AVVISO

[Avviso: lezione di "recupero" lunedì 24 maggio
ore 13 in presenza e su MS Team
Prof. L. Tortora de Falco]

(l'aula verrà comunicata più avanti)

sulla Semantica Denotazionale della Logica Lineare

* MLL
- MALL ←
- MELL

- Il frammento puro moltiplicativo e additivo della logica lineare (MALL)
(senza unità $\top, 0$)

Regole del calcolo dei Sequenti

ad una sola parte

	disg.	cong.
moltiplicativ	$⅋$	$\otimes$
additivo	plus $\oplus$	$\&$ with

via Leggi di De Morgan:

$\forall A,\ A^{\perp\perp} = A$

$(A \otimes B)^{\perp} = A^{\perp} ⅋ B^{\perp}$; $(A ⅋ B)^{\perp} = A^{\perp} \otimes B^{\perp}$

$(A \oplus B)^{\perp} = A^{\perp} \& B^{\perp}$; $(A \& B)^{\perp} = A^{\perp} \oplus B^{\perp}$

· identità $\dfrac{}{\vdash a, a^{\perp}}$ a literal

$$\dfrac{\vdash \Gamma, A \qquad \vdash \Delta, A^{\perp}}{\vdash \Gamma, \Delta}\ \text{cut}$$

· regole moltiplicative

$$\dfrac{\vdash \Gamma, A \qquad \vdash \Delta, B}{\vdash \Gamma, \Delta, A \otimes B}\ \otimes \qquad \dfrac{\vdash \Gamma, A, B}{\vdash \Gamma, A ⅋ B}\ ⅋$$

regole invertibili

synchron connettivi positivi true non-det.

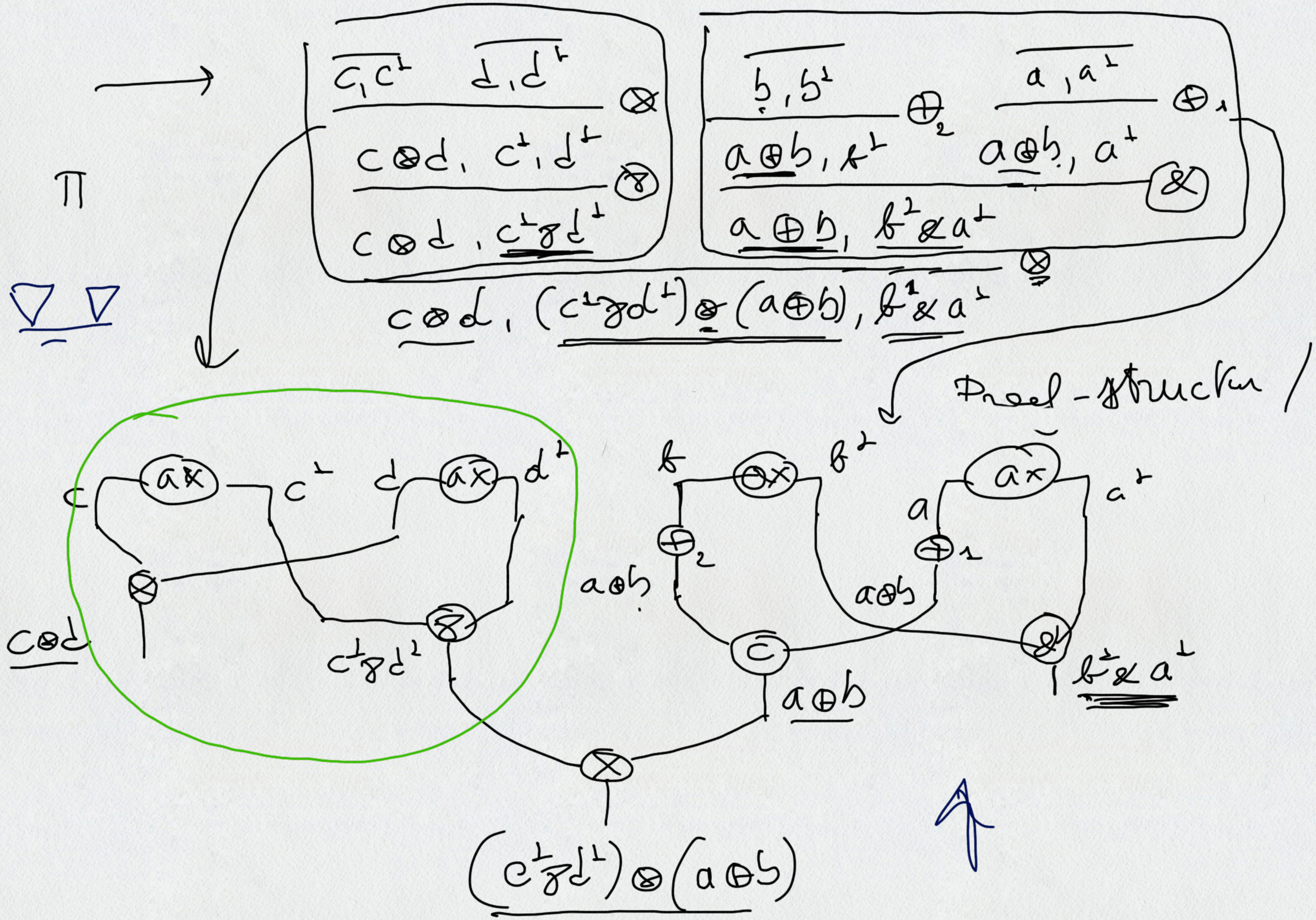
regole additive:

conn. asynchron dont care non-det.

$$\dfrac{\vdash \Gamma, A \qquad \vdash \Gamma, B}{\vdash \Gamma, A \& B}\ \& \qquad \dfrac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B}\ \oplus_1 \qquad \dfrac{\vdash \Gamma, B}{\vdash \Gamma, A \oplus B}\ \oplus_2$$

Proof net

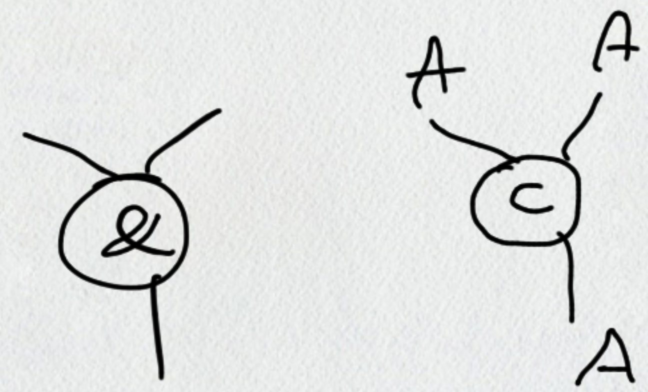
"caratterizzazione geometrica" dell'equivalenza denotazionale di due dimostrazioni dello stesso sequente.



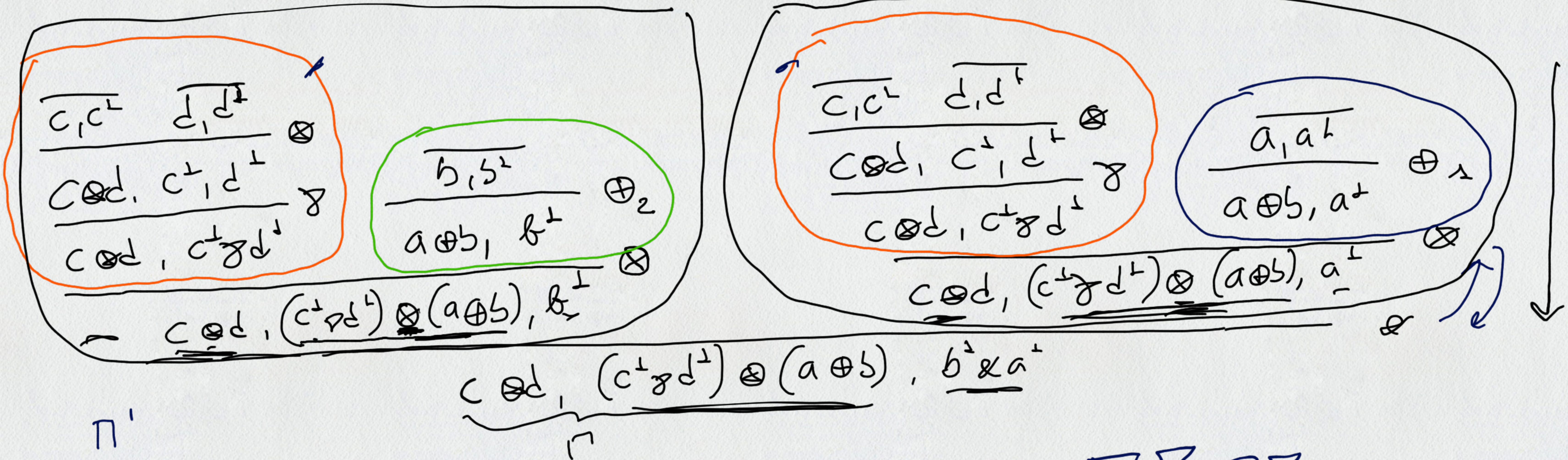
per ind. nell'albero delle derivazioni

Proof-structure / reti (proof net).

sovrapp.

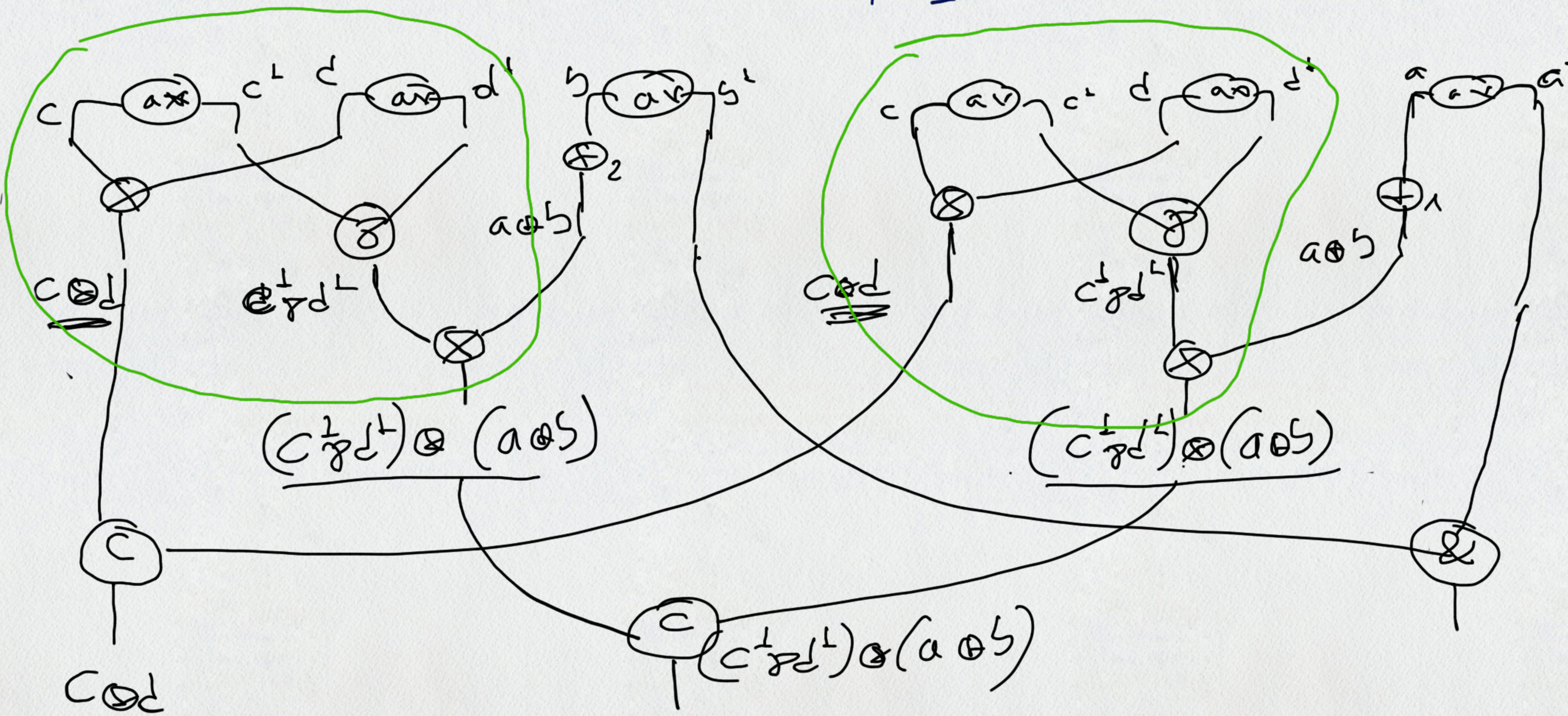


if ... then ... else.



