Issues on the dynamics of 1D quantum magnets

Xenophon Zotos

Department of Physics, University of Crete
Leibnitz Institut/IFW, Dresden

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issues

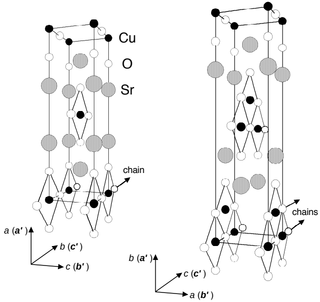
- *novel - magnetic - mode of heat transport*
- *fractal Drude weight*
- *magnetothermal transport*
- *DTN - a route to XXZ*
- *TBA vs. low energy effective theories*

motivation

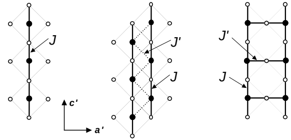
- thermal transport in 1D quantum magnets
  spin Seebeck effect
- dynamic heat transport
- transient grating spectroscopy
- spinon THz spectroscopy

A novel mode of thermal transport [1]

Sr_2CuO_3 $SrCuO_2$



Sr_2CuO_3 $SrCuO_2$ $Sr_{14}Cu_{24}O_{41}$



$$J'/J \sim 10^{-4}$$
$$J \sim 2'400K$$

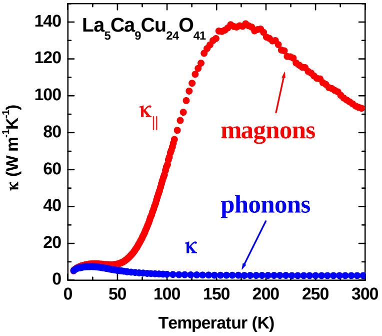
$Cu(C_4H_4N_2)(NO_3)_2$

$J \sim 10K$

Cs_2CuCl_4

$S = 3/2, \quad J \sim 10K$

[1] A. Revcolevschi, C. Hess, B. Büchner, A. Sologubenko, H.R. Ott, Y. Koike



magnetic thermal transport

- highly directional
- electrically insulating
- ”metallic” $J \sim \epsilon_F$
- ”mechanical” - switching

[2] C. Hess

flash method[3]

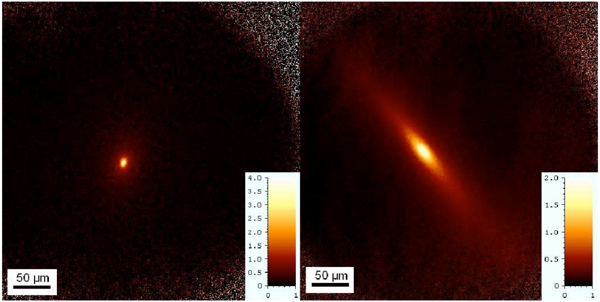
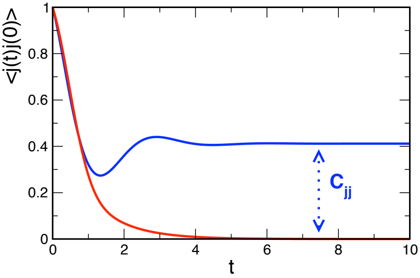


Figure 2: Left panel: Thermal image on the ab plane of $La_5Ca_9Cu_{24}O_{41}$ showing very localized symmetric heating after a 40 ms heat pulse. Right panel: similar image on the ac plane showing a highly asymmetric streaked heating pattern due to the large magnon heat conductivity in the (diagonal) c-direction.

[3] P. van Loosdrecht

integrable transport and Drude weights - a conjecture [4]



$$D \sim \beta C_{jj}$$

$$\sigma'' = \frac{D}{\omega}\Big|_{\omega\to 0}$$

$$\sigma'_{dc} \sim \beta \int_0^\infty dt \langle j(t) j \rangle$$

[4] W. Kohn 1964

Mazur inequality[5]

$$[Q_n, H] = 0, \quad \langle Q_m Q_n \rangle = \delta_{mn}$$

$$\langle j(t) j \rangle_{t\to\infty} \sim C_{jj} \geq \sum_n \frac{\langle j Q_n \rangle^2}{\langle Q_n^2 \rangle}$$

