1 Cut-reduction: Teorema dell'ubriaco (C) 1

$\Pi_1 = \Pi^1_{rev}$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(s), P(s)}\,ax}{\neg P(s), P(u), P(s)}\,w}{\neg P(s), \forall z P(z), P(s)}\,\forall}{\neg P(s) \vee \forall z P(z), P(s)}\,\vee}{\vdash \exists y(\neg P(y) \vee \forall z P(z)), P(s)}\,\exists}{\vdash \exists y(\neg P(y) \vee \forall z P(z)), \forall x P(x)}\,\forall$$

Π_1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash \neg P(w), P(w)}\,ax}{\vdash \neg P(w), \neg P(v), P(w)}\,w}{\vdash \exists x \neg P(x), \neg P(v), P(w)}\,\exists}{\vdash \exists x \neg P(x), \neg P(v), \forall z P(z)}\,\forall}{\vdash \exists x \neg P(x), \neg P(v) \vee \forall z P(z)}\,\vee}{\vdash \exists x \neg P(x), \exists y(\neg P(y) \vee \forall z P(z))}\,\exists}{\exists y \ldots, \exists y \ldots}\,ax}{\exists y(\neg P(y) \vee \forall z P(z))}\,c$$

Iglob

↓ cut (S1) Π_1 è già Rev.

S1 ⇝ (C)

Π_1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(s), P(s)}\,ax}{\neg P(s), P(u), P(s)}\,w}{\neg P(s), \forall z P(z), P(s)}\,\forall}{\neg P(s) \vee \forall z P(z), P(s)}\,\vee}{\exists y(\neg P(y) \vee \forall z P(z)), P(s)}\,\exists}{\exists y(\neg P(y) \vee \forall z(P(z)), \forall x P(x)}\,\forall$$

sostituzione [w/s]

Π_2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(w), P(w)}\,ax}{\neg P(w), \neg P(v), P(w)}\,w}{\exists x \neg P(x), \neg P(v), P(w)}\,\exists}{\exists y(\neg P(y) \vee \forall z P(z)), \neg P(v), P(w)}\,cut\ (C)\ w/s}{\exists y \ldots \quad \vdots \quad \neg P(v), \forall z P(z)}\,\forall$$

$$\dfrac{\dfrac{\exists y \ldots \quad \neg P(v) \vee \forall z P(z)}{\exists y \ldots, \exists y \ldots}\,\exists}{\exists y \ldots}\,c$$

$\Pi_1[w/s]$

segue →

riduco taglio logico (C)

2

Π_1 $\Pi_1[w/s]$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(w), P(w)}\,ax}{\neg P(w), P(u), P(w)}\,w}{\neg P(w), \forall z P(z), P(w)}\,\forall u}{\neg P(w) \vee \forall z P(z), P(w)}\,\vee}{\exists y(\neg P(y) \vee \forall z P(z)), P(w)}\,\exists \qquad \dfrac{\dfrac{}{\neg P(w), P(w)}\,ax}{\neg P(w), \neg P(v), P(w)}\,w}{\exists y(\neg P(y) \vee \forall z P(z)), \neg P(v), P(w)}\,cut$$

⋮ δ un passo (S2)

$\exists y \ldots$

eseguo un passo (S) seguo le occorrenze di $P(w)$ e $\neg P(w)$ nell'A.S.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(w), P(w)} \qquad \dfrac{}{\neg P(w), P(w)}}{\neg P(w), P(w)}\,cut\ ax}{\neg P(w), P(u), P(w)}\,w}{\neg P(w), \forall z P(z), P(w)}\,\forall u}{\neg P(w) \vee \forall z P(z), P(w)}\,\vee}{\exists y(\neg P(y) \vee \forall z P(z)), P(w)}\,\exists}{\exists y(\neg P(y) \vee \forall z P(z)), P(w), \neg P(v)}\,w}{\vdots\ \delta}$$

$\exists y \ldots$

infine riduco il taglio ax e ottengo Π' cut-free