Accademia dei Lincei

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Slow transport in disordered quantum systems

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Based on work with/by

B.Altshuler, V.Oganesyan, D.Abanin, D.Huse, M.Mueller, C.Laumann, A.Chandran, R.Nandkishore, D.Abanin, V.Ros, J.Imbrie, F. Pollmann, M.Znidaric and many others

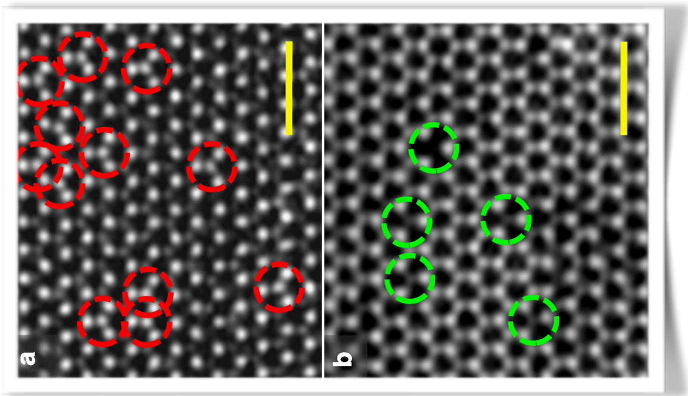
Summary

- Effects on disorder on transport in quantum systems
- Localization: suppression of transport
- Nature of the delocalised phase: modes of slow transport
- Implications for the MBL-ETH phase transition?

Effect of disorder

In a perfect crystal transport is ballistic: momentum is a good quantum number

A perfect crystal is an abstraction: disorder is everywhere



ballistic ⟶ diffusive

Effect of disorder

For conduction electrons disorder can be included (in a semiclassical approximation) in the Boltzmann equation for $f(x,p)$

$$\frac{\partial f}{\partial t} + v\nabla_x f + F\nabla_p f = \int dp' w(p,p')\left(f(x,p') - f(x,p)\right)$$

where a Born approximation gives

$$w(p,p') = 2\pi |V_{p',p}|^2$$

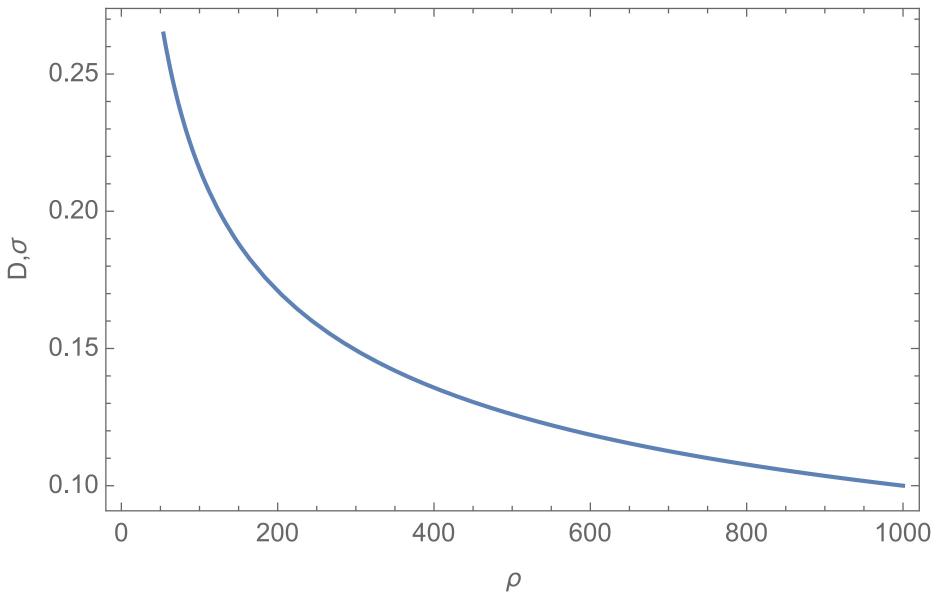
This eventually gives diffusive transport

Effect of disorder

conductivity $\sigma = J/E$

Einstein's relation $\sigma \propto D$

Semiclassical result: always diffusive, slower as disorder increases

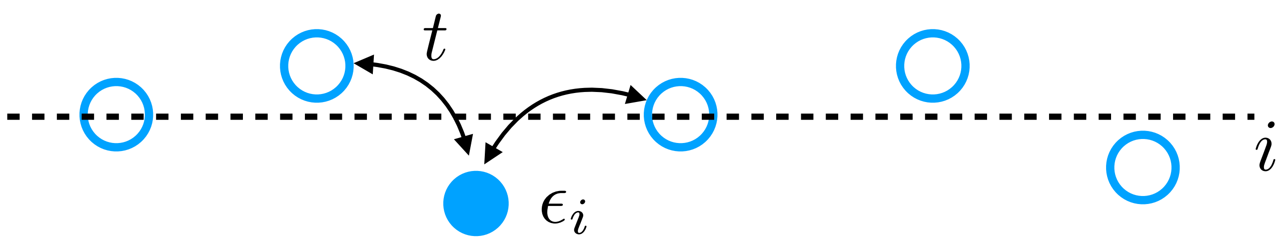


density of scatterers, disorder

Effect of disorder

Anderson argued that for sufficiently strong disorder, diffusive transport must be completely suppressed

From experiments on electrons in the impurity band in a semiconductor



$$H = -t\sum_i c_i^\dagger c_i + \text{h.c.} + \sum_i \epsilon_i c_i^\dagger c_i$$