[5] P. Mazur 1969

$S = 1/2$ Heisenberg model

$$H = J\sum_{l=1}^{L} S_l^x S_{l+1}^x + S_l^y S_{l+1}^y + \Delta S_l^z S_{l+1}^z - hS_l^z$$

- $J > 0$ antiferromagnet
- $\Delta < 1$ *easy-plane*
- $\Delta > 1$ *easy-axis*
- $\Delta = cos(\pi/\nu)$

- $[S^z, H] = 0$
- $j_S = J\sum_l (S_l^x S_{l+1}^y - S_l^y S_{l+1}^x)$

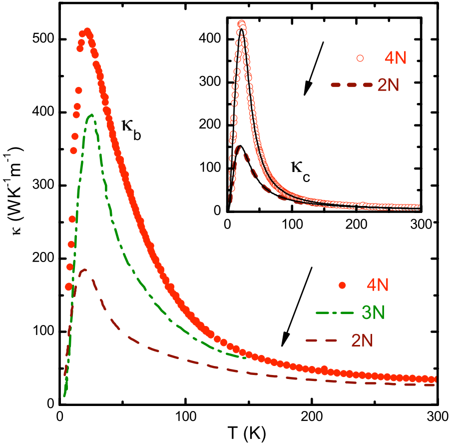
Bethe ansatz integrable model

conservation laws [6]

$Q_3 = j^E$

[6] X.Z., F. Naef, P. Prelovšek 1997

Sr_2CuO_3 from 2N to 4N



conservation laws [7]

$Q_3 = j_E$

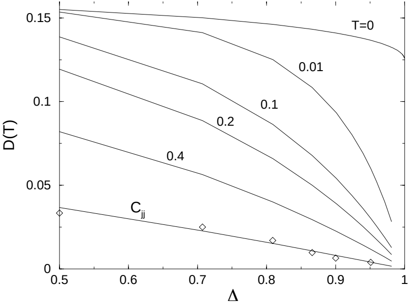
- $\kappa(\omega) = D_{th}\delta(\omega)$
- $D_{th} = \beta^2 < j_E^2 >$
- $\kappa_{dc} = D_{th}\tau$

spin Drude weight - D_s

- $D_s(T) \geq \frac{\beta}{2L}\frac{\langle j_S Q_3\rangle^2}{\langle Q_3^2\rangle}$
- $\beta \to 0$
- $D_s(T) \geq \frac{\beta}{2}\frac{8\Delta^2 m^2(1/4-m^2)}{1+8\Delta^2(1/4+m^2)}$
- $m = \langle S^z\rangle$

[7] X.Z., F. Naef, P. Prelovšek 1997

$0 < \Delta < 1, \quad m = 0$, TBA [8]



$$
\begin{aligned}
D_0 &= \frac{\pi}{8} \frac{\sin(\pi/\nu)}{\frac{\pi}{\nu}(\pi - \frac{\pi}{\nu})} \\
C_{jj} &= 1 - \frac{\sin(2\pi/\nu)}{2\pi/\nu}
\end{aligned}
$$

$$
\begin{aligned}
0 &< \Delta < 1 \\
D_s(T) &= D_0 - cT^{2/(\nu-1)}
\end{aligned}
$$

[8] S. Fujimoto, N. Kawakami 1998, XZ 1999

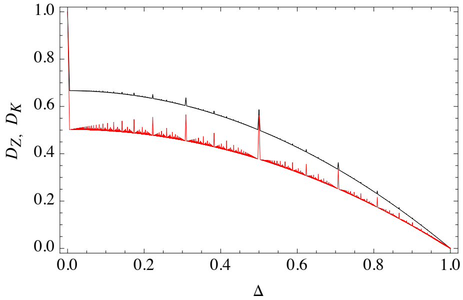
m=0

- alternative BA - no strings[9]
- quasi-local conservation laws[10]
- numerical simulations - ED, DMRG, ”typicality” [11]
- Generalized Hydrodynamics (GHD) [12]
- Drude weights from GHD [13]

