho}^5_{\mathbf{p}} ; \hat{\psi}^-_{\mathbf{k}+\mathbf{p}^\pm_F} \hat{\psi}^+_{\mathbf{k}+\mathbf{p}^\pm_F} \rangle = \pm \langle \hat{\rho}_{\mathbf{p}} ; \hat{\psi}^-_{\mathbf{k}+\mathbf{p}^\pm_F} \hat{\psi}^+_{\mathbf{k}+\mathbf{p}^\pm_F} \rangle (1 + O(\mathbf{k}, \mathbf{p}))$$

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- The lattice chiral density is not exactly local: therefore, one has to couple it to the A field via the Peierls substitution, in order to ensure full lattice gauge invariance, and denote by $\rho^5_x(A)$ the gauge-invariant chiral density.

Background e.m. field

- The generating function of correlations can be written as a Grassmann integral as

$$e^{W(\mathbf{A},\mathbf{A}^5,\phi)} = \int P(d\psi)\, e^{V(\psi)+B(\mathbf{A},\psi)+(A^5_\mu, j^5_\mu(A))+(\psi,\phi)},$$

V contains the interaction and ν term. We call $\Gamma^5_{\mu,\mu_1,..\mu_n}$ and $\Gamma_{\mu,\mu_1,..\mu_n}$ the derivatives with respect to $A^5_\mu, A_{\mu_1}, ..$ and $A_\mu, A_{\mu_1}, ..$ at $A = 0$.

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- By Gauge invariance

$$\partial_\alpha W(A_\mu, +\partial_\mu \alpha, A^5_0, \phi e^{i\alpha}) = 0$$

  so that $\mathbf{p}_\mu \Gamma_{\mu,\mu_1,..\mu_n} = 0$ implying the conservation of the current $< \mathbf{p}_\mu j_\mu >_A = 0$ in presence of an e.m. field. Of course no conservation holds for the "chiral current"; there is no associated symmetry.

Background e.m. field

- Response in A

$$\langle \hat{\rho}^5_{\mathbf{p}} \rangle_{\mathbf{A}} = i\Gamma^5_{0,\nu}(\mathbf{p})\hat{A}_{\nu,\mathbf{p}} + \frac{i}{2}\int d\mathbf{p}_1 d\mathbf{p}_2\, \Gamma^5_{0,\nu,\sigma}(\mathbf{p}_1,\mathbf{p}_2)\delta\hat{A}_{\nu,\mathbf{p}_1}\hat{A}_{\sigma,\mathbf{p}_2} + ...$$

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- $\Gamma^5_{\mu,\mu_1,\ldots,\mu_n}$ are derivatives of W, that is $\langle \hat{\jmath}^5_{\mu,\mathbf{p}}; \hat{\jmath}_{\nu,+\mathbf{p}_1}; \hat{\jmath}_{\sigma,\mathbf{p}_2}\rangle +$

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- In the non interacting case $\mathbf{p}_1+\mathbf{p}_2=(p_0,0)$, one has (Nielsen and Ninomiya (1983):

$$p_0\Gamma^5_{0,\nu,\sigma}(\mathbf{p}_1,\mathbf{p}_2) = \frac{e^2}{\hbar^2}\frac{1}{2\pi^2}p_{1,\alpha}p_{2,\beta}\varepsilon_{\alpha\beta\nu\sigma}$$

  This is same expression of the anomaly for $\bar\psi\gamma_\mu\gamma_5\psi$ for massless QED (but for Weyl semimetal strictly speaking is not an anomaly but a simulation).

Main Result

- What happens in presence of interaction $U \neq 0$? The interaction produce non-universal modiifcation in the physical quantities; the Fermi points are shifted and are given by $\pm p_F + b_\pm U + \ldots$, with $b_\pm \neq 0$, and the interacting Fermi velocities are given by $v_i = v_i^0 + a_i U$... Such quantities are expressed by series in U with non trivial coefficients.

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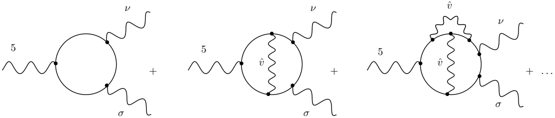

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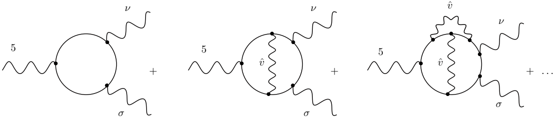

- The contributions of such terms is $\frac{e^2}{\hbar^2}\frac{1}{2\pi^2}p_{1,\alpha}p_{2,\beta} + AU + CU^2 + ...$ Do such corrections cancel or not?