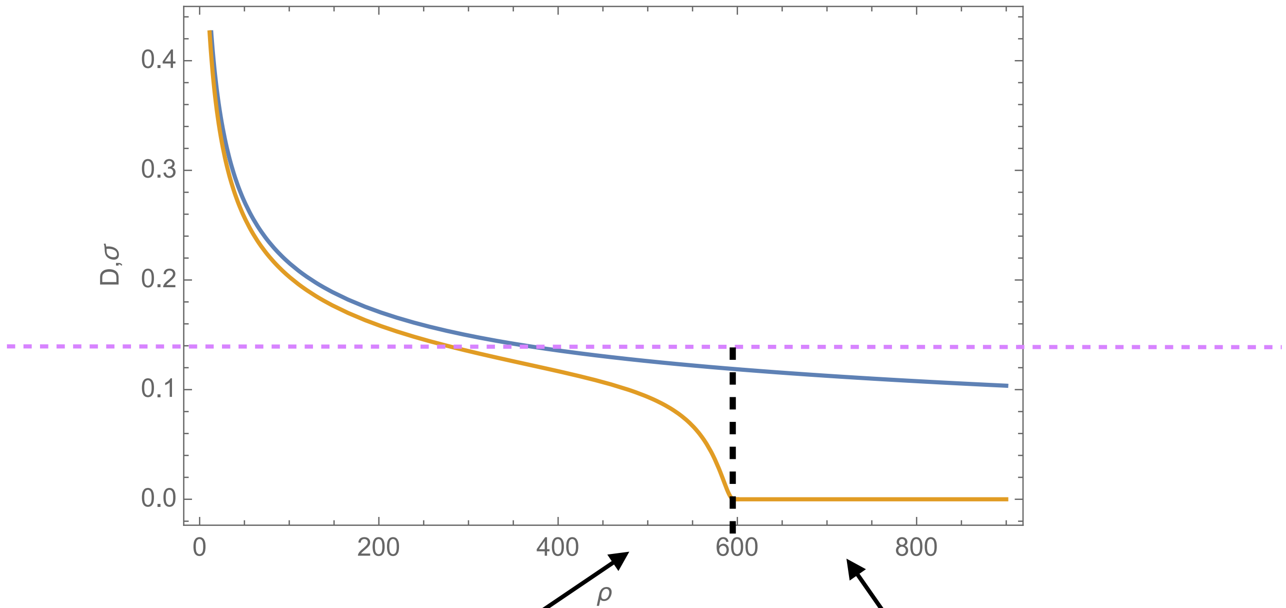
$\epsilon_i \in (-W, W)$

P.W. Anderson, 1958

Changing the ratio *t/W* we get to a phase transition

Effect of disorder

Due to quantum effects the diffusion constant must disappear altogether



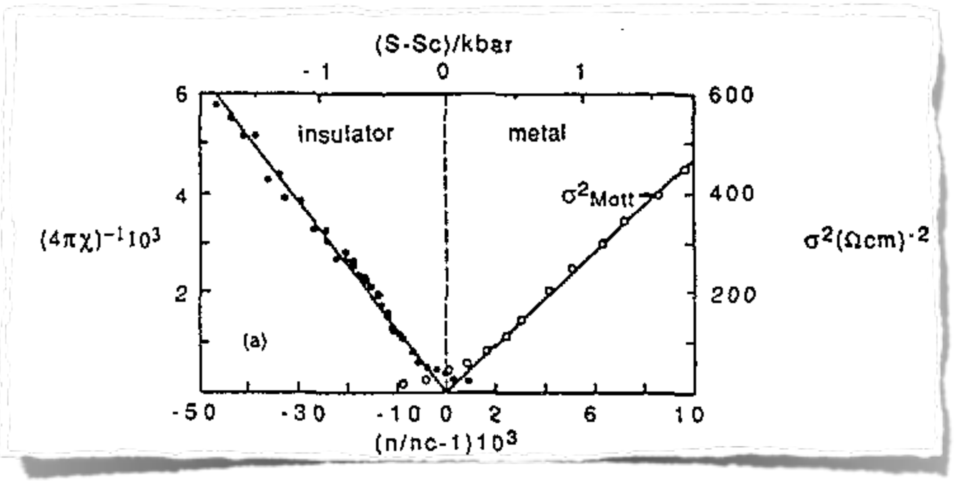
$$\sigma \sim \frac{e^2}{\hbar}$$

Conductive

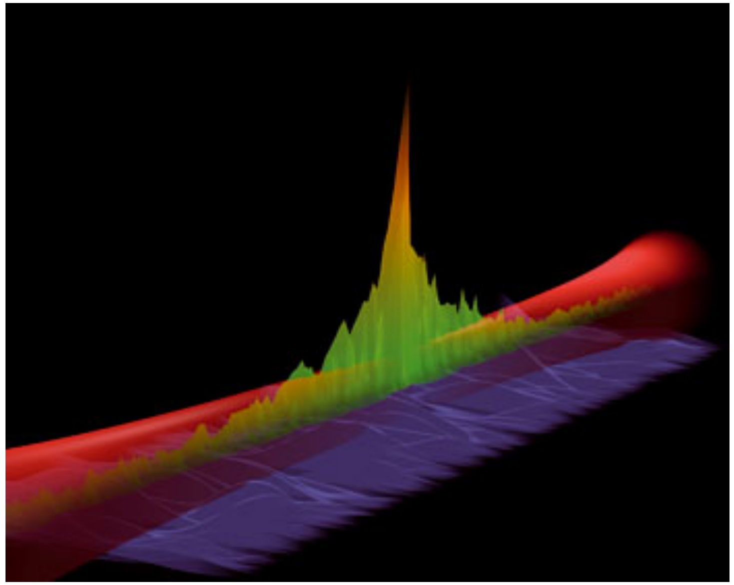
Localized

Localization

Observation in doped semiconductors



Direct observation in optical lattices



Huge body of theoretical/mathematical work

Localization

What happens when we introduce interactions?

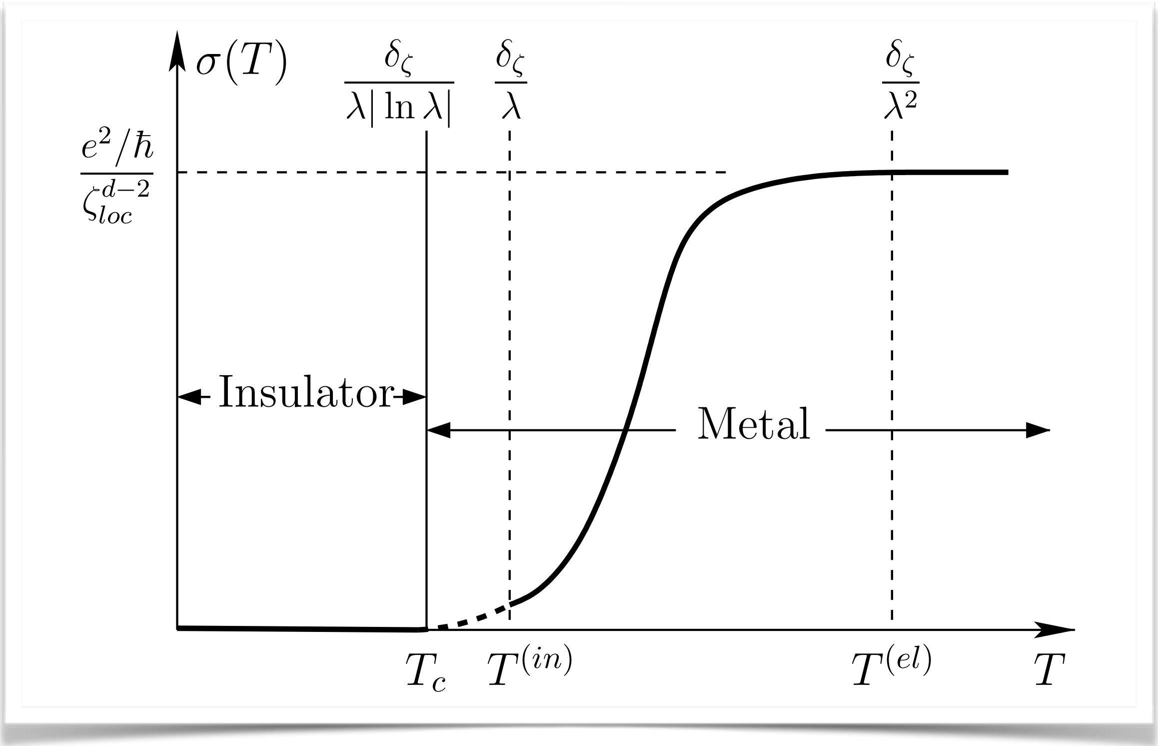
$$H = -t \sum_i c_i^\dagger c_i + \text{h.c.} + \sum_i \epsilon_i n_i + \sum_{i,j} v(|i-j|) n_i n_j$$

Question asked already in Anderson 1958 but recently finally we think we have the answer:

In 2006 Basko, Aleiner and Altshuler presented a perturbation theory calculation supporting the idea that, for sufficiently small (and short-range interactions), localization survives