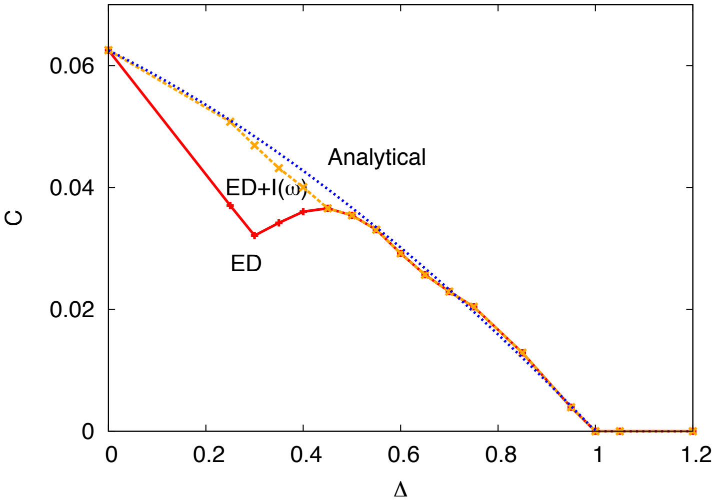
[9] A. Klümper
[10] T. Prosen, R. Pereira, J. Sirker, I. Affleck...
[11] F. Heidrich - Meisner, R. Steinigeweg...
[12] B. Doyon, M. Fagotti...
[13] J. Moore, C. Karrasch, E. Ilievski, J. De Nardis...

$\beta \to 0$ fractal Drude weight

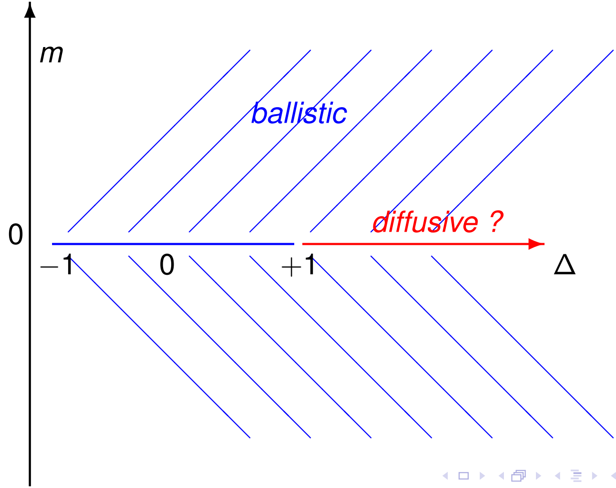
- $\Delta = \cos(\frac{\pi l}{\nu})$
  $D_{ql} = \frac{\sin^2(\frac{\pi l}{\nu})}{\sin^2(\frac{\pi}{\nu})}(1 - \frac{\nu}{2\pi}\sin(\frac{2\pi}{\nu}))$



$\omega < 1/L$ contribution ?



D_s scenario
transient gradient spectroscopy ?



magnetothermal transport [14]

$$j_Q = j_E - h j_S$$

$$\kappa_{th} = C_{QQ} - \beta C_{QS}^2 / C_{SS}$$

$$C_{ij} = D_{ij} \tau_{ij}$$

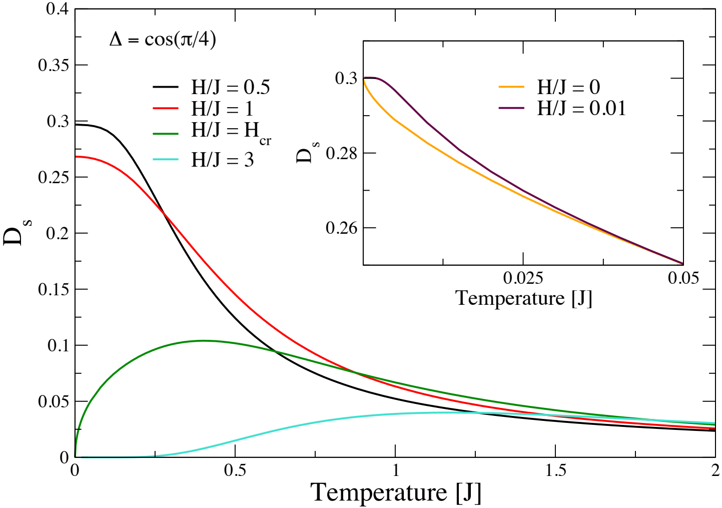
$$\kappa_{th} = (D_{QQ} - \beta D_{QS}^2 / D_{SS}) \tau$$