Π'

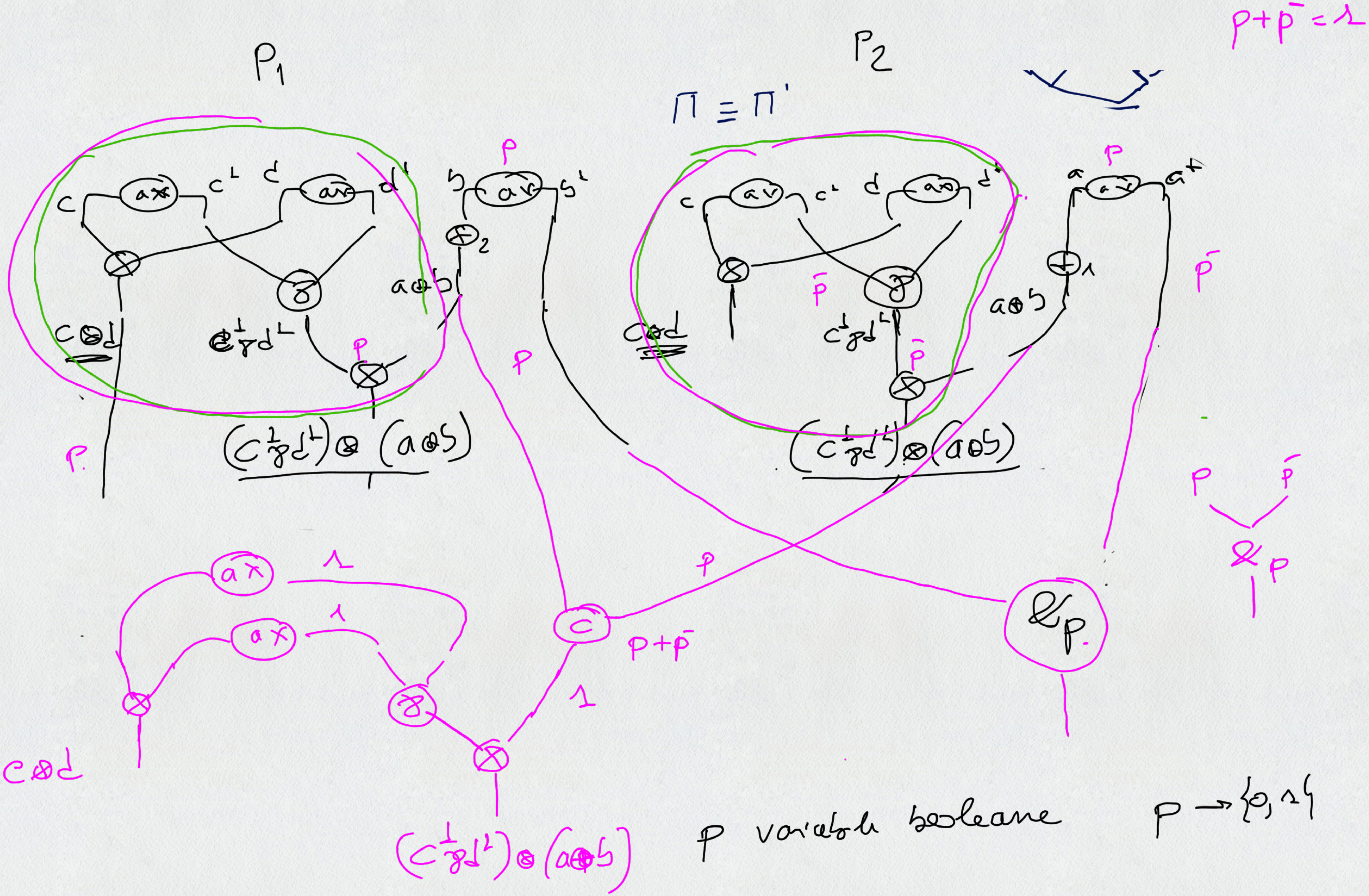
Π ≡ Π'



sovrapposizioni massimali

pari, monomi di variabili booleane.

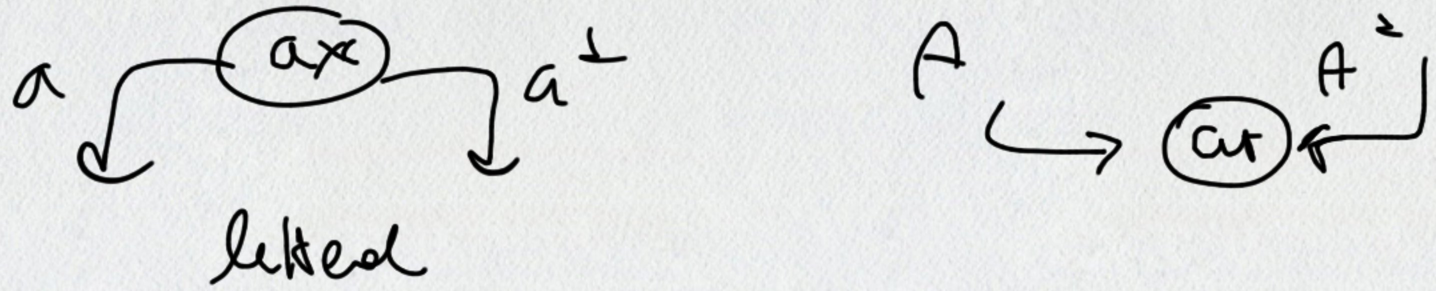
$p, \bar{p} \quad q, \bar{q}$



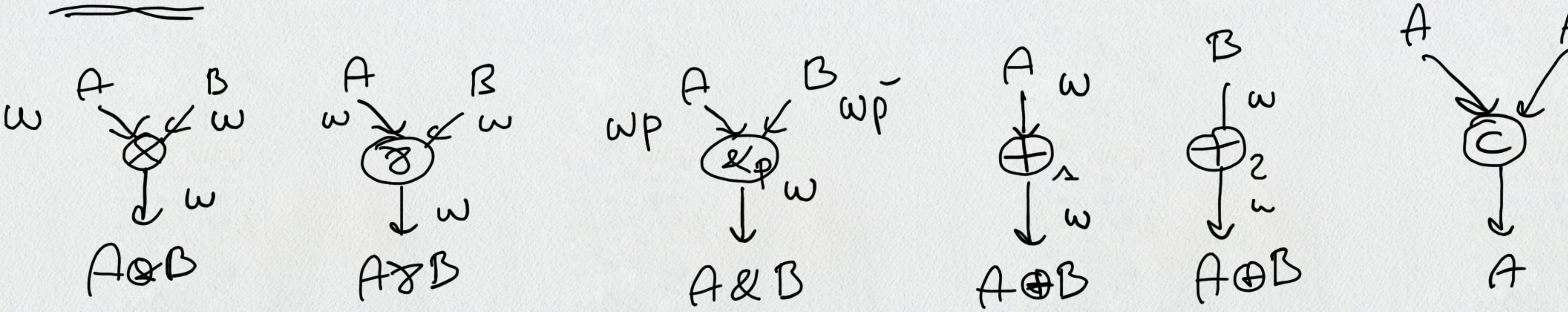
$p + \bar{p} = 1$
P_1
P_2
$\Pi \equiv \Pi'$
$c \otimes d$
$c^\perp \parr d^\perp$
$a \oplus b$
$(c^\perp \parr d^\perp) \otimes (a \oplus b)$
$p + \bar{p}$
$\&_p$
p variabili booleane
$p \to \{0, 1\}$

Proof structure

w peso monomio



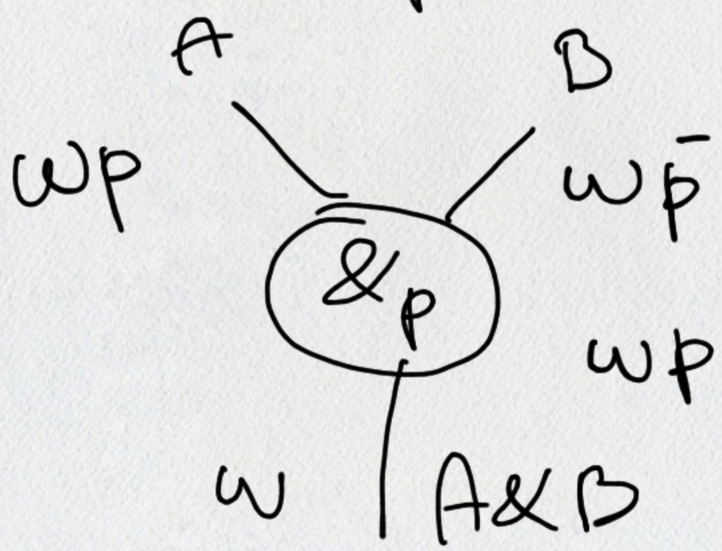
Link



contraction structural

proof structure è un grafo costruito dai links ed è associato a dei pesi monomiali, cioè dei monomi dell'algebra booleana generati a partire dalle variabili booleane associate ai &