Many-body Localization

For sufficiently small interactions



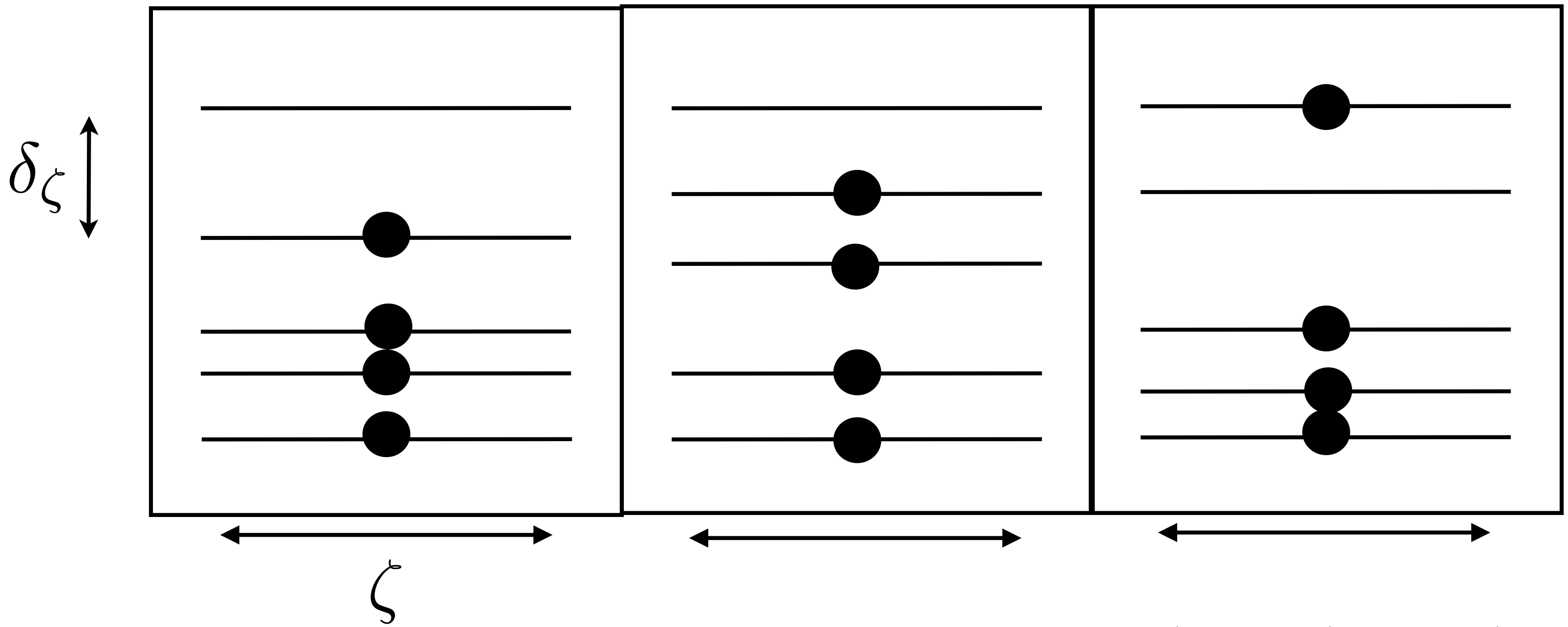
$$\delta_\zeta \sim (\hbar/\zeta)^2/m$$

$$\zeta \sim 1/W^2$$

$$\lambda \lesssim \frac{\delta_\zeta}{T\ln(T/\delta_\zeta)}$$

Many-body Localization

Perturbation theory



$$\Delta E = \Delta E_1 + \Delta E_2 + \Delta E_3 \lesssim \delta_\zeta$$

$$A \sim \frac{\lambda}{\Delta E_1} \frac{\lambda}{\Delta E_1 + \Delta E_2} \frac{\lambda}{\Delta E_1 + \Delta E_2 + \Delta E_3}$$

Many-body Localization

Look at the absence of resonant processes

$\forall c > 0$ check $P(|A_L| > c) \to 0$ as $L \to \infty$

more precisely one can show (in perturbation theory):

$$\exists \xi > 0, x_0 : \quad \forall x > x_0 : \; P(|A_x| < e^{-x/\xi}) \to 1 \quad \text{as} \quad L \to \infty$$

In this way transport is inhibited

Basko, Aleiner, Altshuler (2006), Ros, Mueller, AS (2015), J.Imbrie proved this for a different model (2015)

Many-body Localization

- Absence of transport
- Emergence of local integrals of motion
- Violation of ETH
  - Area law entanglement at high T
  - Memory of initial state in local observables
  - Poisson distribution of gaps in the spectrum
- Slow growth of entanglement

Many-body Localization

Interacting spinless fermions

$$H = -\sum_i c_i^\dagger c_{i+1} + \text{h.c.} + \sum_i \epsilon_i n_i + \sum_i n_i n_{i+1}$$

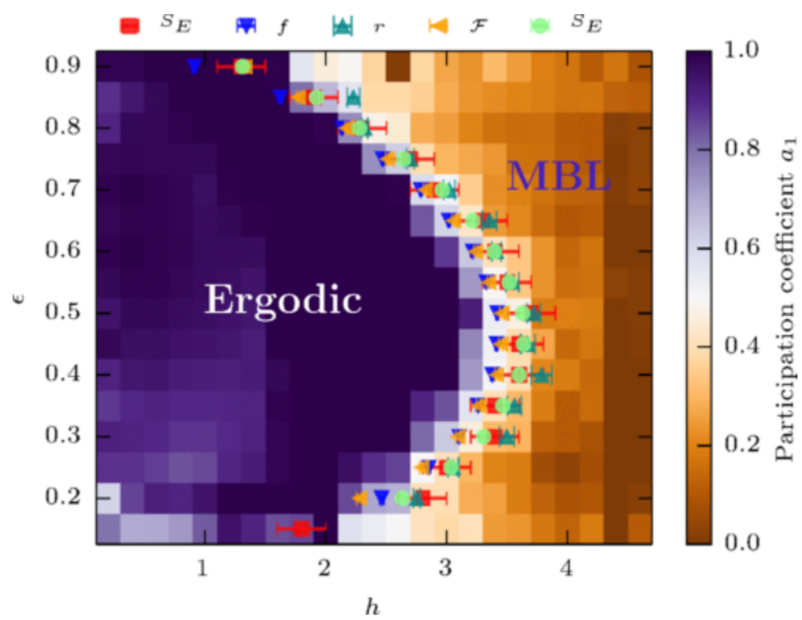
Jordan-Wigner

spin representation

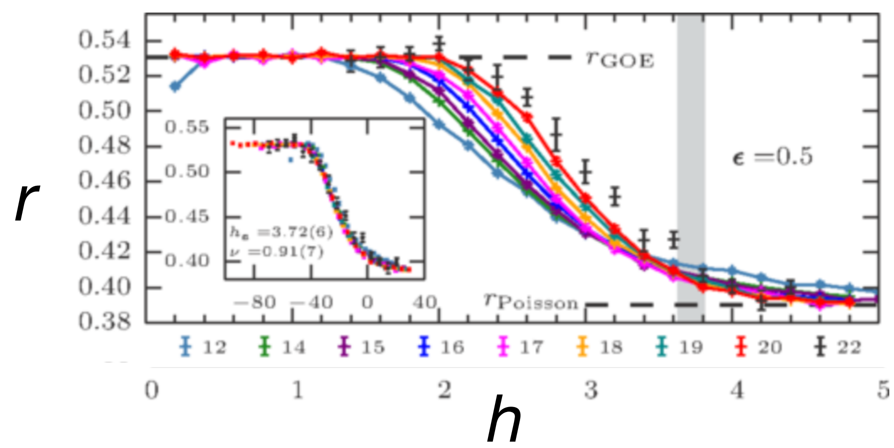
$$H = -\sum_i \vec{s}_i \cdot \vec{s}_{i+1} + \sum_i h_i s_i^z$$

$h_i \in (-h, h)$

F.Alet et al. (2015)



$$r = \frac{\min(\Delta E_a, \Delta E_{a+1})}{\max(\Delta E_a, \Delta E_{a+1})}$$