$$MTC = \beta D_{QS}^2 / D_{SS}$$

$$D_{QQ} = D_{EE} - 2\beta h D_{ES} + \beta h^2 D_{SS}$$

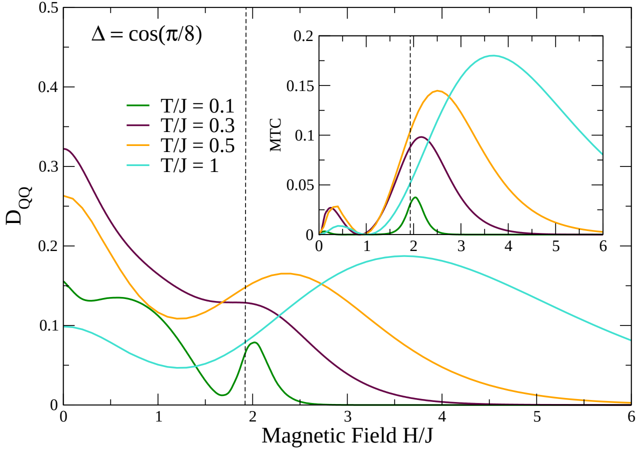
- $D_{EE} = \beta^2 < {j_E}^2 >, D_{ES} = \beta < j_E j_S >$ (QTM - Sakai, A. Klümper)
- D_{SS}

[14] K. Sakai - A. Klümper 2005, C. Psaroudaki, XZ 2015

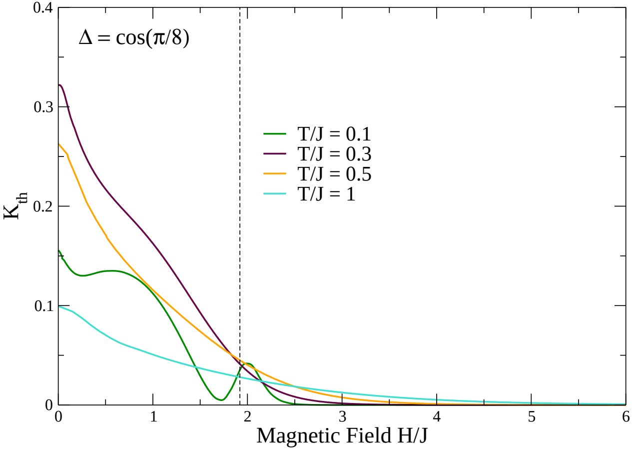


Δ = cos(π/4)
H/J = 0.5
H/J = 1
H/J = H_cr
H/J = 3
H/J = 0
H/J = 0.01
D_s
Temperature [J]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.5
1
1.5
2
0.26
0.28
0.025

$D_{QQ} - MTC$



κ_{th}



$\Delta = \cos(\pi/8)$
T/J = 0.1
T/J = 0.3
T/J = 0.5
T/J = 1
K_{th}
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
Magnetic Field H/J

κ_{exp}[15]

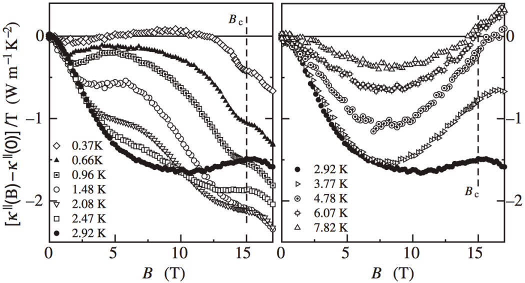
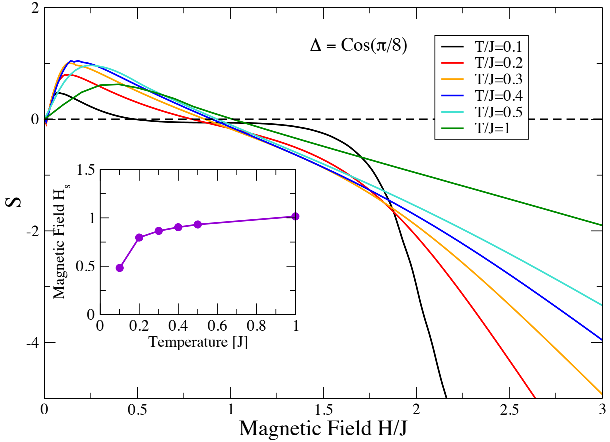