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- **Theorem**(Giuliani Mastropietro Porta arXiv:1907.00682) *There exists $U_0 > 0$, independent of the distance between the Fermi points, such that, if $|U| < U_0$, fixing $\nu = \nu(\lambda)$ and $Z^5 = Z^5(\lambda)$ we have $p_0 \Gamma^5_{0,\nu}((p_0, 0)) = O(p_0^3 \log |p_0|)$ and, if $\mathbf{p}_1 + \mathbf{p}_2 = (p_0, 0)$,*

$$p_0 \Gamma^5_{0,\nu,\sigma}(\mathbf{p}_1, \mathbf{p}_2) = \frac{e^2}{\hbar^2} \frac{1}{2\pi^2} p_{1,\alpha} p_{2,\beta} \varepsilon_{\alpha\beta\nu\sigma},$$

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  up to an error $O(P^3 \log P)$, *with* $P = \max\{|\mathbf{p}_1|, |\mathbf{p}_2|\}$.

- $A_0 = A_1 \equiv 0$, $A_2(t,x) = Bx_1$, $A_3(t,x) = -Et$ we get, at quadratic order, $\partial_t \langle N^5_t(A) \rangle_{\mathbf{A}} = \frac{e^2}{\hbar^2 c} \frac{1}{2\pi^2} EB$ where $N^5 = \sum_x \rho^5_x$

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- Important to understand if is true with Coulomb interactions or strong coupling.
- One can see QFT as an effective theory emerging from some deeper unknown description free of uv divergences and with less symmetries. Weyl semimetals provide an example of emerging QED from a non Lorentz invariant background. The correlations have small corrections (proportional to the ratio between the momentum scale and the lattice) but the anomaly non-renormalization is a robust property: no corrections are present.

RG an effective potential

- The proof of universality combines two main ingredients: (a) invariance under local gauge transformation and Ward Identities; (b) regularity properties of the correlations $\Gamma^5_{\mu,\nu,\sigma}$.
- We perform an exact RG analysis integrating momentum scales of decreasing size $\sim \gamma^h$, $\gamma > 1$. We get a sequence of effective potentials V^h at scale $h = 0, -1, -2, \ldots$ V^h is given by a local part and an irrelevant part, expressed by by sum of *non-local monomials* $\int d\underline{\mathbf{x}} d\underline{\mathbf{y}} W^h_{n,m} \psi^n A^m$

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- Tke kernels $W^h_{n,m}$ are expressed in terms of a convergent power series in U and rcc $\nu_h, Z^h_\mu, Z^{5,h}_\mu$. Technical part; cluster expansion and Gram bound for fermionic determinants ($M_{a,\beta} = (f_\alpha, g_\beta)$, $|\det M| \leq \prod_\alpha ||f_\alpha|| ||g_\alpha||$). In order to achieve convergence Feynman graphs cannot be used; one needs cancellations by anticommutativity (classical trick in constructive QFT: Caianello 1956, Gawedski-Kupianenen 1985)

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- Finiteness of radius of convergence (uniformly in $\sin p_F$) is in (Mastropietro JSP 2015; JPA 2015). The RG has two regimes, the first with dimension $7/2 - 5/4n$ the second with dimension $4 - \frac{3}{2}n$

Boiunds

- It is convenient to separate

$$W_{n,m}^{(h)} = W_{n,m;0}^{(h)} + W_{n,m;1}^{(h)}$$

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- The above bound has immediate implications on the regularity propertiies of the $\hat{\Gamma}_{n,m}$.

Regularity properties and RG

- The anomaly is expressed by the derivative of $\hat{\Gamma}^5_{\mu,\nu,\sigma}$ which is bounded by $\sum_{h\le 0} 2^{-h} 2^h$, which is infinite; $\hat{\Gamma}^5_{\mu,\nu,\sigma}$ is continuous but not differentiable.

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- The only non differentiable term is a renormalized triangle graph with vertices and velocities at scale h; again we can write the propagators as the relativistic part and a rest, and the vertices and velocities as their value at $h = -\infty$ plus a rest. In conclusion the renormalized triangle is a relativistc triangle (non differentiable) and a rest which is differentiable

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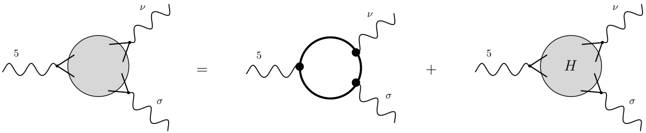

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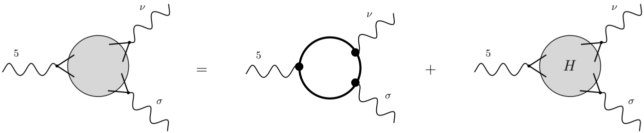

The fact that $H^5_{\mu,\nu,\sigma}$ depend on the irrelevant term implies that is more regular, that is differentiable.