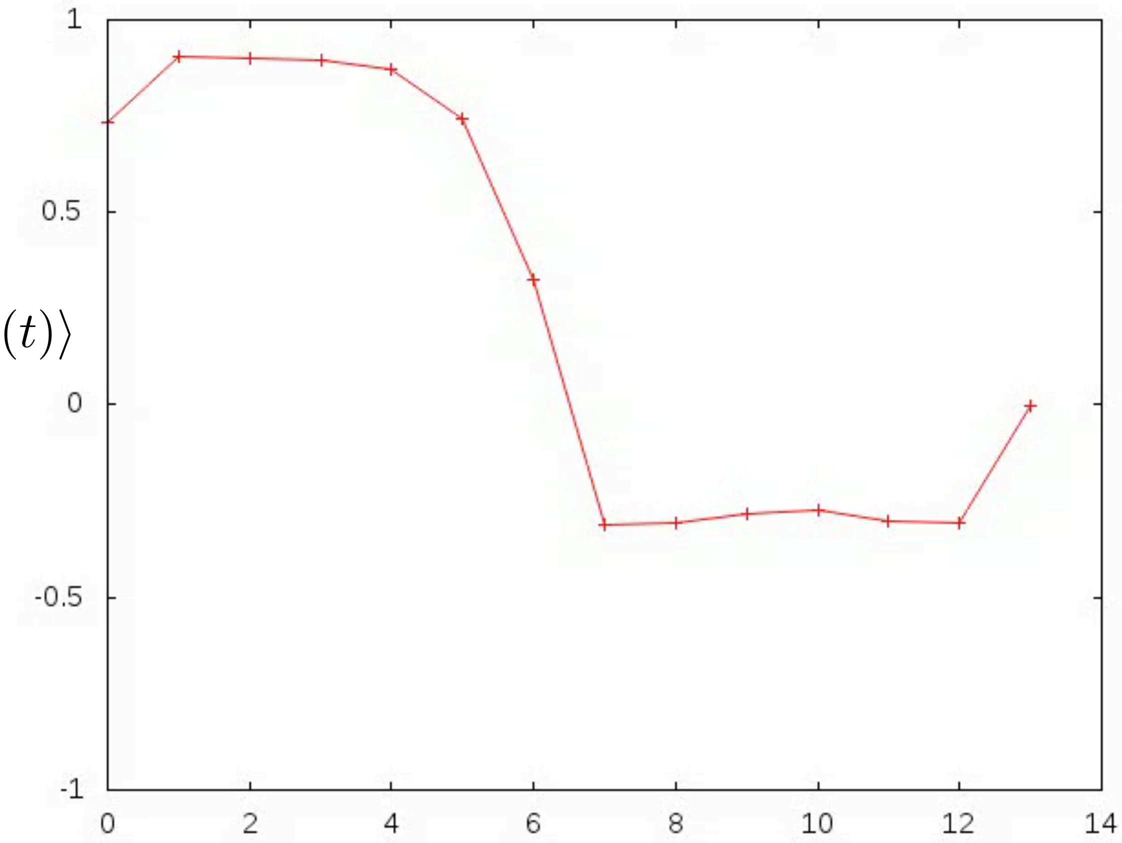


Many-body Localization

ETH

$$\frac{\partial \epsilon}{\partial t} = D\frac{\partial^2 \epsilon}{\partial x^2}$$

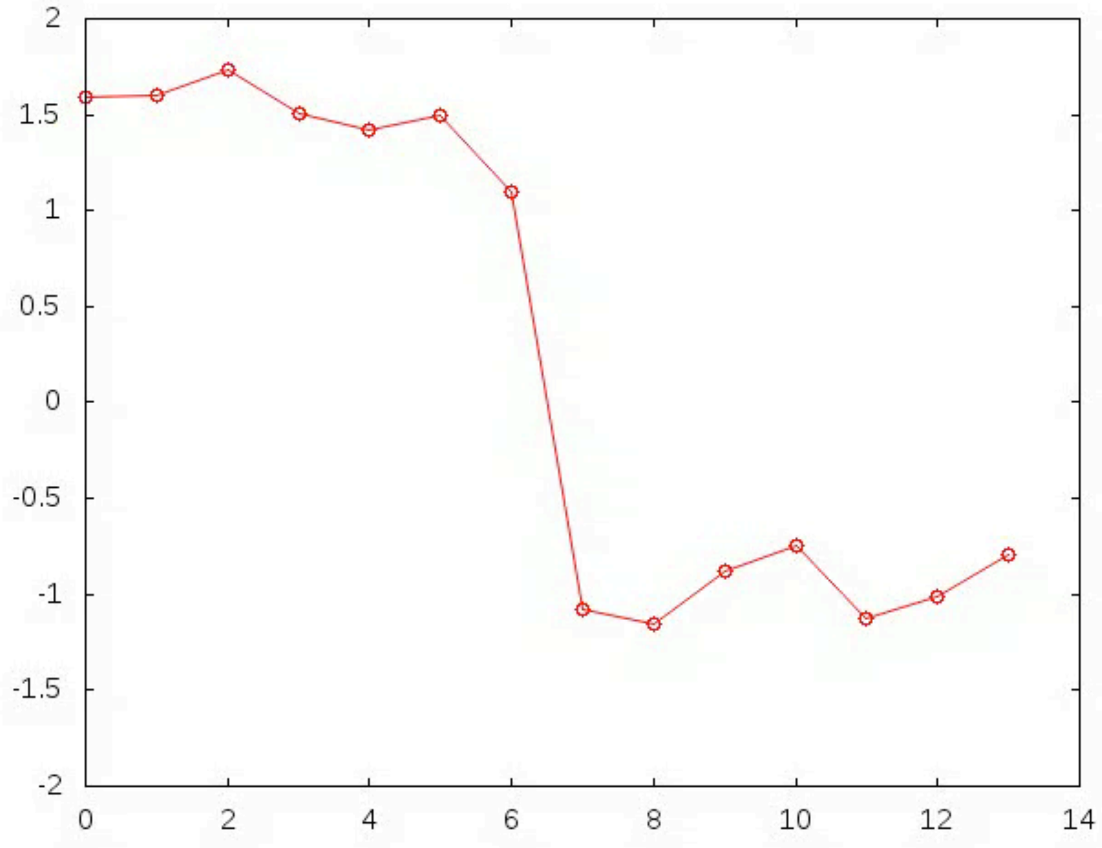
$$\epsilon(x,t) = \langle \psi(t)|H_x|\psi(t)\rangle$$



Many-body Localization

Localized

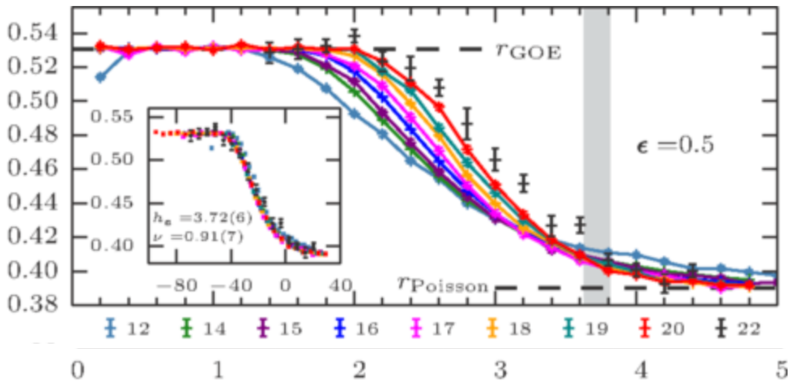
$$\epsilon(x,t) = \langle \psi(t) | H_x | \psi(t) \rangle$$



Many-body Localization

Problems with the phase transition:

Critical exponents are too small



$$r(W, L) = f(L/L_c(W))$$

$$L_c(W) \sim |W_c - W|^{-\nu}$$

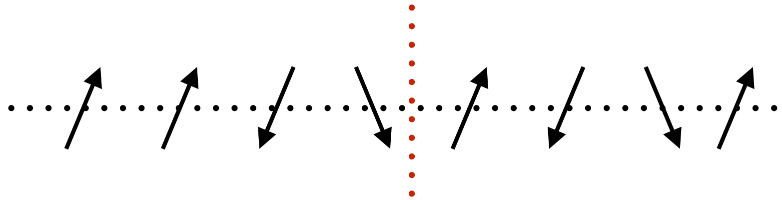
$$\nu \lesssim 1$$

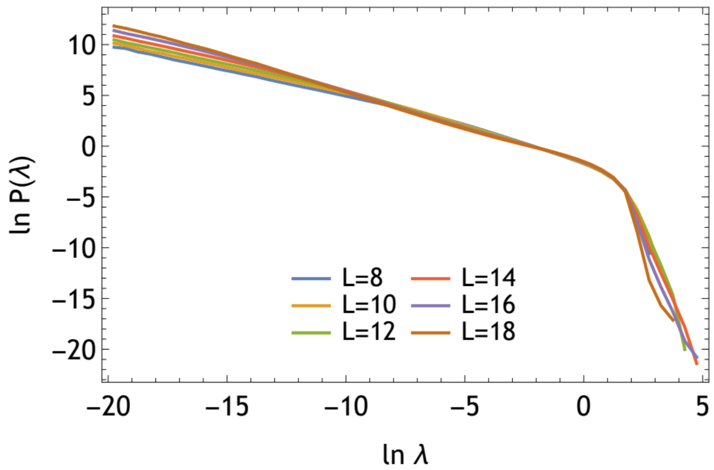
$$\nu \geq 2/d = 2$$

Chandran, Laumann, Oganesyan (2016) based on Chayes, Chayes, Fisher, Spencer (1989)