Figure 4.3.13: Thermal conductivity measured parallel to the chains of $Cu(C_4H_4N_2)(NO_3)_2$ as a function of magnetic field at several fixed temperatures. Figure taken from [196].

[15] T. Lorenz, A. Sologubenco

spin Seebeck coefficient- $S = \nabla h / \nabla T$ [16]

$$S = \frac{\nabla h}{\nabla T} = \beta \frac{D_{ES}}{D_{SS}} - \beta h$$



[16] Sr_2CuO_3 D. Hirobe et al. 2016

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Effective s=1/2 model for S=1 easy - plane quasi-1D antiferromagnet or how to tune Δ[17]

$NiCl_2$-$SC(NH_2)_2$ (DTN) *large D limit*

$$\mathcal{H} = \sum_{n=1}^{N} [J(\mathbf{S}_n \cdot \mathbf{S}_{n+1}) + D(S_n^z)^2 + HS_n^z]$$

$$H_{c_1} = D - 2J + \frac{J^2}{D} + \frac{J^3}{2D^2}, \quad H_{c_2} = D + 4J$$

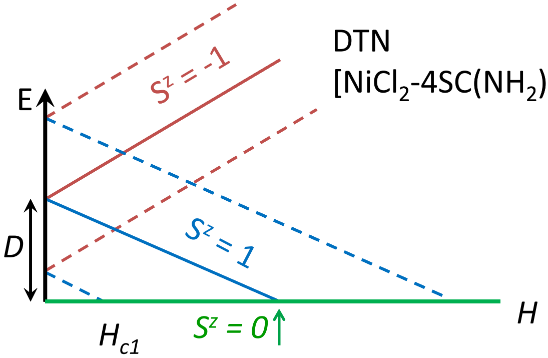
$$H < H_{c_1} \quad S^z = 0$$

$$H > H_{c_2} \quad S^z = -N$$

[17] S. Svyagin, C. Psaroudaki, N. Papanicolaou, G. Karadamoglou, J. Herbrych, XZ

DTN

$[NiCl_2\text{-}4SC(NH_2)_2]$



Effective s=1/2 model

$$S_n^z = 0 \to s_n^z = +1/2$$

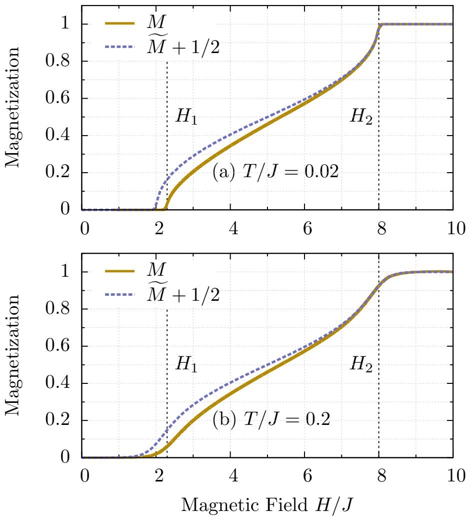
$$S_n^z = -1 \to s_n^z = -1/2$$

$$H_{eff} = \sum_n 2J(s_n^x s_{n+1}^x + s_n^y s_{n+1}^y + \Delta s_n^z s_{n+1}^z) + h s_n^z$$