conclusioni di PS hanno valore 1

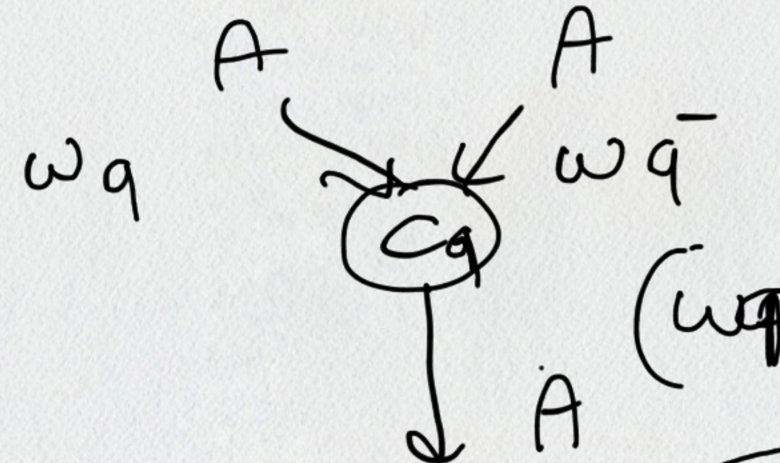


$wp + w\bar{p} = w \cdot (p + \bar{p}) = w \cdot 1 = w$

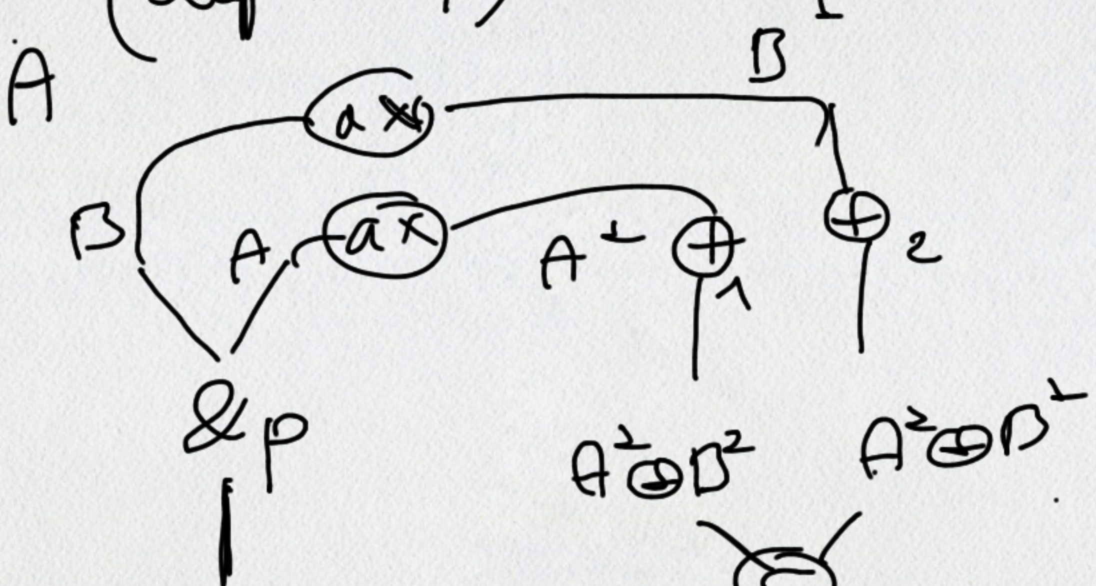
p è var. booleana propria de &

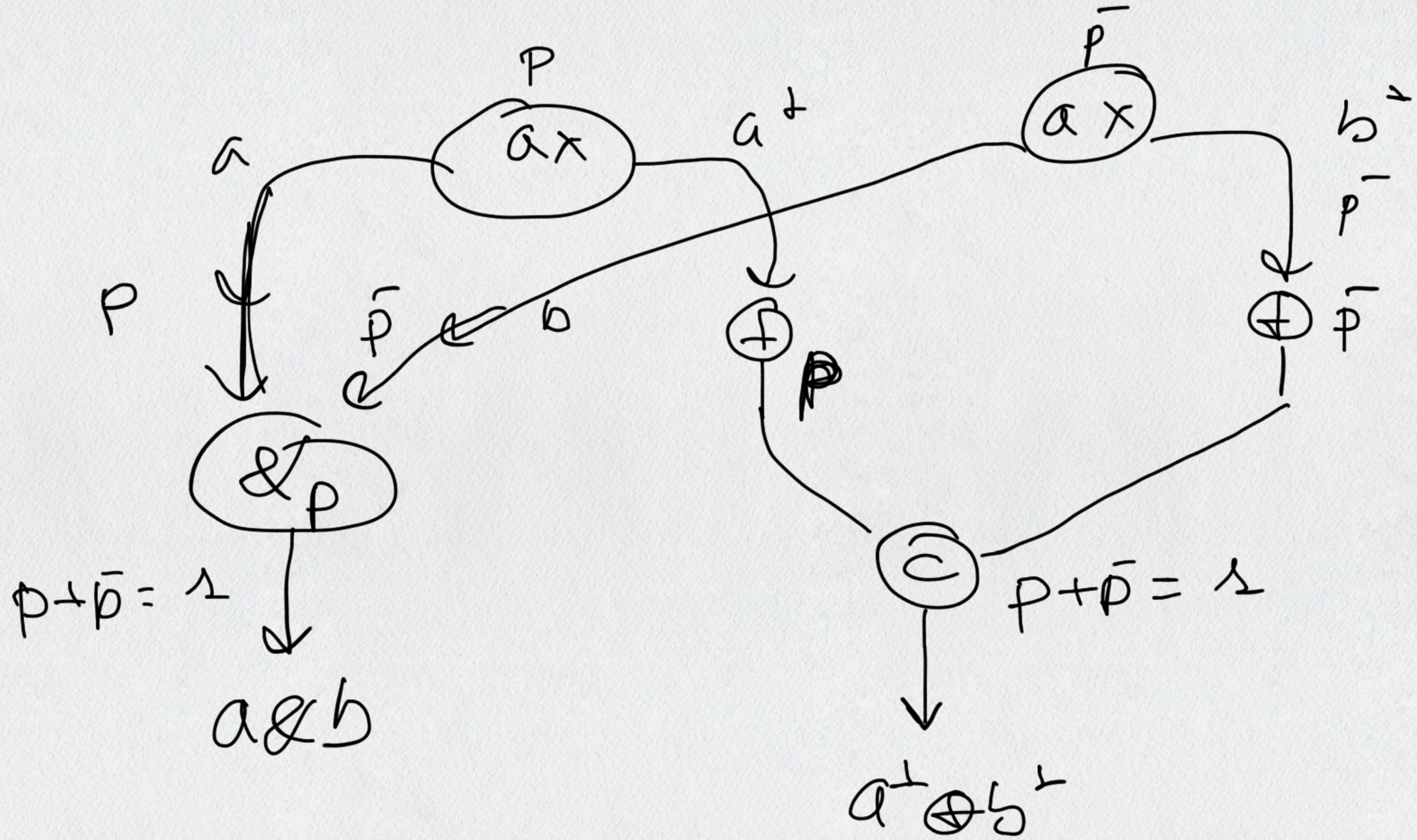
p o negato $\bar{p}$

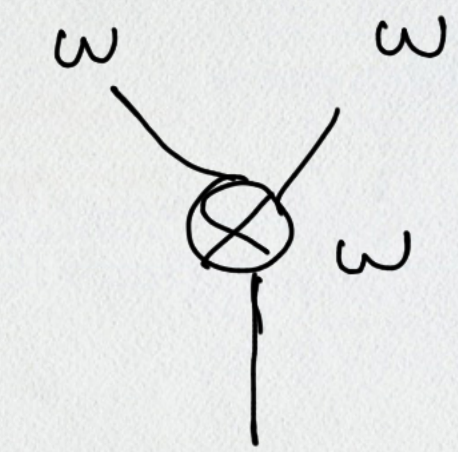
$w = p \cdot \bar{q} \cdot r \, \bar{t}$



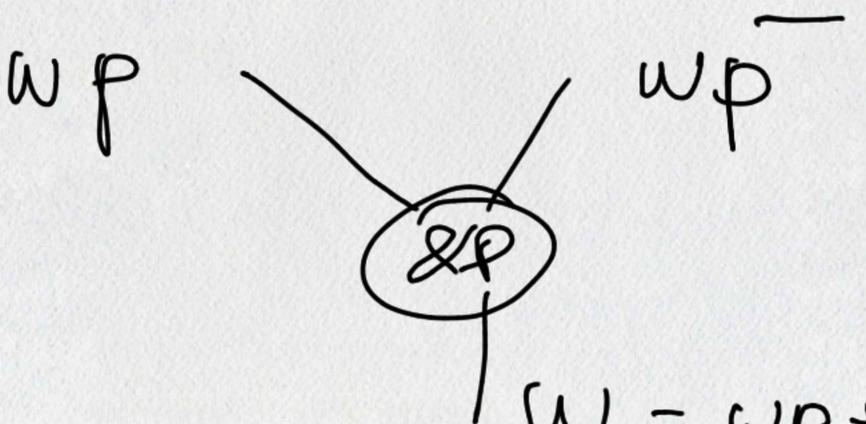
$(wq + w\bar{q}) = w$



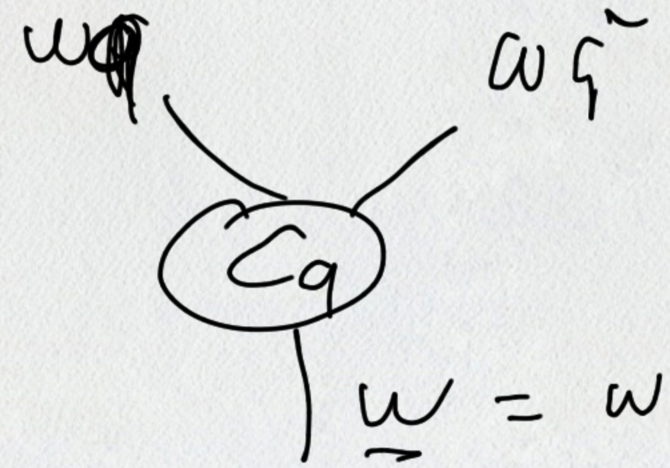




$w p + w \bar{p} = 0$



$$\underline{w} = wp + w\bar{p} = w(p+\bar{p})$$
$$\quad\quad 1$$



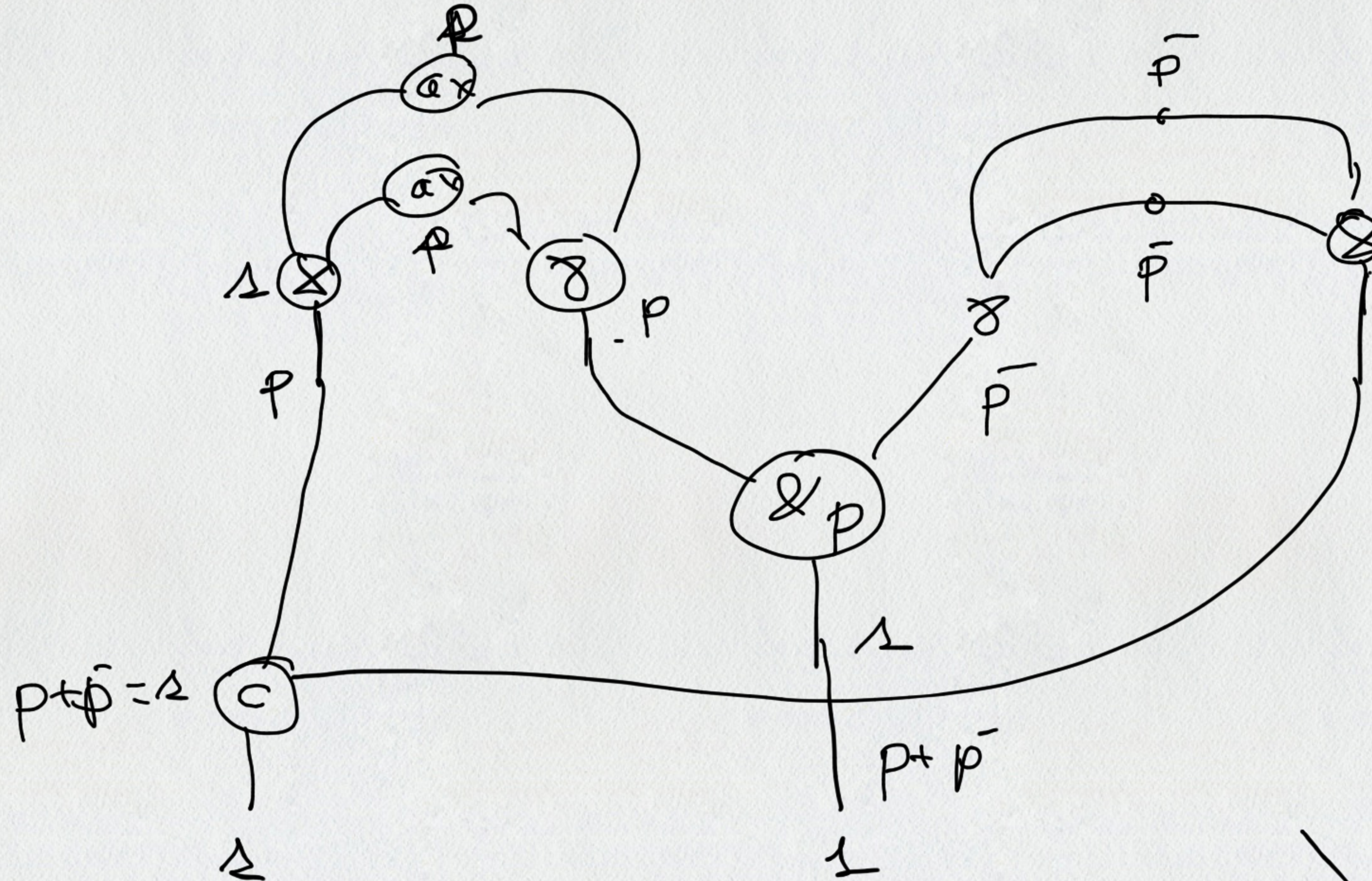
$$\underline{w} = wq + w\bar{q}$$
$$w(q+\bar{q})$$
$$\quad 1$$