Many-body Localization

$$H = -\sum_i \vec{s}_i \cdot \vec{s}_{i+1} + \sum_i h_i s_i^z$$



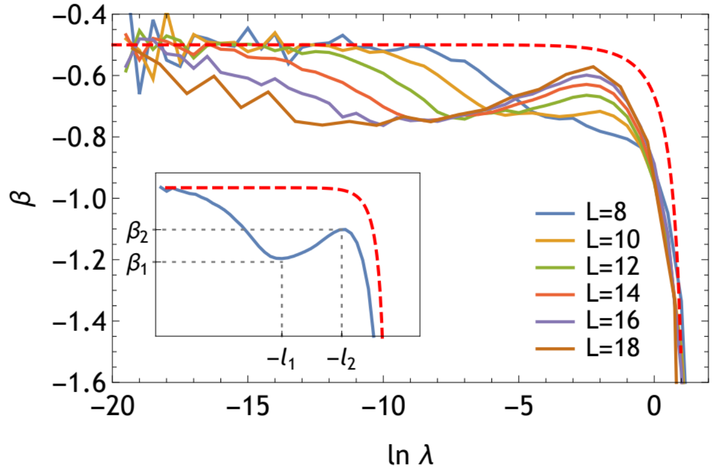


$$|E\rangle = \sum_a \sqrt{\lambda_a} |\psi_a^1\rangle |\psi_a^2\rangle$$

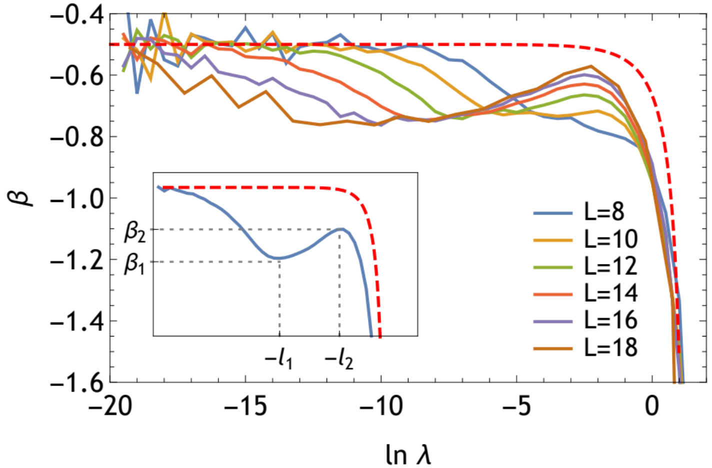
Extract a length scale

F.Pietracaprina, et al. 2017

$$\beta = \frac{\partial \ln P}{\partial \ln \lambda}$$



Many-body Localization

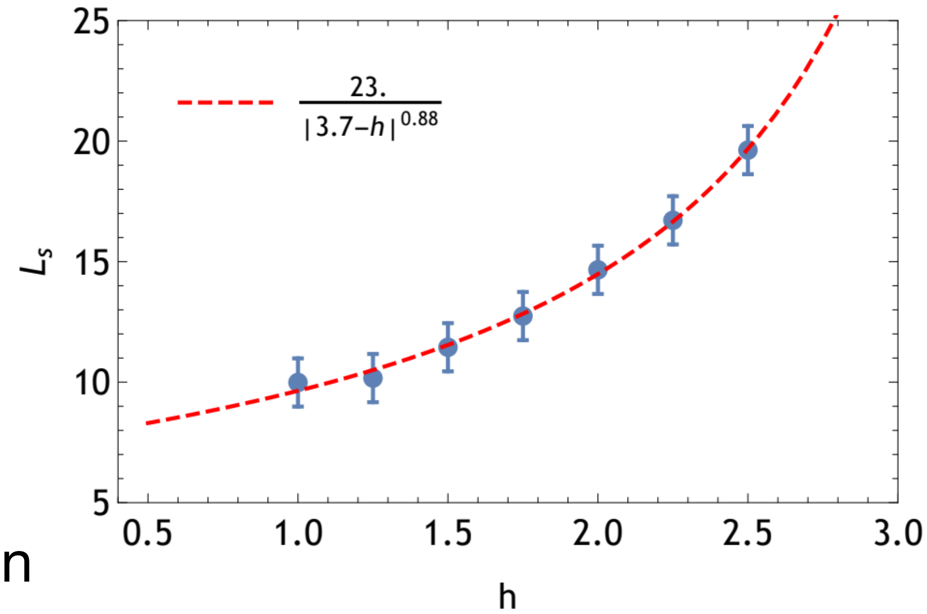


For any disorder saddle point appears at some *L*

$$\nu = 0.9 \pm 0.1$$

$$L_s \to \infty, \quad h \to h_{MBL}$$

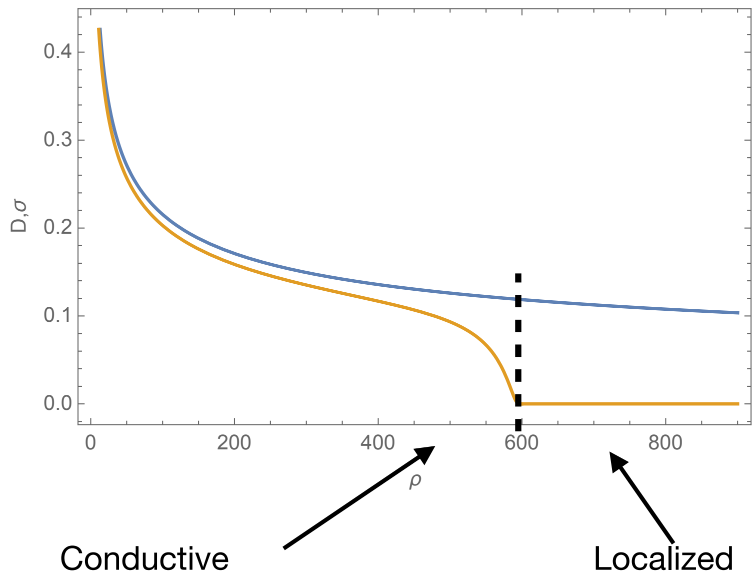
Large finite size effects?



Analogy with Anderson model on RRG/Bethe lattice

Delocalized region

$$\sigma = J/E \qquad J = \sigma E = \sigma\ \Delta V/L$$



When conductivity is zero, is it localized?

Not necessarily!