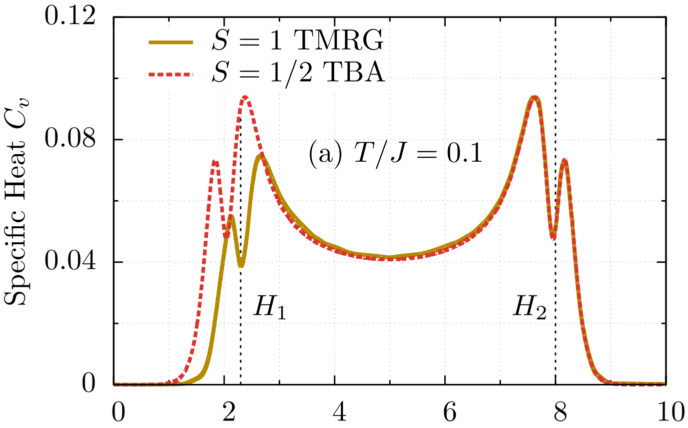
where

$\Delta = 1/2, \quad h = -J - D + H, \quad h_c = \pm 2J(\Delta + 1)$

magnetization



specific heat



1D Heisenberg Hamiltonian

$$H = \sum_{i=1}^{N} J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) - hS_i^z,$$

- $\Delta = \cos\theta$
- $\theta = \pi/\nu$
- $(\theta = \frac{\pi}{\nu_1 + 1/(\nu_2 + 1/\nu_3 + \ldots)})$
- excitations, strings of order $\mathrm{n}_j = j$, parity λ_j, $\quad j = 1, \nu$

Thermodynamic Bethe Ansatz (TBA - C.N.Yang, C.P.Yang, 1969) M. Takahashi and M. Suzuki, 1972

$$\epsilon_j = \epsilon_j^{(0)} + h\mathrm{n}_j + T\sum_k \lambda_k T_{jk} \circ \ln(1 + e^{-\beta\epsilon_k}), \quad j = 1, ..., \nu$$

- ϵ_j thermal string energies
- x rapidity
- T_{jk} phase shifts
- $a \circ b = \int a(\mathrm{x} - \mathrm{y})b(\mathrm{y})d\mathrm{y}$
- $e^{\beta\epsilon_j} = \rho_j^h/\rho_j$
- $n_k = \rho_k/(\rho_k + \rho_k^h)$.

dressed excitations [18]

$$\frac{\partial p_j}{\partial \mathrm{x}} = \frac{dp_j^{(0)}}{d\mathrm{x}} - \sum_k \lambda_k T_{jk} \circ n_k \frac{\partial p_k}{\partial \mathrm{y}}$$

$$E_j = \epsilon_j^{(0)} - \sum_k \lambda_k T_{jk} \circ \; n_k E_k$$

$$j_j^{\epsilon} = j_j^{\epsilon(0)} - \sum_k \lambda_k T_{jk} \circ n_k j_k^{\epsilon}.$$

$$Q_j = Q_j^{(0)} - \sum_k \lambda_k T_{jk} \circ n_k Q_k, \quad Q_j^{(0)} = \mathrm{n}_j, \quad Q_j = \partial \epsilon_j / \partial h$$

- $h = 0, \;\; Q_j = 0, \;\; j = 1, \nu - 2, \;\; Q_{\nu-1} = -Q_\nu = \nu/2$
- physically, uniform change of the S^z component of the magnetization by ± 1, e.g. in ESR experiments

[18] Doyon, Fagotti and collaborators

mean quantities

$$E = \sum_j \int d\mathrm{x} \rho_j \epsilon_j^{(0)} = \sum_j \lambda_j \int \frac{dp_j^{(0)}}{2\pi} n_j E_j.$$

$$J_E = \sum_j \int d\mathrm{x} \rho_j j_j^{\epsilon(0)} = \sum_j \lambda_j \int \frac{dp_j^{(0)}}{2\pi} n_j j_j^{\epsilon}$$

$$v_j = \frac{\partial \epsilon_j}{\partial p_j}$$

$$J_E = \sum_j \lambda_j \int \frac{dp_j^{(0)}}{2\pi} n_j (v_j E_j).$$

thermodynamics

$$f = -T \sum_j \lambda_j \int \frac{dp_j^{(0)}}{2\pi} \ln(1 + e^{-\beta \epsilon_j}).$$