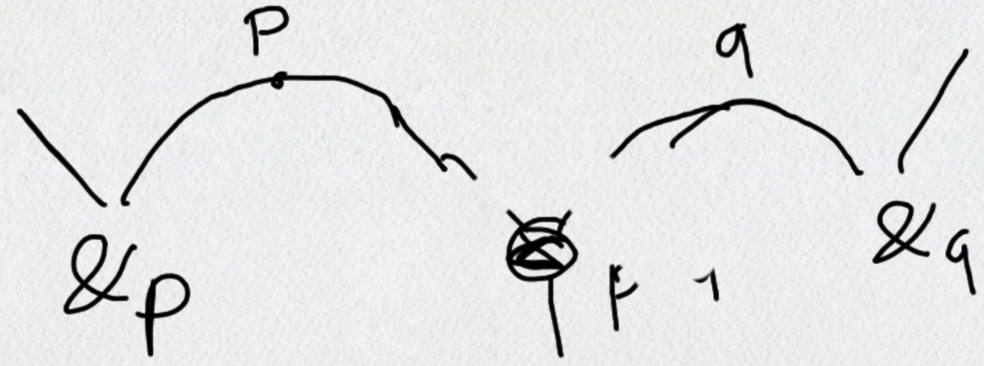
$\boxed{p+\bar{p}}$

$p \vee \neg p = 1$

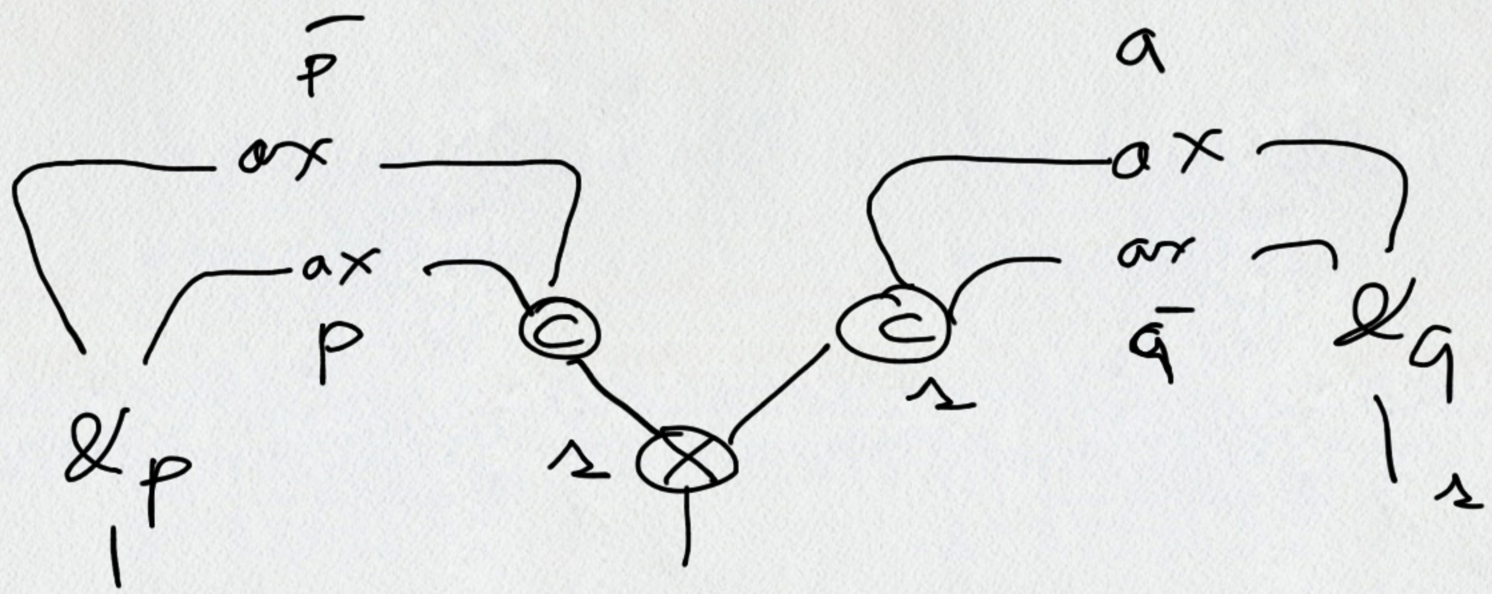
$\boxed{p \cdot \bar{p}}$

$p \wedge \neg p = 0$

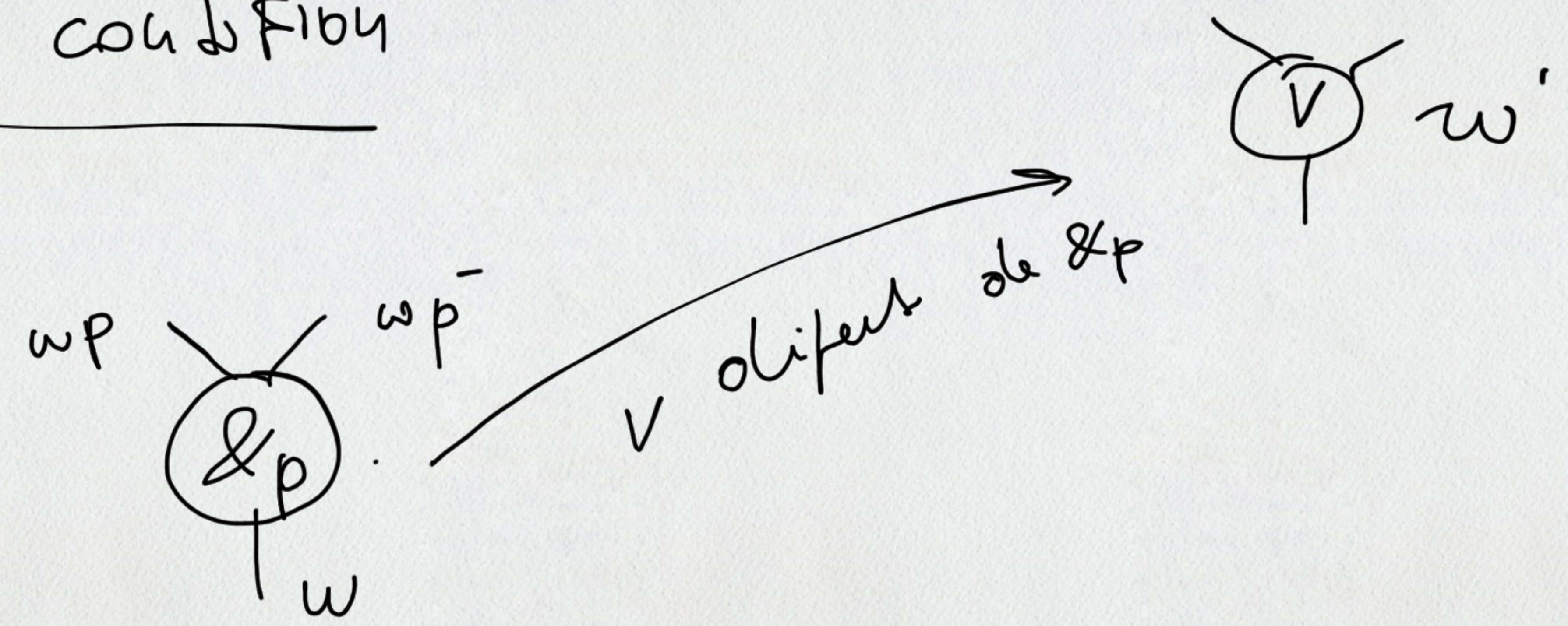




ex.



dependency condition



def. dipendenza

w' dipende da p se p o $\bar{p}$ occorre in w'

se v dipende da &p allora i pesi w' di v e w di &p sono nella seguente relazione $w' \leq w$

$w' = 1$

$w = 0$

$1 \nleq 0$

$w' \nleq w$

$qp < p$

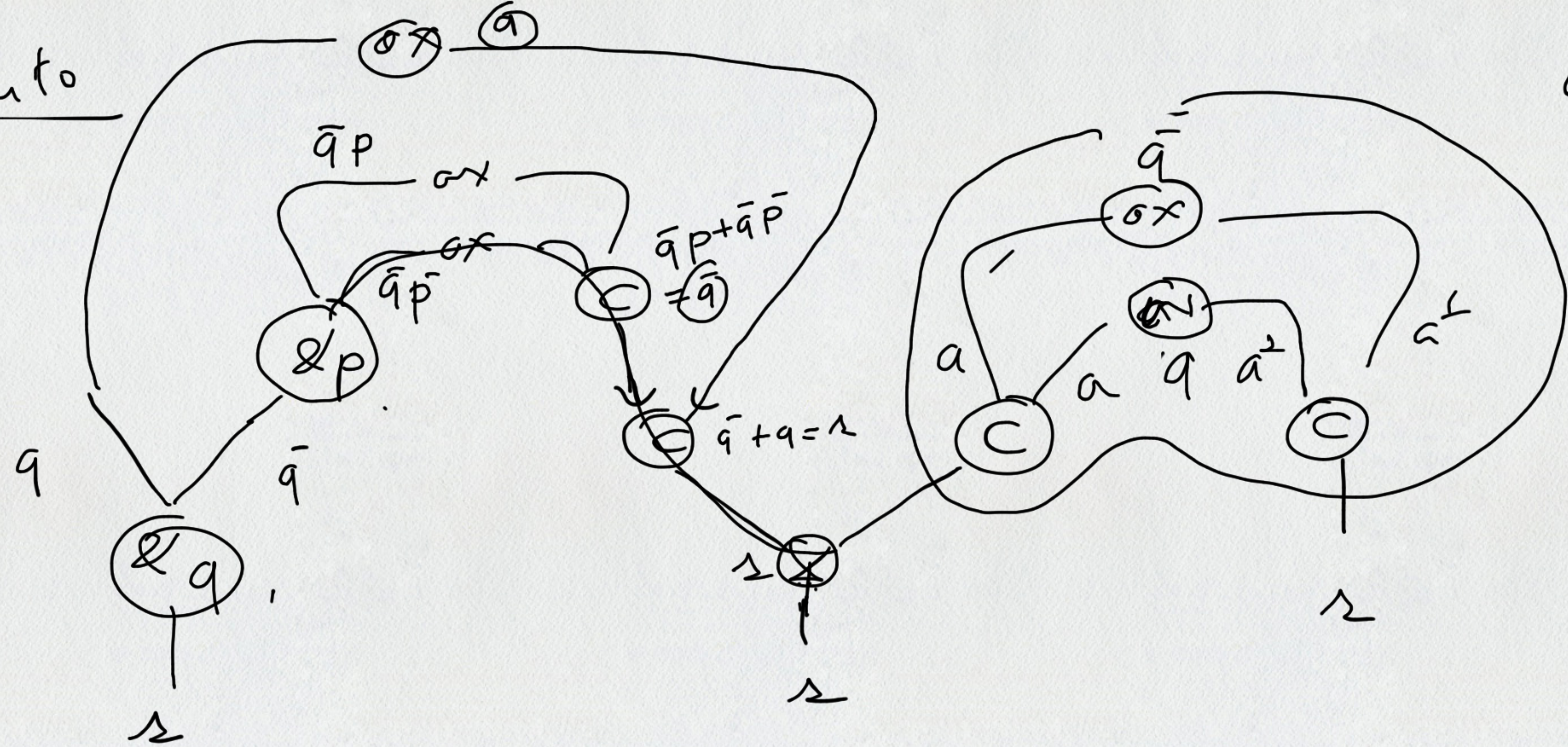
$1 \cdot 1 < 1$

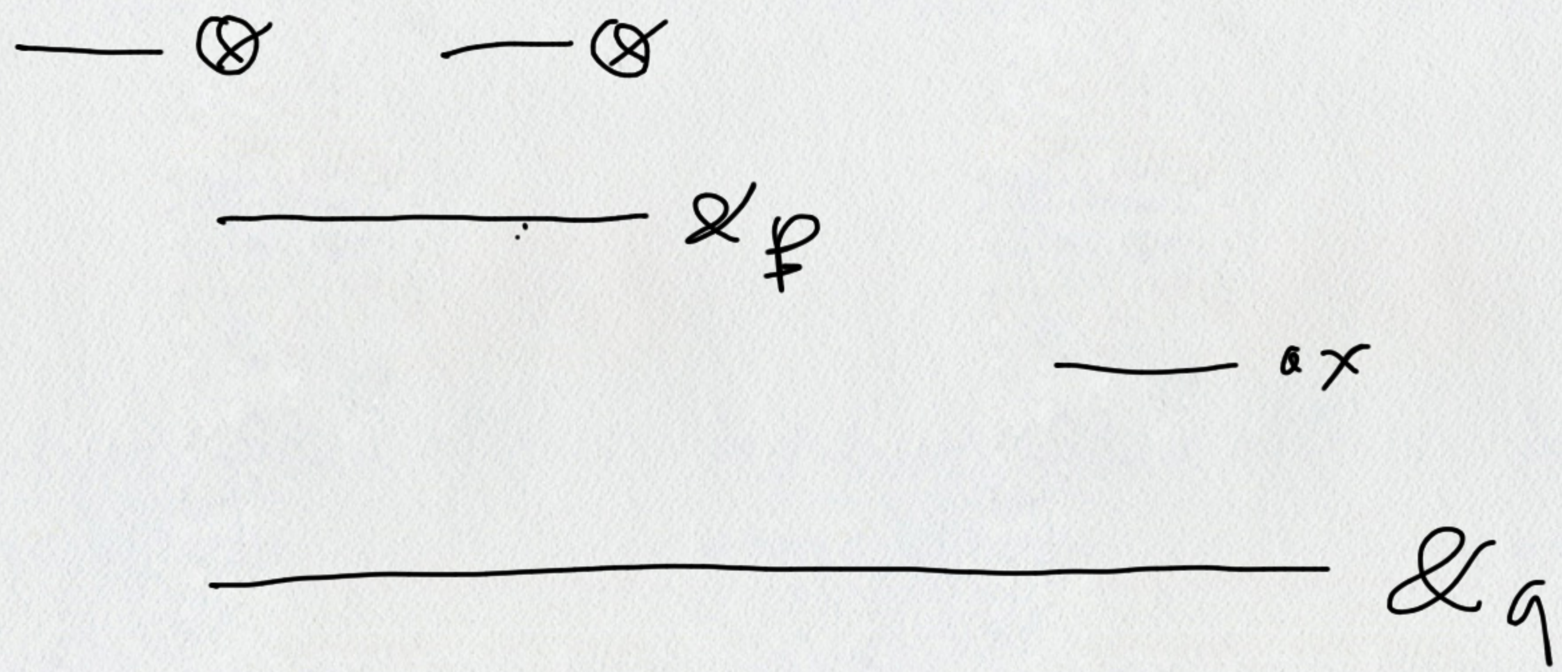
$q = 0$ $\quad 0 < 1$

w'

Esempio concreto

Esempio



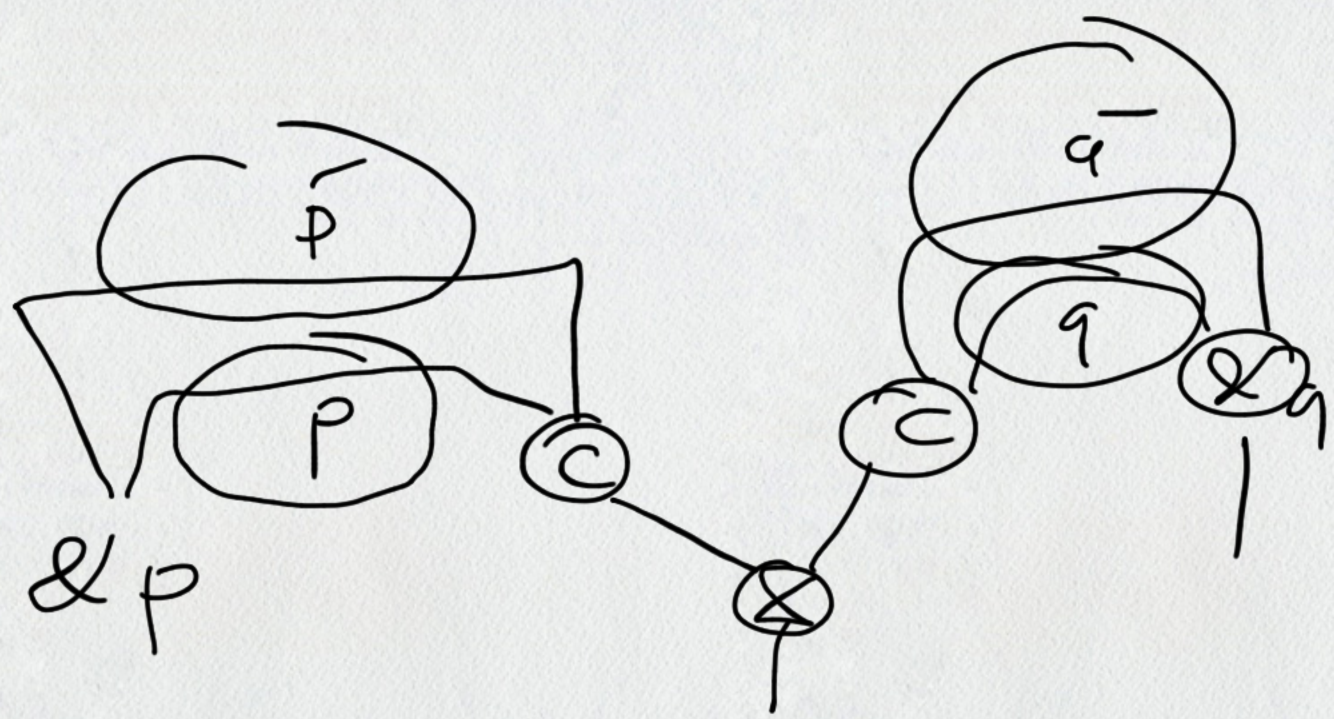


Criterio di correttezza

π proof structure di MALL

- valutazione booleana $\varphi : p \mapsto \{0,1\}$ es $\varphi(p) = 1 \Rightarrow \varphi(\bar{p}) = 0$
- $\varphi(\pi)$ produce un sottografo non nullo ($\neq 0$) chiamato SLICE di π $\quad \varphi : \&_p \mapsto \{0,1\}$

Es.