$$J \sim L^{-\gamma} \qquad \gamma > 1$$

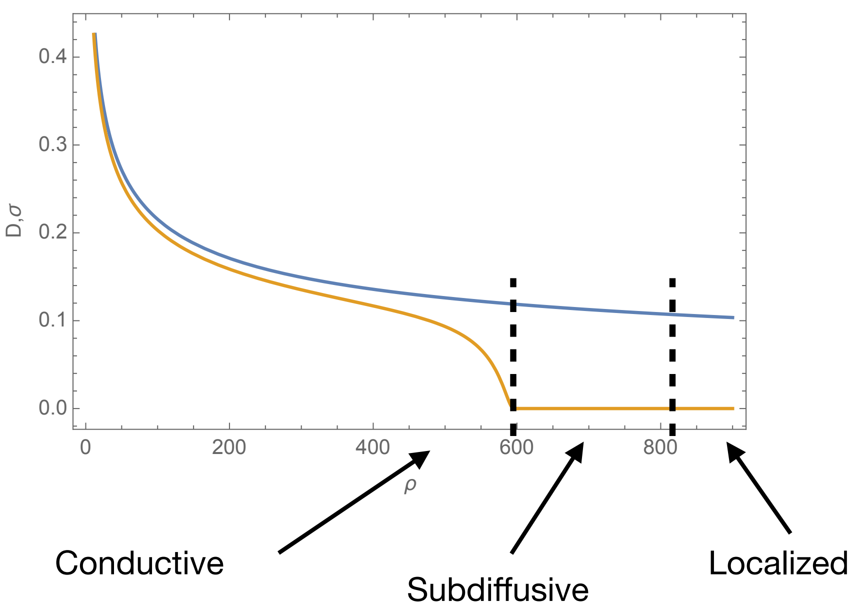
non-ohmic or subdiffusive dynamics

$$R \sim L^{\gamma}$$

This is not observed in Anderson model

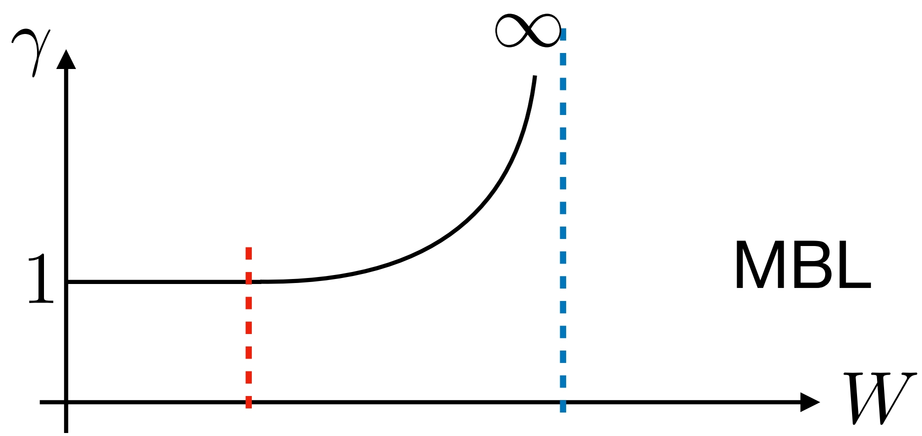
Delocalized region

The delocalized region of the interacting model



$$H = -\sum_i c_i^\dagger c_{i+1} + \mathrm{h.c.} + \sum_i \epsilon_i n_i + \sum_i n_i n_{i+1}$$

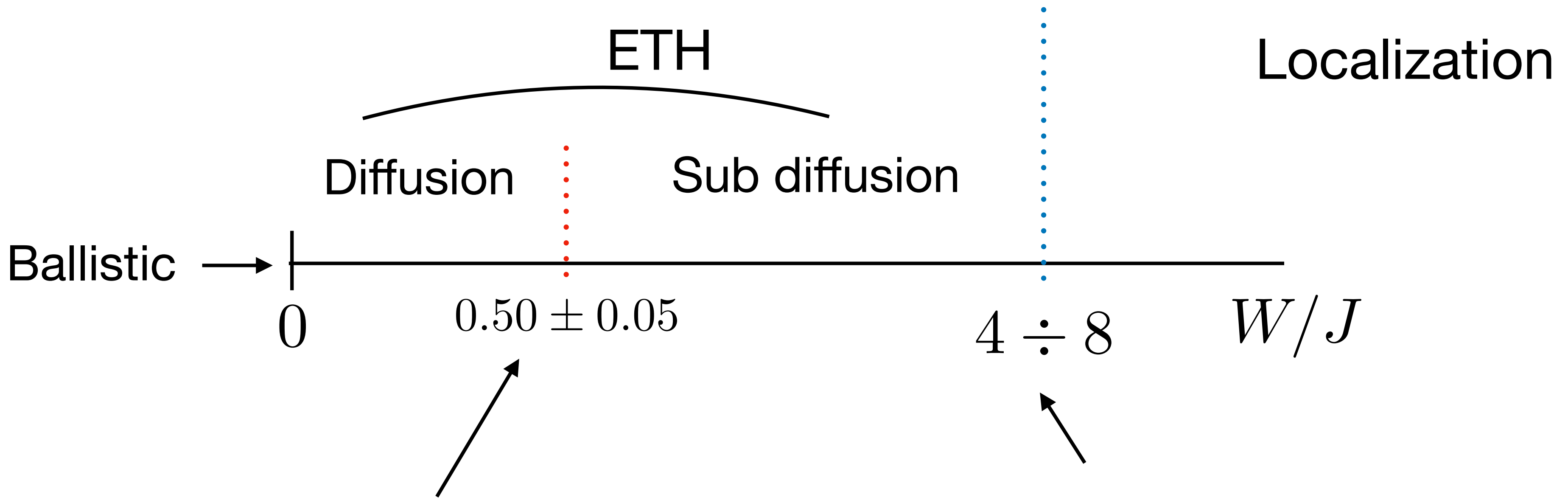
$$R \sim L^\gamma$$



Delocalized region

$H = -\sum_i \vec{s}_i \cdot \vec{s}_{i+1} + \sum_i h_i s_i^z$ the dynamical phase diagram is

$W/J \gtrsim 4$ localized at infinite temperature



open system transport: Znidaric, Varma, AS (2016),

eigenstates: Luitz, Bar Lev (2015)

eigenstates: Alet, Luitz, and others (2014)

Delocalized region

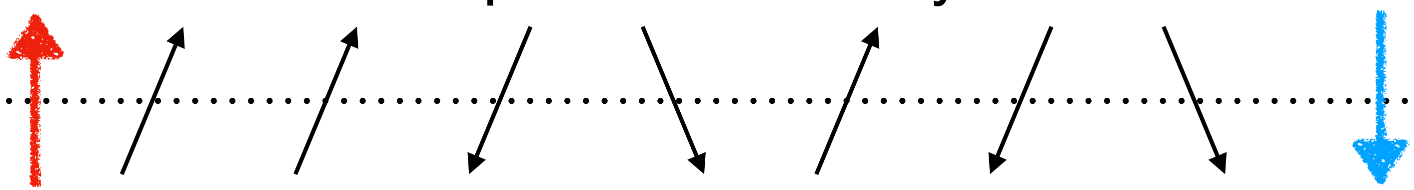
Transport using KGS/Lindblad

$$\frac{d\rho}{dt} = i\left[\rho, H\right] + \frac{1}{4}\kappa \sum_{k=1}^{4} \left(\left[L_k \rho, L_k^\dagger \right] + \left[L_k, \rho L_k^\dagger \right] \right)$$

$$H = -\sum_i \vec{s}_i \cdot \vec{s}_{i+1} + \sum_i h_i s_i^z \qquad L_{1,2} = \sqrt{1 \pm \mu}\sigma_1^\pm, \quad L_{3,4} = \sqrt{1 \pm \mu}\sigma_L^\mp$$