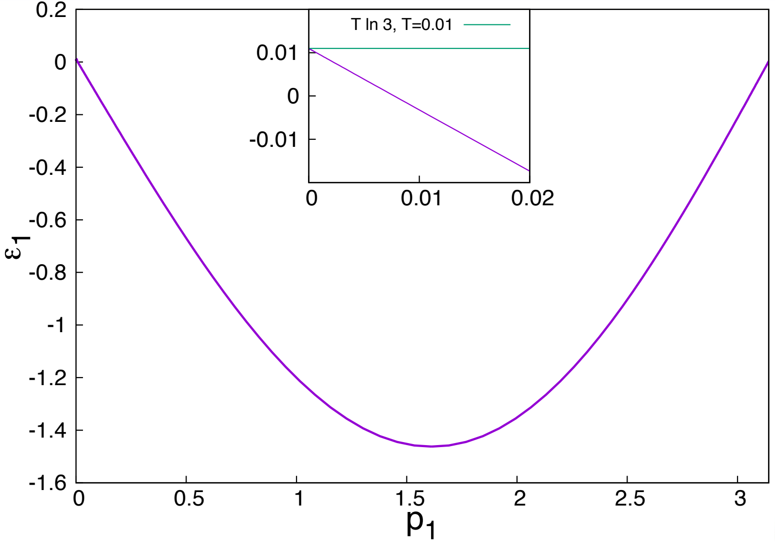
$$c(T) = \frac{\partial \epsilon}{\partial T} = \beta^2 \sum_j \int \frac{dp_j}{2\pi} n_j (1 - n_j) E_j^2.$$

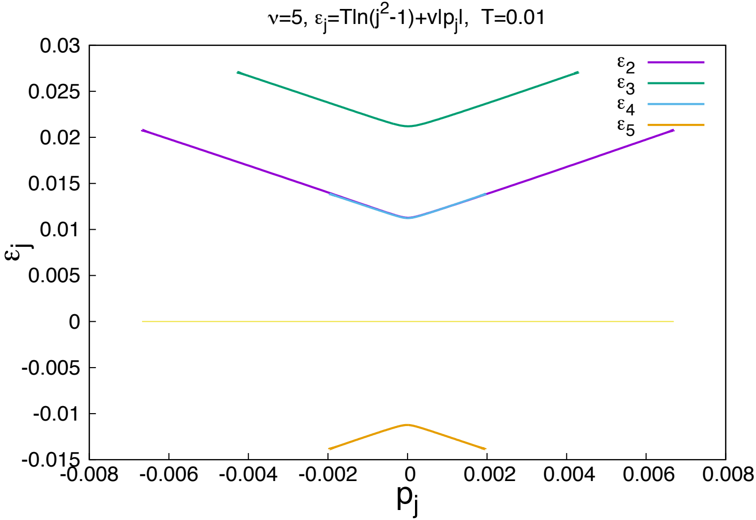
$$\chi(T) = \frac{\partial m}{\partial h}|_{h \to 0} = \beta \sum_j \int \frac{dp_j}{2\pi} n_j (1 - n_j) Q_j^2.$$

$T \to 0$

$$
\begin{aligned}
T &= 0 \\
\epsilon_1 &= -v \sin p_1, \quad 0 \leq p_1 < \pi \\
\epsilon_j &= 0, \quad j > 1 \\
v &= J\frac{pi}{2}\frac{\sin\theta}{theta}
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_1 \quad &\simeq T\ln 3 - v\sin p_1, \quad 0 \leq p_1 \leq \pi \\
\epsilon_j \quad &\simeq T\ln(j^2-1) + v|p_j|, \quad j = 2, ..., \nu - 2 \\
|p_j| \quad &\leq p_j^{max}, \quad p_j^{max} = \frac{T}{v}\ln\left(\frac{(j+1)^2-1}{j^2-1}\right) \\
\epsilon_{\nu-1} \quad &\simeq T\ln(\nu-2) + v|p_{\nu-1}| \\
|p_{\nu-1}| \quad &< p_{\nu-1}^{max}, \quad p_{\nu-1}^{max} = \frac{T}{v}\ln\left(\frac{\nu-1}{\nu-2}\right) \\
\epsilon_\nu \quad &= -\epsilon_{\nu-1}
\end{aligned}
$$


T ln 3, T=0.01
0.01
0
-0.01
0
0.01
0.02
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
ε_1
0
0.5
1
1.5
2
2.5
3
p_1