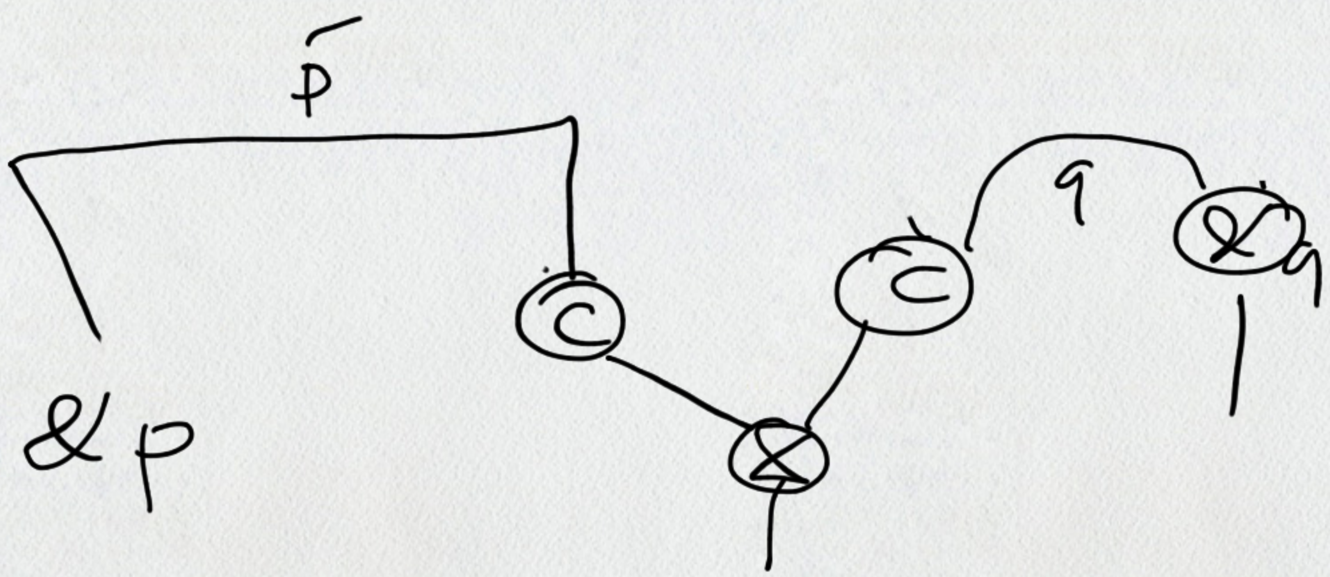


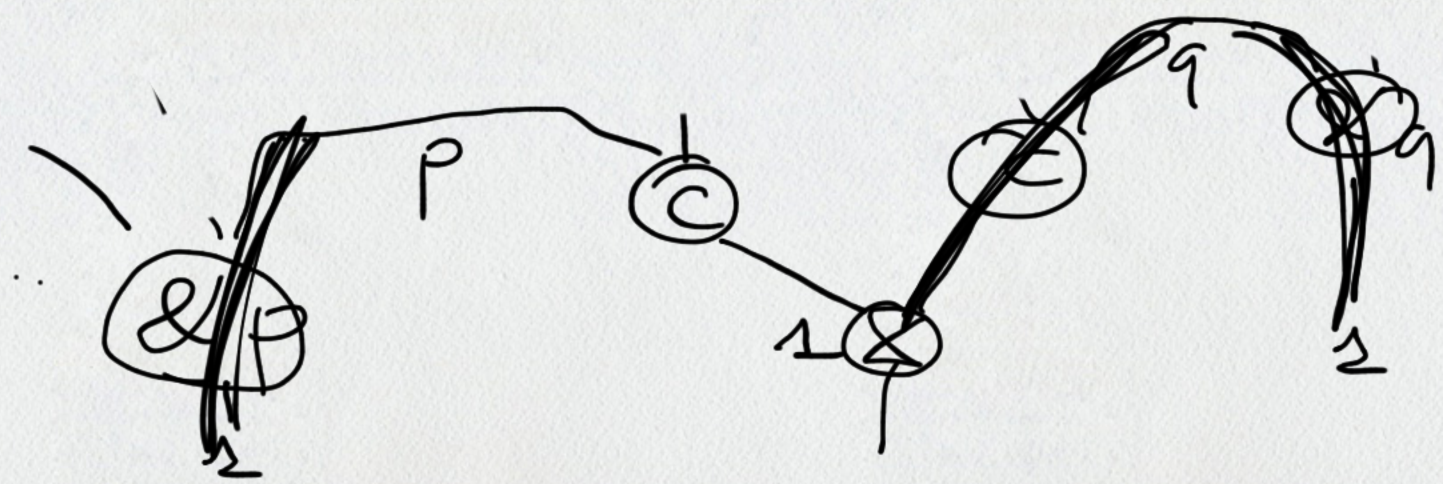
$$\varphi : \begin{cases} p \to 1 \\ \bar{p} \to 0 \\ q \to 1 \\ \bar{q} \to 0 \end{cases}$$

slice $\varphi(\pi)$

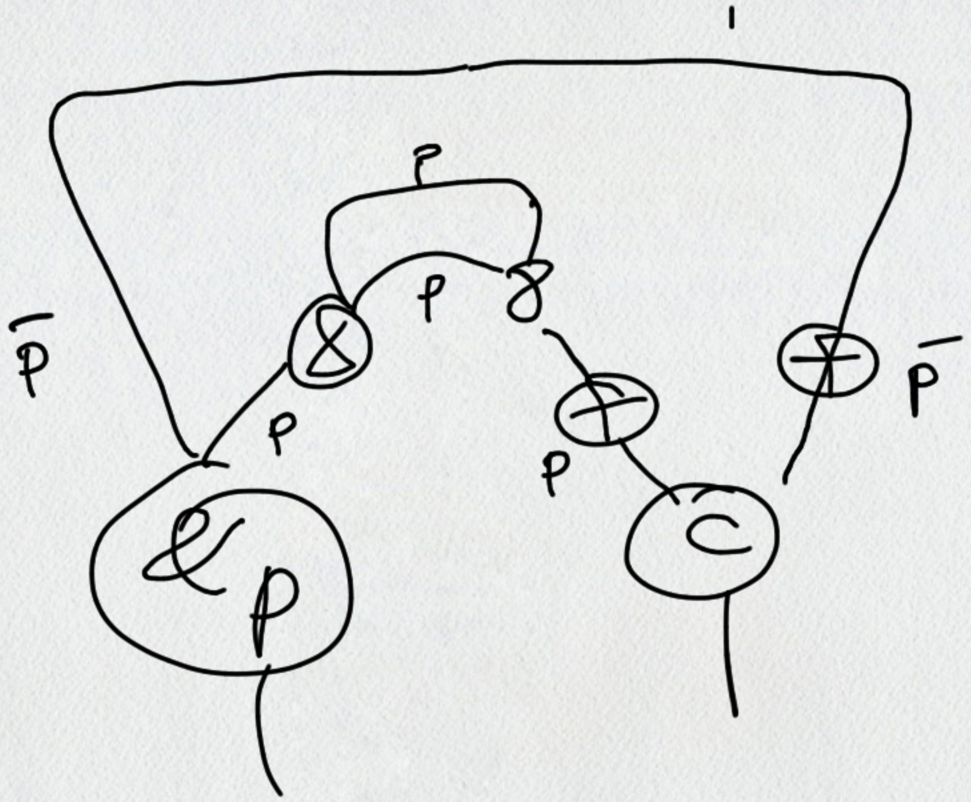
↓

$$\varphi' = \begin{cases} p \to 0 \\ \bar{p} \to 1 \\ q \to 1 \\ \bar{q} \to 0 \end{cases}$$





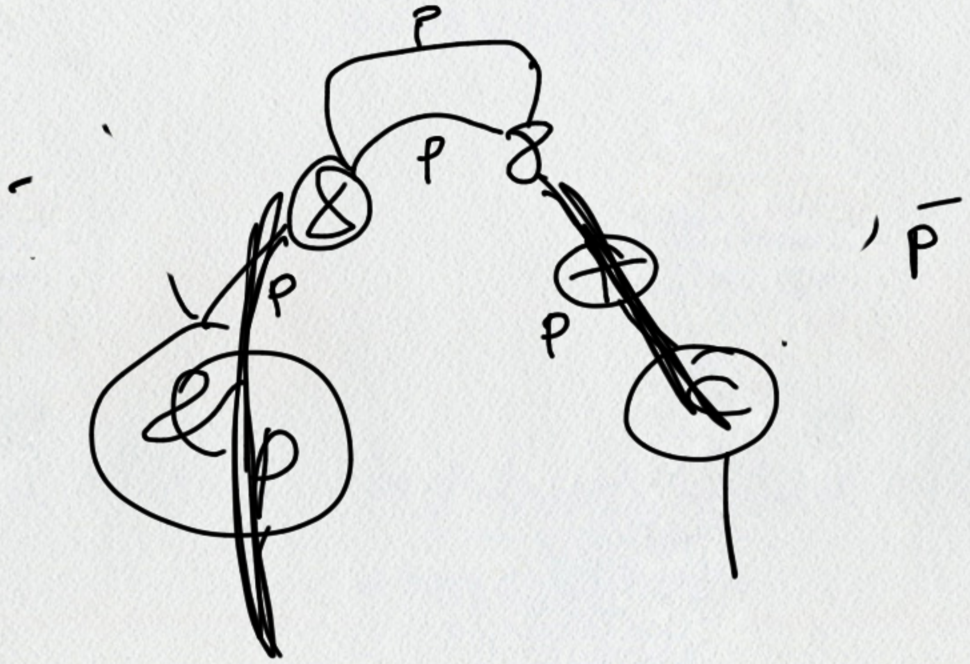
Π

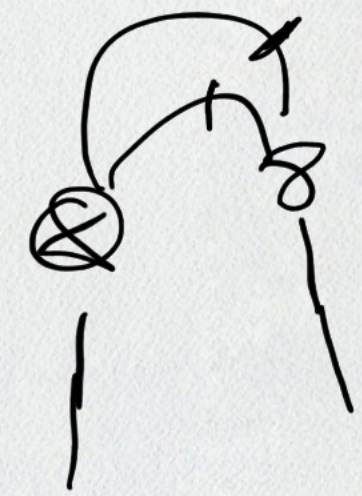


$$\varphi \begin{cases} p = 1 \\ \bar{p} = 0 \end{cases}$$

sile $\varphi(\Pi)$

$\varphi(\Pi)$:



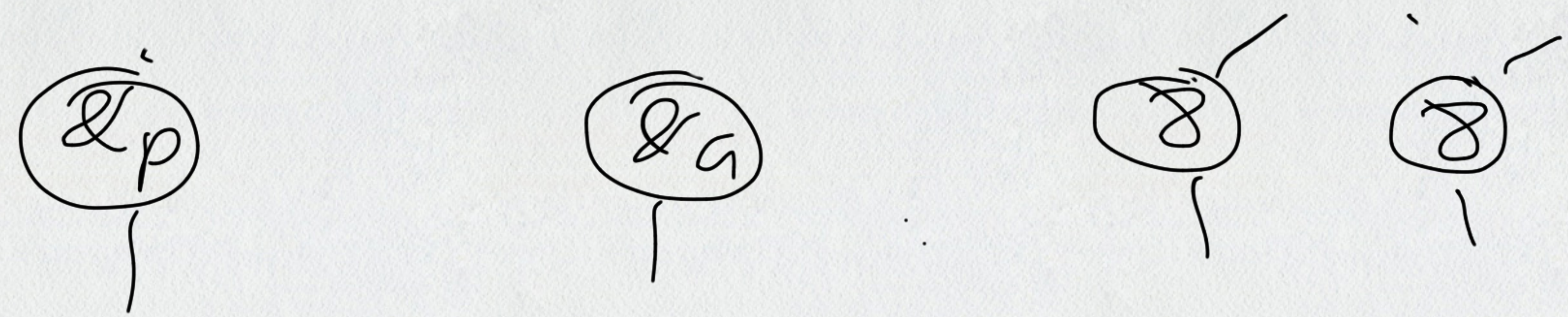


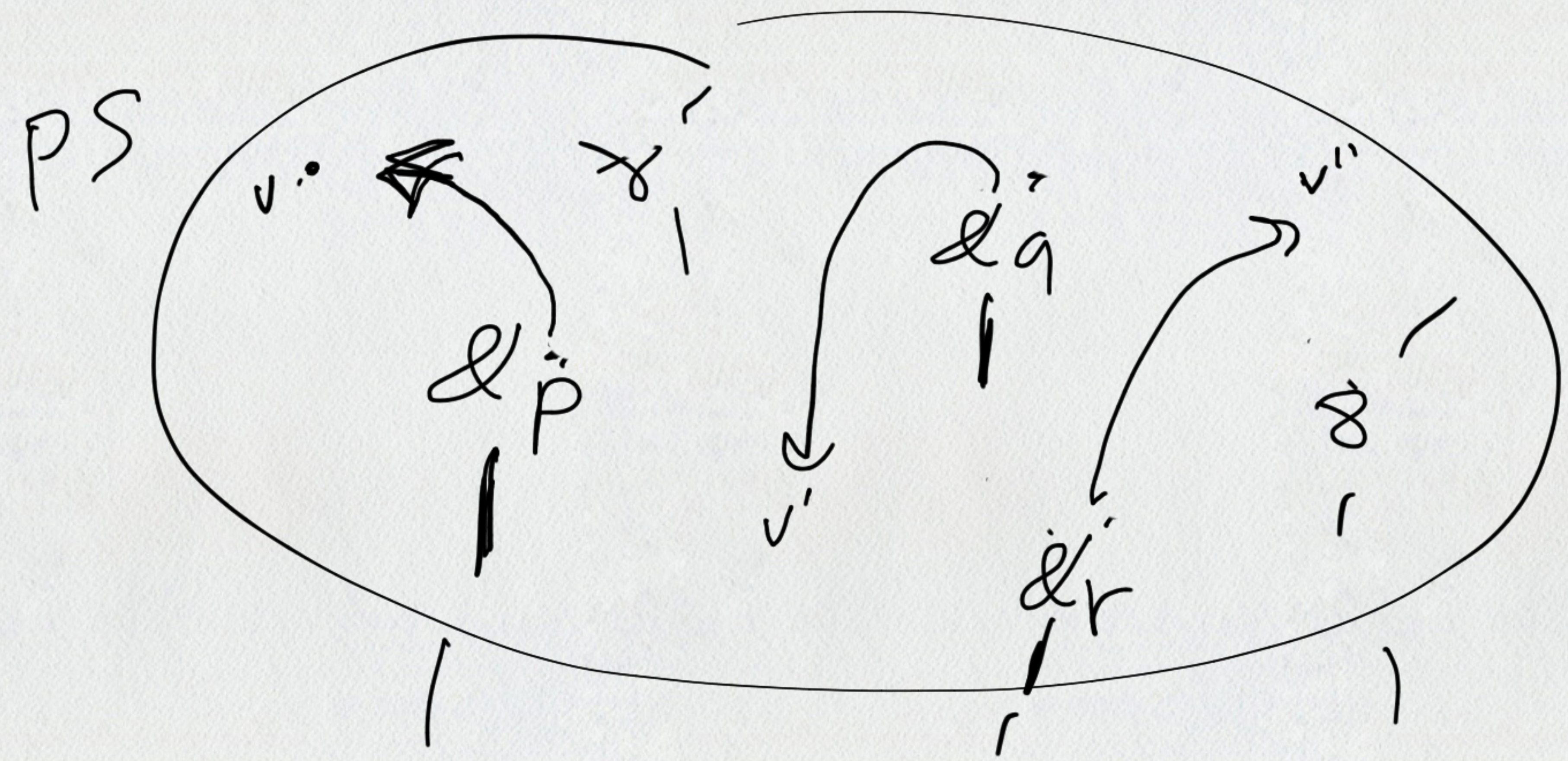
switching moltiplicativo: fissata una valutazione, φ, e considerata la slice φ(Π) indotta da φ

stacco una premessa di ogni ⅋-link della slice

switching additivo: (switching)

- presa una slice φ(Π) e preso uno switching moltiplicativo
- elimino l'unica premessa attaccata ai &-link del nodo &p vero e aggiungo un jump (salto, arco) da un qualsiasi nodo v dipendente da p



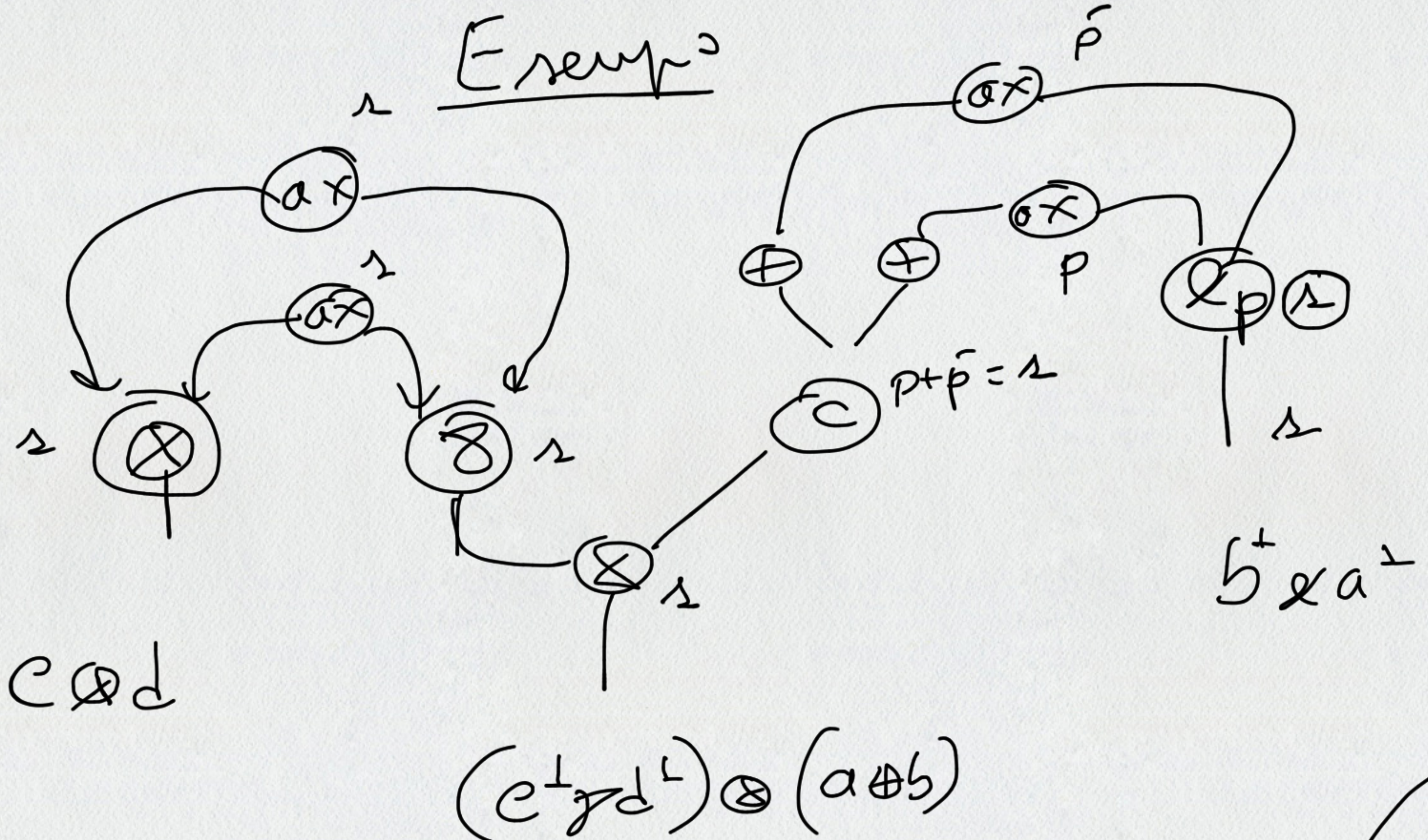


$$\varphi \begin{cases} p = q = r = 1 \\ \bar{p} = \bar{q} = \bar{r} = 0 \end{cases}$$

se ogni switching additivo è ACC $\Rightarrow$ la PS è corretta.

Def. <u>proof net di MALL</u>: una proof structure Π di concl. Γ è corretta (è una PN) <u>sse</u> $\forall$ valutazione φ, ogni additive switching di $\varphi(\Pi)$ (slice) è A.C.C.