$$j_i = s_i^x s_{i+1}^y - s_i^y s_{i+1}^x$$

Non-equilibrium steady state

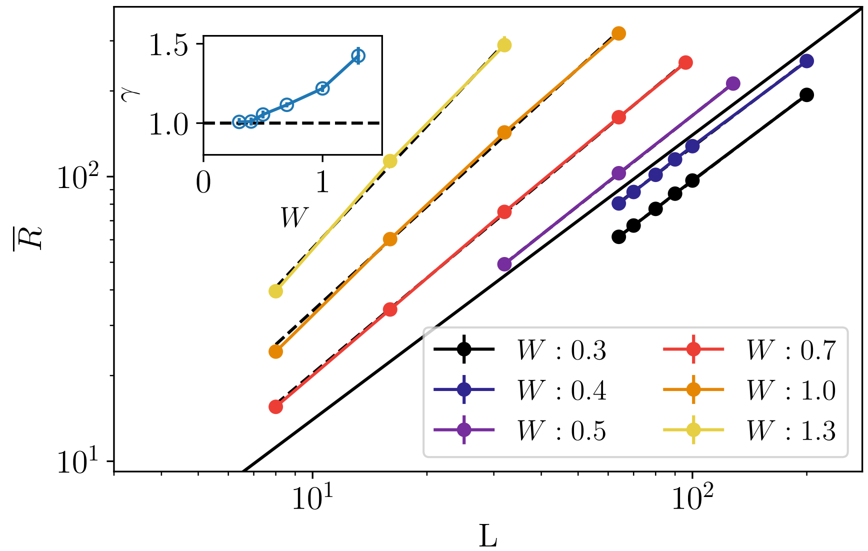


$$\kappa = 1, \quad \mu = 10^{-3} \qquad j_i = j_j = J$$

Delocalized region

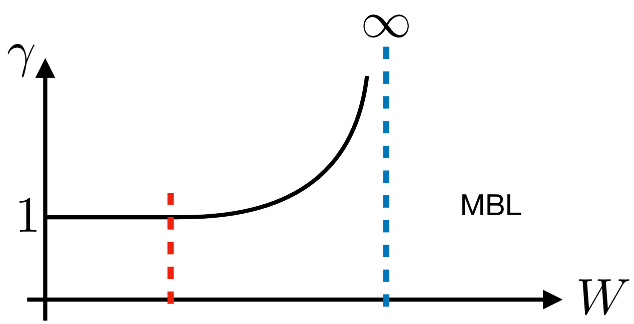
$$R = \mu/J$$

$$W = 0.7$$

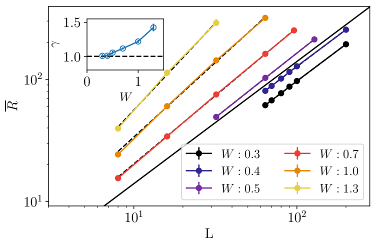


$$\boxed{\gamma = 1.12 \pm 0.02}$$

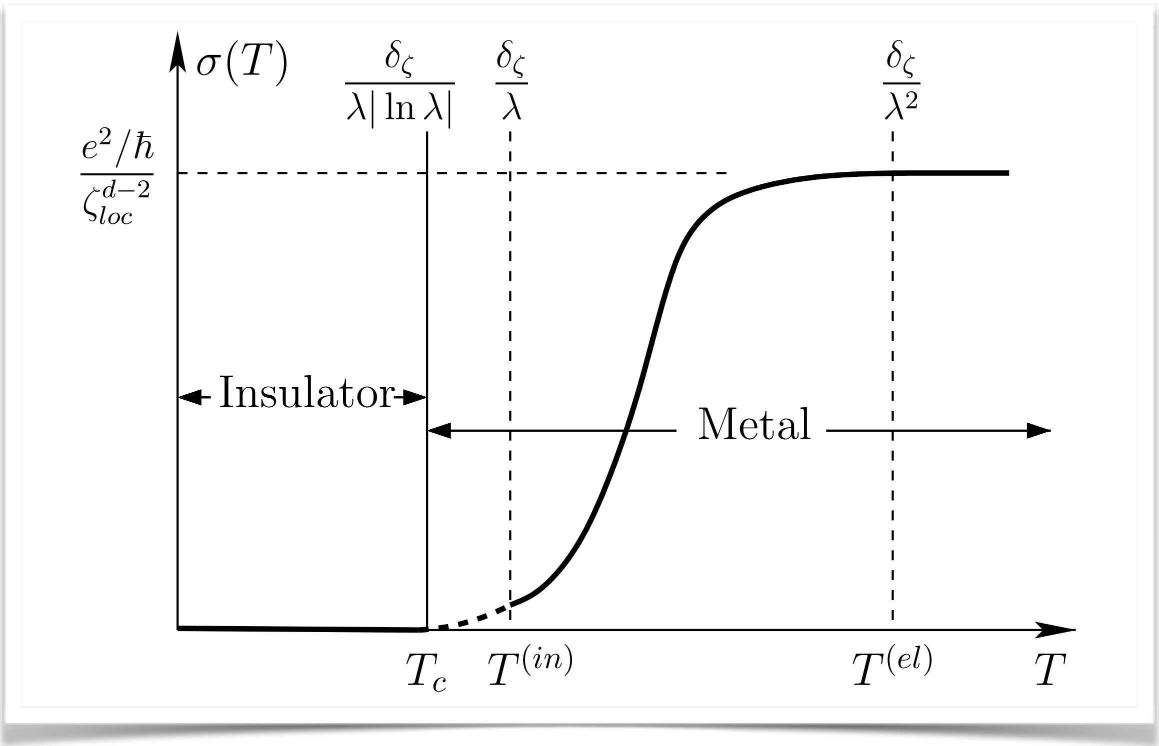
$$R \sim L^{\gamma}$$



Delocalized region



Where in BAA diagram is this region?

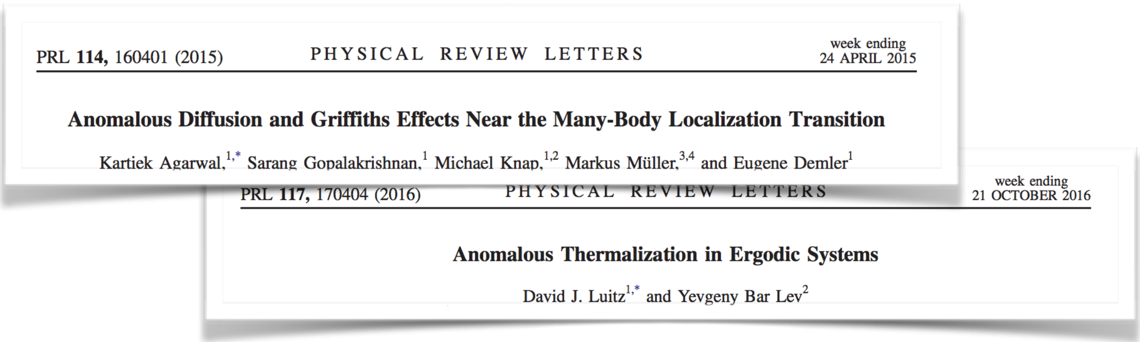


underestimate
or
overestimate?

peculiarity of
1d?

Delocalized region

Subdiffusion has been observed before



PRL **114,** 160401 (2015) PHYSICAL REVIEW LETTERS week ending 24 APRIL 2015

Anomalous Diffusion and Griffiths Effects Near the Many-Body Localization Transition

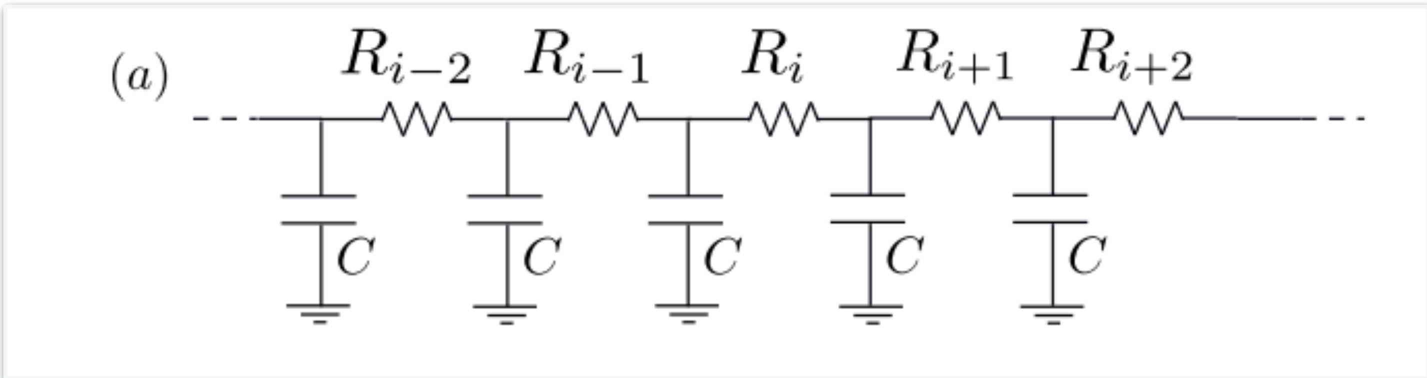
Kartiek Agarwal,[1,*] Sarang Gopalakrishnan,[1] Michael Knap,[1,2] Markus Müller,[3,4] and Eugene Demler[1]

PRL **117,** 170404 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

Anomalous Thermalization in Ergodic Systems

David J. Luitz[1,*] and Yevgeny Bar Lev[2]