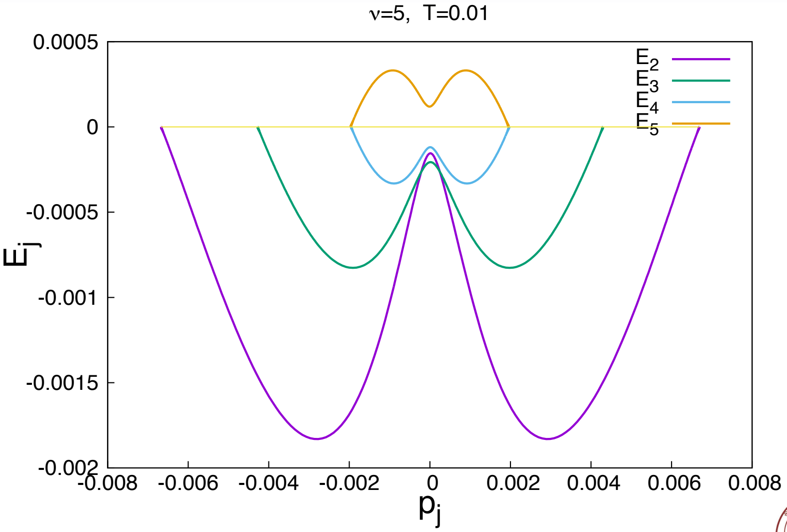

ν=5, ε_j=Tln(j^2-1)+v|p_j|, T=0.01
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
ε_j
ε_2
ε_3
ε_4
ε_5
-0.008
-0.006
-0.004
-0.002
0
0.002
0.004
0.006
0.008
p_j

$$
\begin{aligned}
T &= 0 \\
E_1 &= \epsilon_1, \quad E_j = 0, \quad j > 1
\end{aligned}
$$

$$
\begin{aligned}
E_1 &\sim -v|\sin p_1| \\
E_j &\sim -T|\sin(\frac{\pi p_j}{p_j^{max}})|, \quad j > 1 \quad |p_j| \leq p_j^{max}
\end{aligned}
$$

a note

$$
\begin{aligned}
f(p) &= T \ln g - \epsilon_p \\
n_h = 1 - n &= \frac{g}{g + e^{\beta \epsilon_p}}
\end{aligned}
$$


ν=5, T=0.01
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
E_j
-0.008
-0.006
-0.004
-0.002
0
0.002
0.004
0.006
0.008
p_j
E_2
E_3
E_4
E_5

$$\begin{aligned} \epsilon_{spinon} &= v|\sin p|, \quad -\pi < p \leq \pi \\ c_{spinon} &\simeq \frac{\pi}{3}\frac{1}{\beta v}, \quad (\pi/3 \simeq 1.047) \end{aligned}$$

$$\begin{aligned} c^{(1)}_{TBA} &\simeq \beta^2 2\int_0^{+\infty} \frac{dp_1}{2\pi}\frac{(vp_1)^2}{4\cosh^2\frac{\beta(T\ln 3 - vp_1)}{2}} \simeq 1.234\frac{1}{\beta v} \\ c^{(j)}_{TBA} &\sim \frac{1}{\beta v}\frac{1}{j^3} \end{aligned}$$

$$\chi = \frac{1}{\pi v}K, \quad K = \frac{1}{2}\frac{1}{1-1/\nu}$$

Drude weights - revisited[19]

- $D_{th} = \frac{\beta^2}{2} \sum_j \lambda_j \int \frac{dp_j}{2\pi} n_j(1-n_j)(v_j E_j)^2$
- $T \to 0, \quad v_j \to v, \quad D_{th} = \frac{v^2}{2} c$

- $D_s = \frac{\beta}{2} \sum_j \lambda_j \int \frac{dp_j}{2\pi} n_j(1-n_j)(v_j Q_j)^2$
- $T \to 0, \quad D_s = \frac{v^2}{2} \chi = \frac{1}{2\pi} vK$ (Shastry-Sutherland)

[19] Klümper, Sakai, XZ

perspectives

- theory vs. experiment
- impurity scattering
  phonon scattering - microscopic theory
- TBA vs. low energy effective theories