Esempio



$p \leq 1$

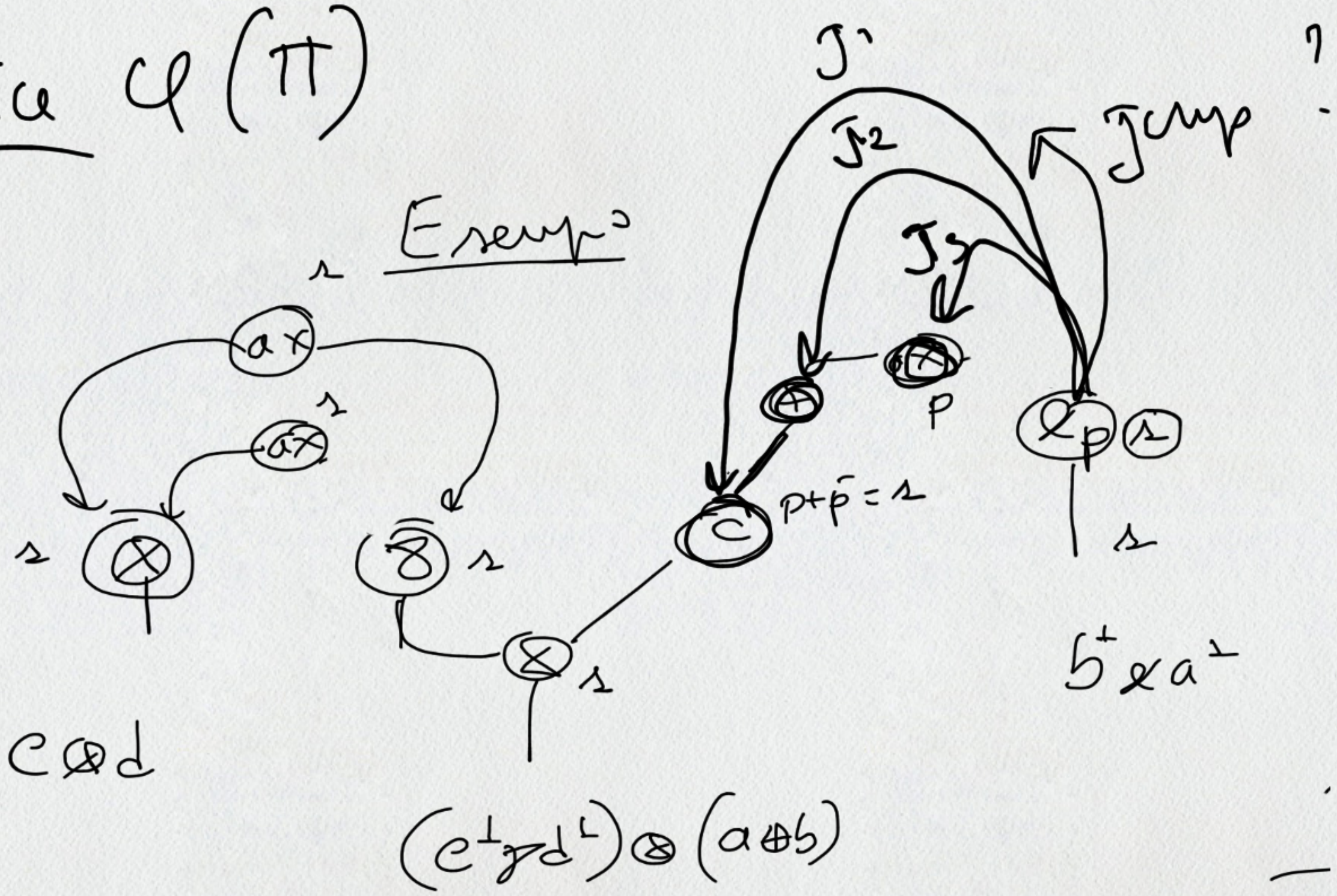
$\bar{p} \leq 1$

$p+\bar{p} \leq 1$

$$\frac{\varphi : p \to 1}{\varphi' : p \to 0}$$

slice $\varphi(\pi)$

Esempio

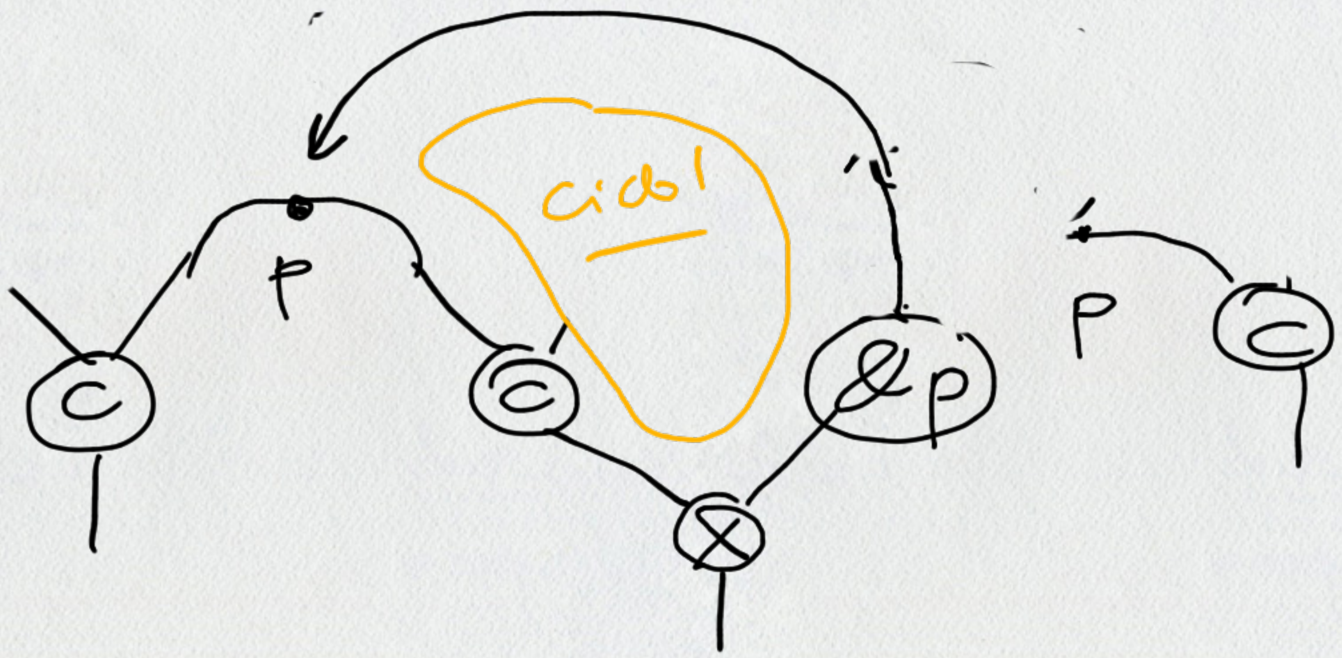


ACC

Controesempio PS non corretta

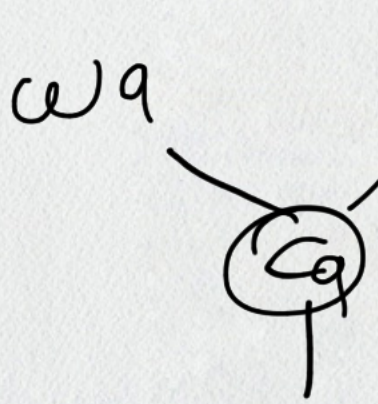
$\varphi : p \to s$

$\varphi(\pi)$ slice

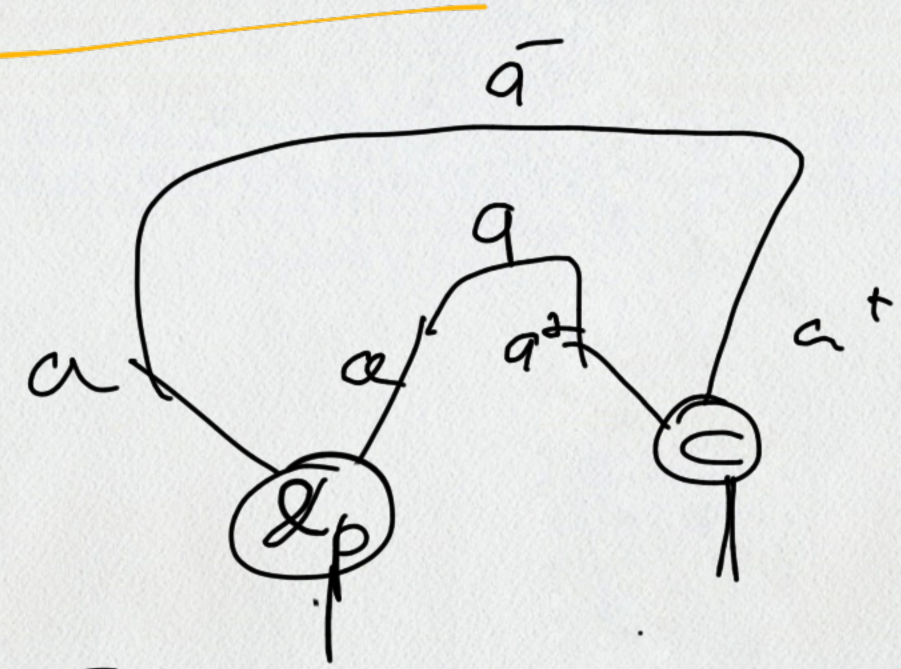


Ace

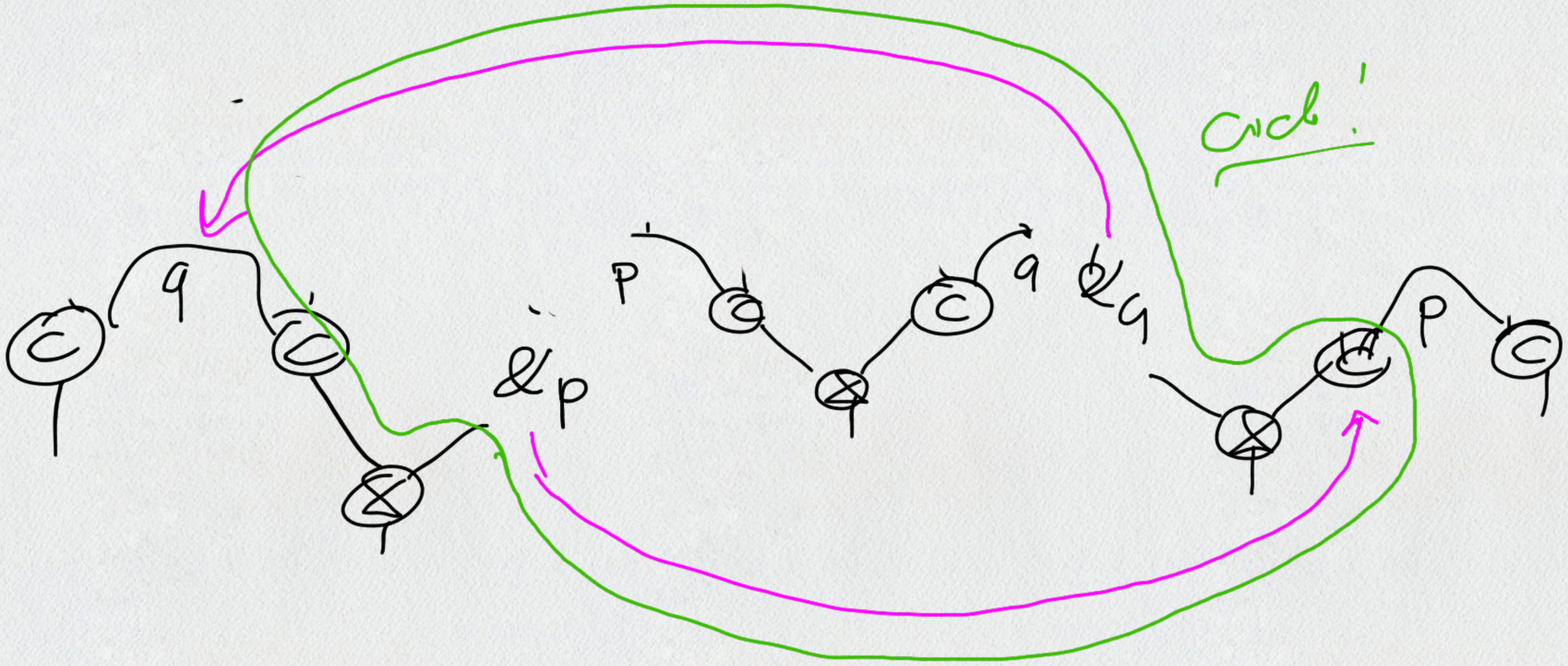
non è corretta!



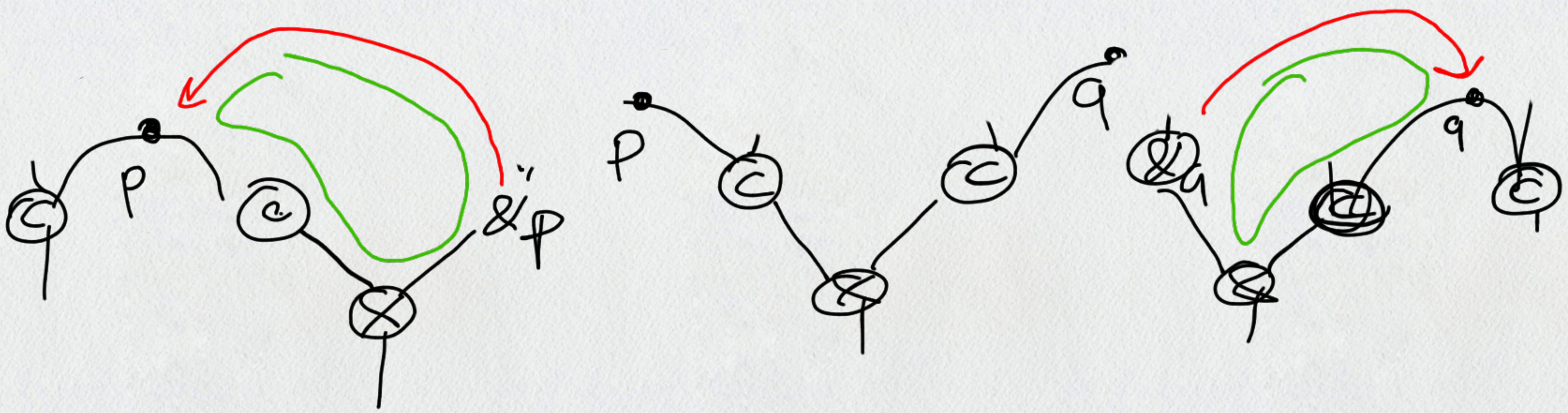
$\omega q + \omega \bar{q} = \omega$



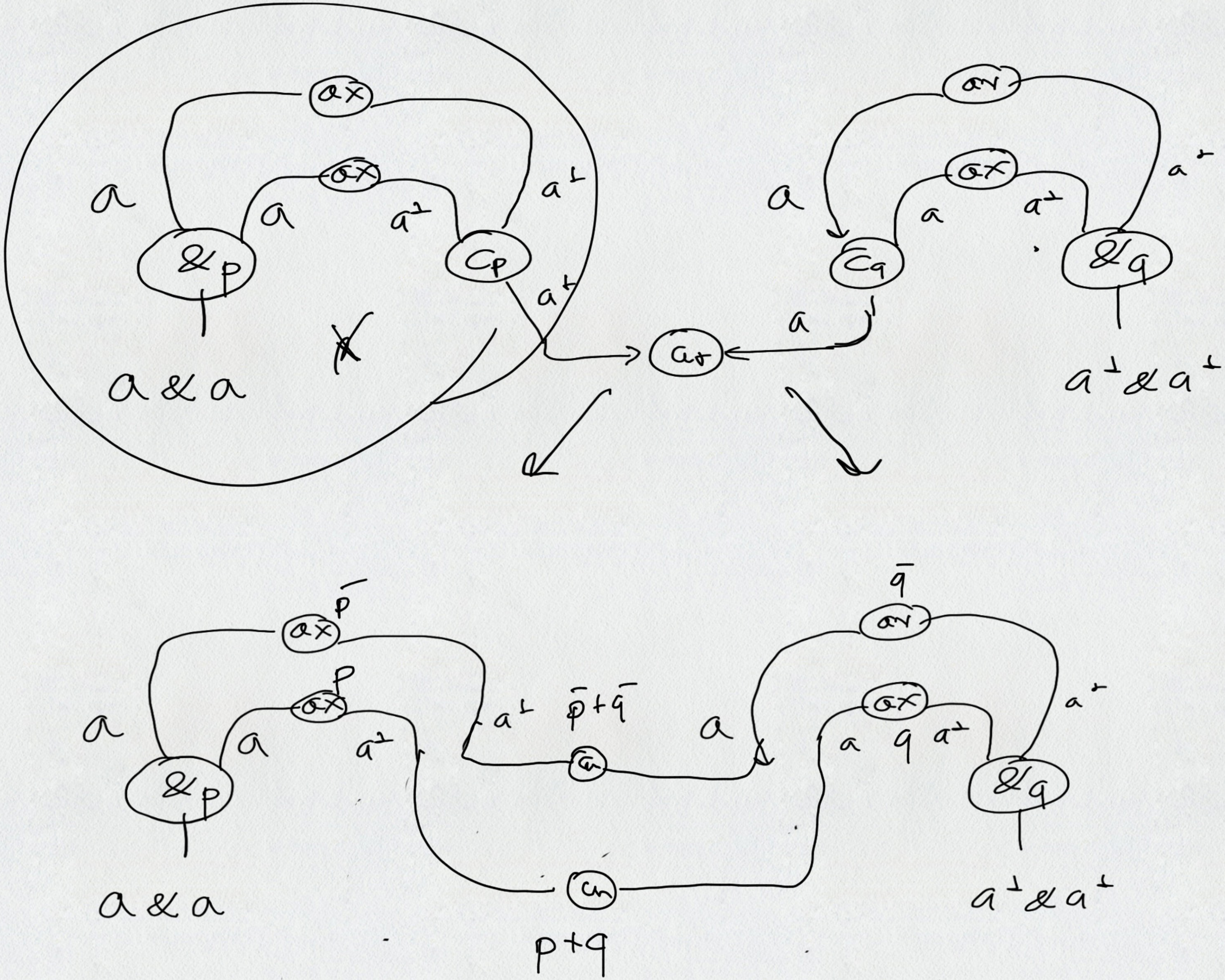
$$\dfrac{\dfrac{\bar{q}}{\vdash a, a^{\perp}} \qquad \dfrac{q}{\vdash a, a^{\perp}}}{a \& a,\ a^{\perp}}\ \&_p$$