Renormalization Group analysis

- We now combine the use of Ward Identities (WI) with the regularity properties of $\Gamma^{5,rel}_{\mu,\nu,\sigma}$ and $H^5_{\mu,\nu,\sigma}$.
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- $\bar{p}_\alpha = v_\alpha p_\alpha$ and note that $v_\alpha v_\beta v_\nu v_\sigma \varepsilon_{\alpha\beta\nu\sigma} = v_1 v_2 v_3 \varepsilon_{\alpha\beta\nu\sigma}$.

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- Open problem (at least on a rigorous side); long range interaction or disorder. The decomposition used above cannot be used.

1d non integrable chains

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- We perform a two-regime RG analysis (Bonetto Mastropietro AHP 2018, EPL 2018); in the infrared linear regime the scaling dimension is $2 - n/2$ (quartic terms marginal), in the ultraviolet quadratic one is $3/2 - n/4$ (quartic terms relevant)

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- However the cut-off breaks the local phase symmetry and produces an extra term in the WI for this interacting QFT theory (the relativistic anomaly).

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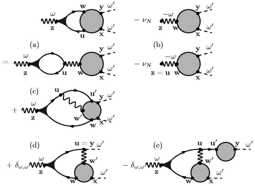

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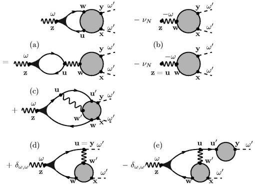

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- One get a closed expression for the relativistic dominant part correlations (non differentiable).

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where (r is the distance from criticality)

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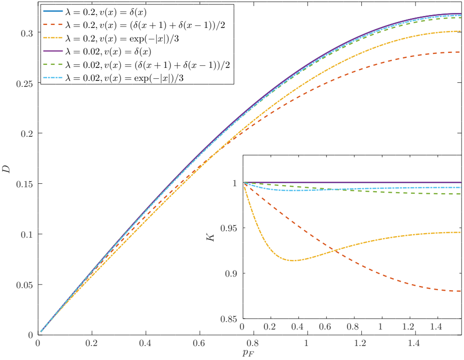
- Moreover at criticality one gets the non interacting values

$$K \to 1 \quad D/D_0 \to 1$$

as $r \to 0$. μ_c is shiifted by the interaction (see e.g. Zotos et al (2016)). Main point: the $O(\lambda r)$ are bounds on convergent series.

Spinless case



D and K as function of density (or magnetic field), both in Heisenberg or non solvable cases. D/D_0 and K tend to 1: Features found in the solvable case persists up to the criticapl point.

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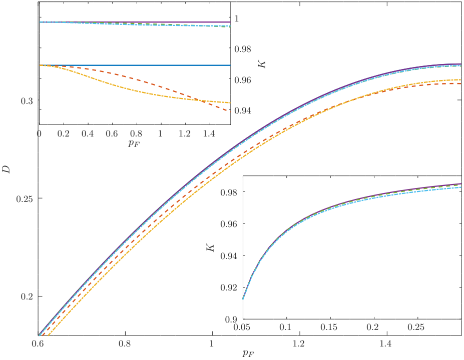
  ν_ρ, ν_4 are the anomalies of the emerging theory.
- One cannot take the $r \to 0$ limit; however for λ small one can see that K does not tend to the non interacting value 1 but D becomes close to D_0.

Spinful case



Contrary to the spinless case, we cannot get $p_F = 0$. K show the tendency to a strongly interacting fixed point while D is close to the non interacting value. Cfr the behavior of the Hubbard model by Bethe ansatz (e.g. Schultz 1993)

Small divisors and Quantum Many body Physics

- Another case when irrelevant terms are crucial; interacting Aubry-Andre' model (XXZ chain with quasi random disorder)

$$H = -\varepsilon\Big(\sum_{x\in\Lambda}(a^+_{x+1}a_x + a^+_{x-1}a^-_x) +$$
$$\sum_{x\in\Lambda} u\cos(2\pi(\omega x+\theta))a^+_x a^-_x + U\sum_{x,y} v(x-y)a^+_x a^-_x a^+_y a^-_y$$

with $v(x-y) = \delta_{y-x,1} + \delta_{x-y,1}$.