+more recent works by V.Khemani et al



The proposed explanation relais on the presence of rare regions of unusually large resistance

Delocalized region

sum of iid random variables

$$P(R) \sim R^{-\alpha} \qquad 1 < \alpha \leq 2$$

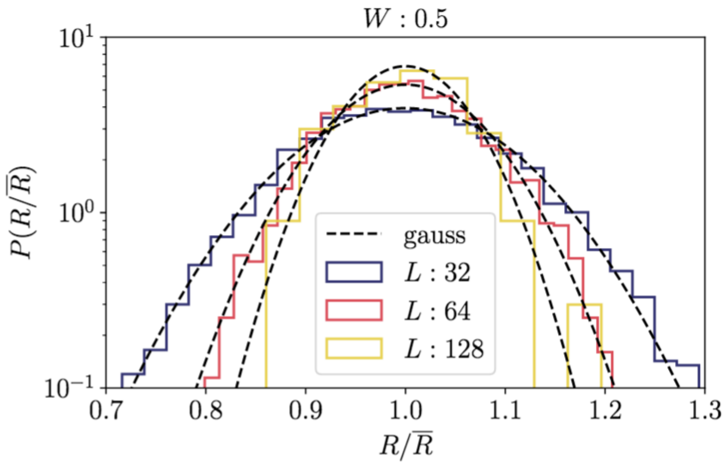
central limit does not work

$$R = \sum_{i=1}^{L/\xi} R_i \sim L^{\gamma} \qquad \gamma = \frac{1}{\alpha - 1}$$

So this explanation relies on long tails

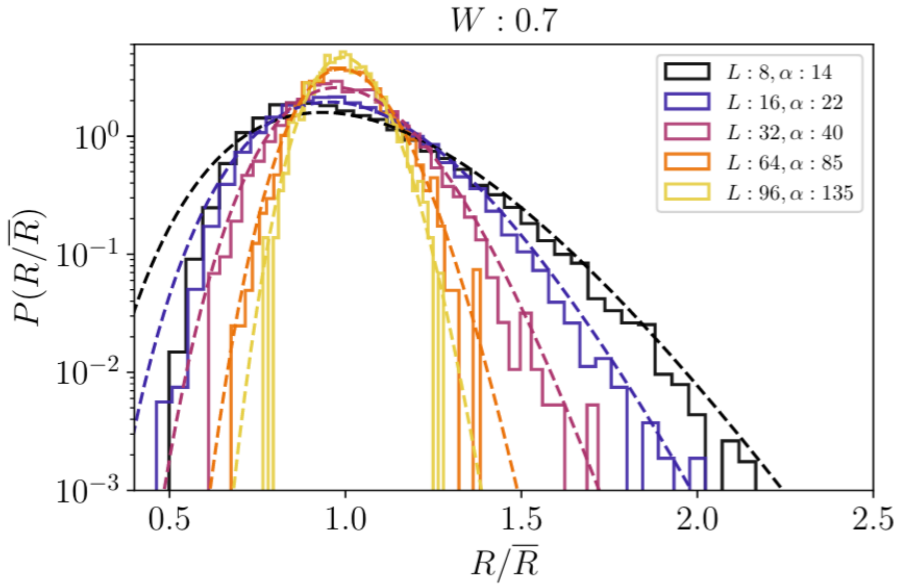
We tested this hypothesis in 1909.09507

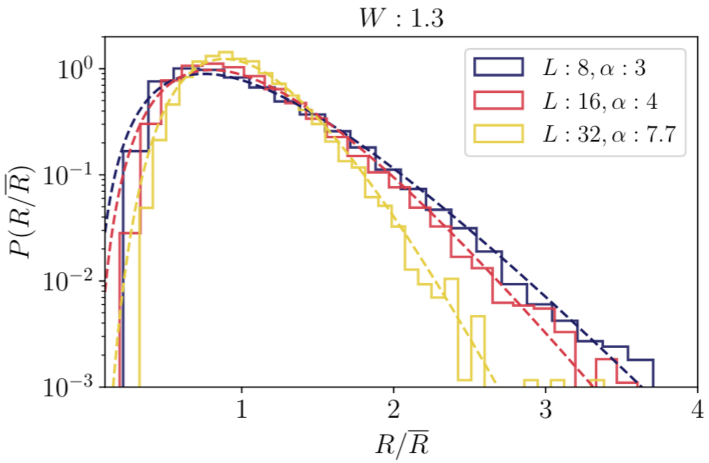
Delocalized region



$$P(R/\overline{R}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(R/\overline{R}-1)^2}{2\sigma^2}}$$

$$P(R/\overline{R}) \propto (R/\overline{R})^{\alpha} e^{-(1+\alpha)(R/\overline{R})}$$

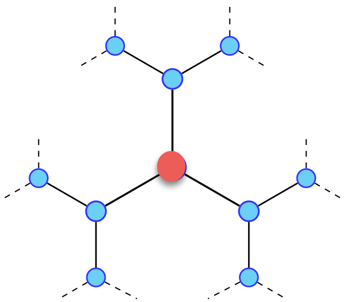




Exponential decay!

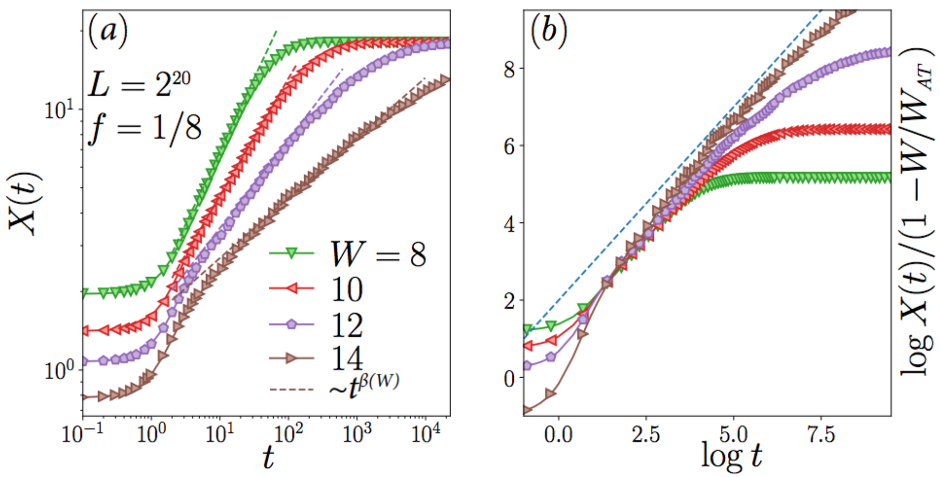
No long tails!

Another example of subdiffusion

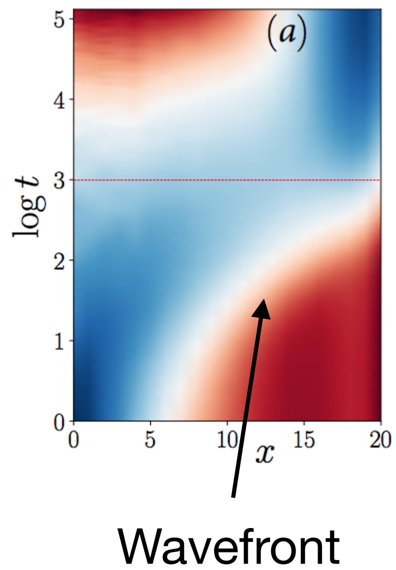


Anderson model on the Bethe lattice

$$H = -\sum_{(i,j)} c_i^\dagger c_j + \text{h.c.} \; + \sum_i \epsilon_i c_i^\dagger c_i$$



Hydrodynamics with subdiffusion?



G.De Tomasi, S. Bera, AS, I.M. Khaymovich 1908.11388

Conclusion

- Quantum dynamics generally *slower* than classical dynamics in disordered systems
- “Slow” in quantum dynamics comes in different flavors
- The “subdiffusion” flavor is mysterious
- It has important bearings on the critical point of MBL-ETH

KT scaling/SDRG motivated works