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- ω irrational.
- Umklapp terms (marginal or relevant) are effectly irrelevant except at half filling, as they do not fill momentum conservation from the Fermi points; Due to incommensurability however momenta are almost conserved, hence it is not clear if this improvement with respect to scaling dimension holds.

The Aubry-Andre' model

- In the non interacting case $U = 0$ the states are obtained by the antisymmetrization (fermions) of the eigenfunctions of almost Mathieu equation

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- For almost every ω, θ the almost Mathieu operator has a)for $\varepsilon/u < \frac{1}{2}$ only pps with exponentially decaying eigenfunctions (Anderson localization);
  b)for $\varepsilon/u > \frac{1}{2}$ purely absolutely continuous spectrum (extended quasi-Bloch waves)

Molecular limit

- $\varepsilon = U = 0$ molecular limit $H = \sum_x (\cos 2\pi(\omega x) - \mu) a^+_x a^-_x$

$$< \mathbf{T} a^-_{\mathbf{x}} a^+_{\mathbf{y}} > |_0 = \delta_{x,y} \bar{g}(x, x_0 - y_0)$$

$$\bar{g}(x, x_0 - y_0) = \frac{1}{\beta} \sum_{k_0} \frac{e^{-ik_0(x_0 - y_0)}}{-ik_0 + \cos 2\pi(\omega x) - \cos 2\pi(\omega \bar{x})}$$

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- We assume Diophantine conditions; KAM (Kolmogorv Arnol Moser) methods.

$$||\omega x|| \geq C_0 |x|^{-\tau} \quad (*)$$

$$||\omega x \pm 2\omega \bar{x}|| \geq C_0 |x|^{-\tau} \quad \forall x \in \mathbb{Z}/\{0\} \quad (**)$$

Localized regime

Theorem 1.

In the spinless interacting Aubry-Andre' model, assuming (*) and $\bar{x}$ *verifying (**) if* $u = 1$, $\mu = \cos 2\pi(\omega\bar{x}) + \nu$ *for small* ε, U *and suitable* ν, *for any* N, $L = 1/T = \infty$

$$|<\mathbf{T} a^-_{\mathbf{x}} a^+_{\mathbf{y}}>| \leq C e^{-\xi|x-y|} \log(1+\min(|x||y|))^\tau \frac{1}{1 + (\Delta|x_0 - y_0)|)^N} (***)$$

with $\Delta = (1 + \min(|x|, |y|))^{-\tau}$, $\xi = |\log(\max(|\varepsilon|, |U|))|$.
Assuming (*) and $\bar{x}$ *half integer the same holds with* Δ *replaced by* $\sigma = O(\varepsilon^{2\bar{x}})$

Exponential decay in coordinates signala persistence of localization in presence of interactions.
Mastropietro Phys. Rev. Lett. 115, 180401 (2015); Comm. Math. Phys. 342, 1, 217-250 (2016); Comm. Math. Phys. (2017)

Some idea of the proof

- We integrate higher scales getting a sequence of effective potentials. According to power counting, the theory is non renormalizable ; all effective interactions have positive dimension, $D = 1$.

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- Methods coming from direct proof of convergence of Lindstedt series for tori of quasi integrale systems
- The relevance of all terms suggest that localization (the umpertured case) is broken.

Some idea of the proof

- The non resonant terms are irrelevant. The idea is that if two propagators have similar (not equal) small size (non resonant subgraphs) , then the difference of their coordinates is large and this produces a "gain" as passing from x to $x+n$ one needs n vertices. That is if $(\omega x_1')_{\rm mod 1}\sim (\omega x_2')_{\rm mod 1}\sim \Lambda^{-1}$ then by the Diophantine condition
$$2\Lambda^{-1}\ge ||\omega(x_1'-x_2')||\ge C_0|x_1'-x_2'|^{-\tau}$$
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- The resonant terms are vanishing by Pauli principle
- Therefore perturbation theory is convergent for small t. In contarst delocalized behavior is found for large ε.

Conclusions

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- No difference between integrable and non integrable chains even in regimes where the irrelevant terms dominate (close to criticality) for $T = 0$. Of course $T \neq 0$ is a major problem.
- In presence of quasi-random disorder, again the localizationn at $T = 0$ persists due to irrelevance of terms almost connecting Fermi points; number theoretical (Diophantine) properties ensure such irrelevance. Again major problem understand the role of such irrelevant terms at $T \neq 0$.