$$\varphi \left\{ p = q = r \right.$$



$$\varphi : p = q = r$$



a
a
a⊥
a⊥
&p
Cp
a⊥
a & a
a
Cq
&q
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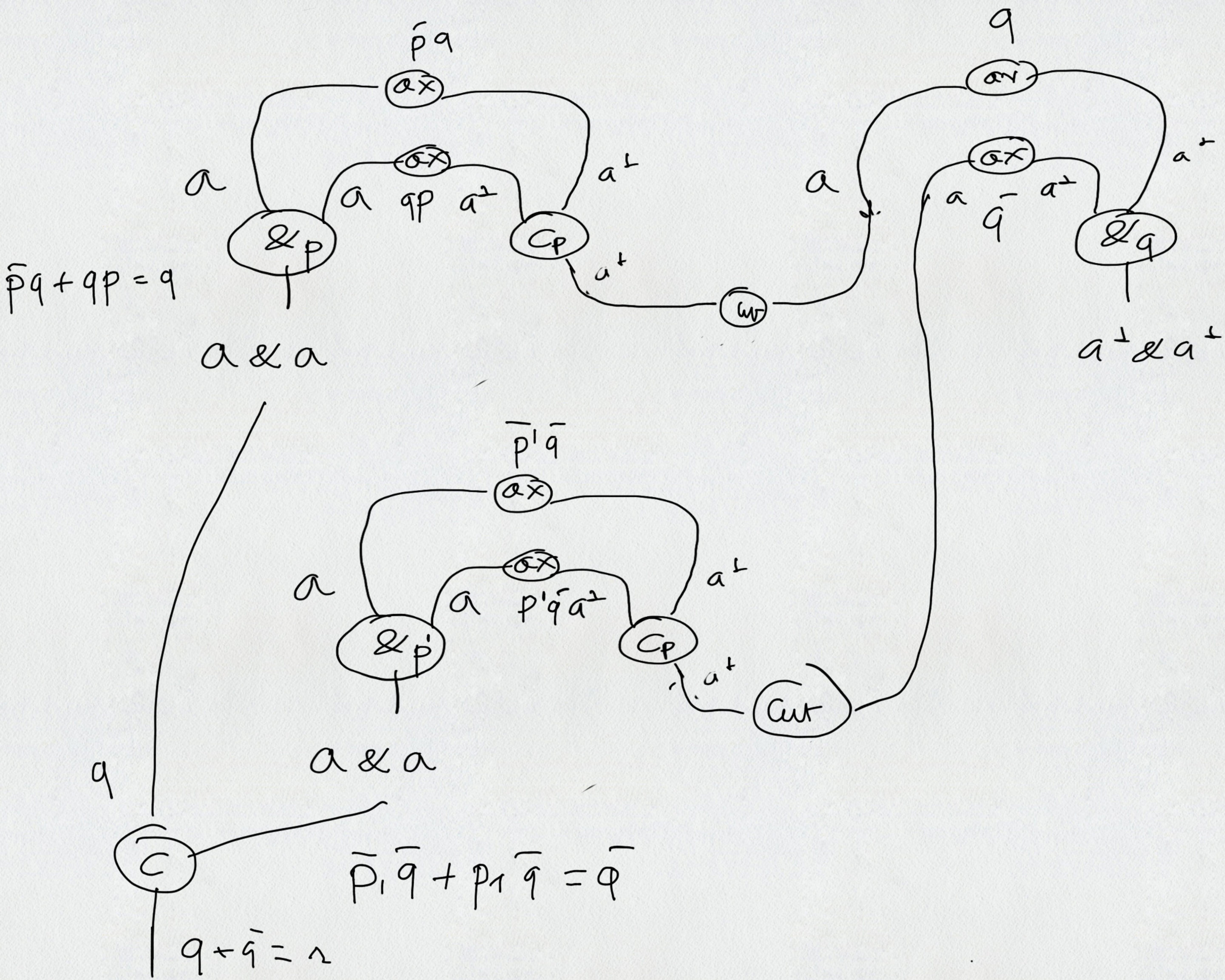
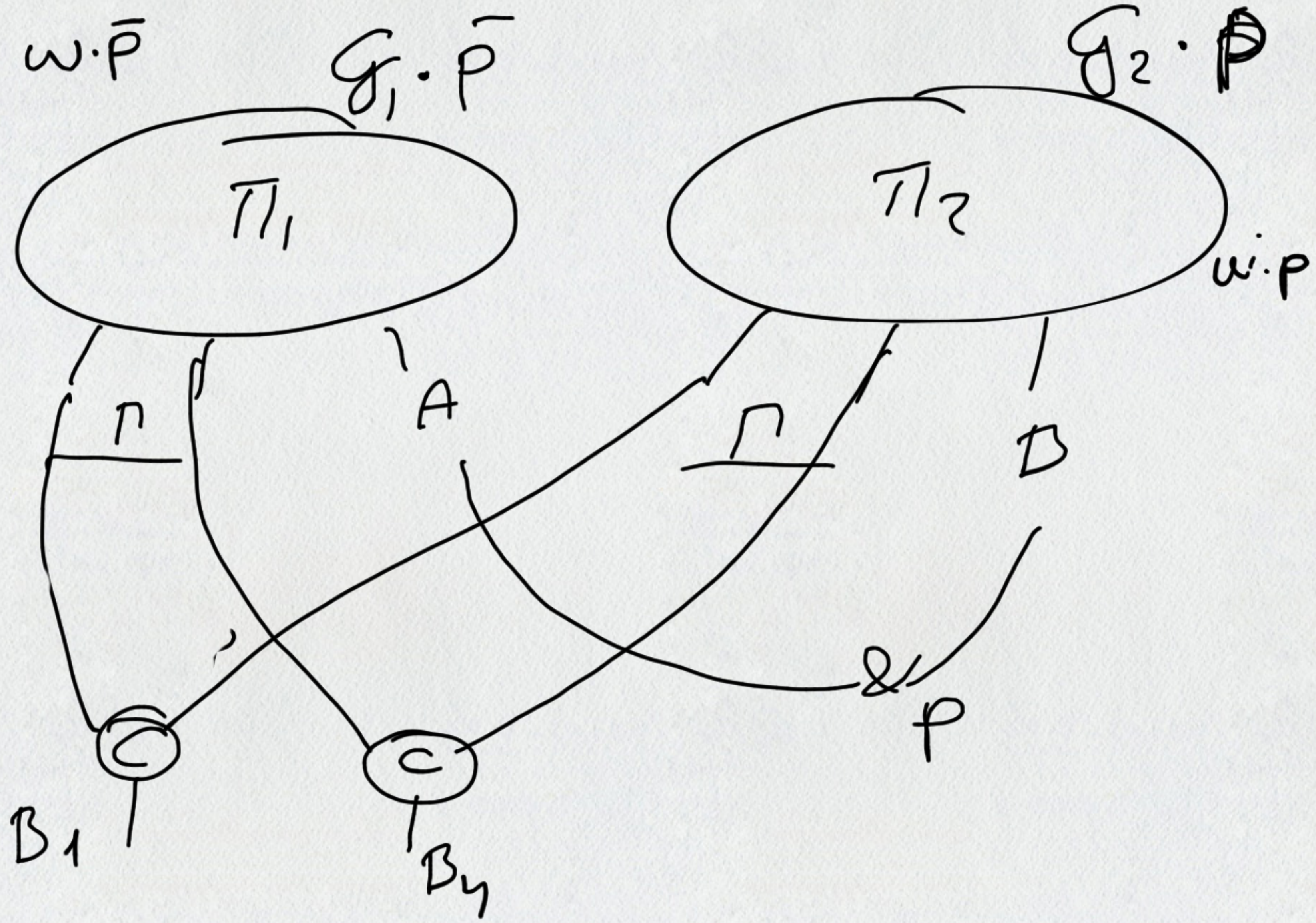
a⊥
a⊥ & a⊥
p̄
p
a
a
a⊥
a⊥
p̄+q̄
a
q̄
a
q
a⊥
a⊥
&p
&q
a & a
p+q
a⊥ & a⊥

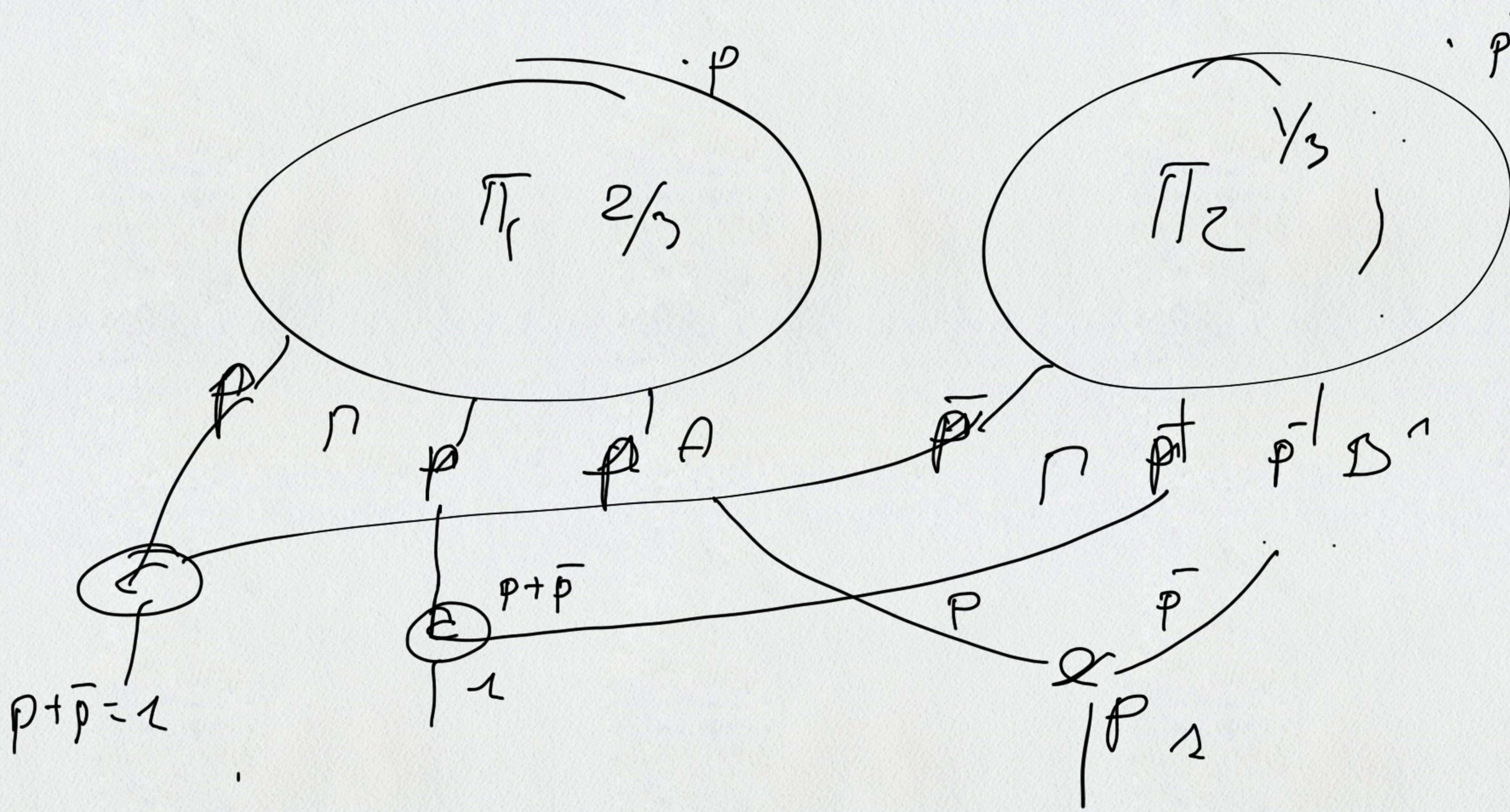


p̄q
ax
a
a
qp
a⊥
&p
Cp
p̄q + qp = q
a & a
cut
q
ax
ax
a
a
q̄
a⊥
&q
a⊥ & a⊥
p̄'q̄
ax
ax
a
a
p'q̄a⊥
&p'
Cp
cut
a & a
q
C̄
p̄₁q̄ + p₁q̄ = q̄
q + q̄ = 1

$$\cfrac{\cfrac{\Gamma, A \qquad \Gamma, B}{\Gamma, A \& B}}{\Gamma, A \& B} \subseteq$$

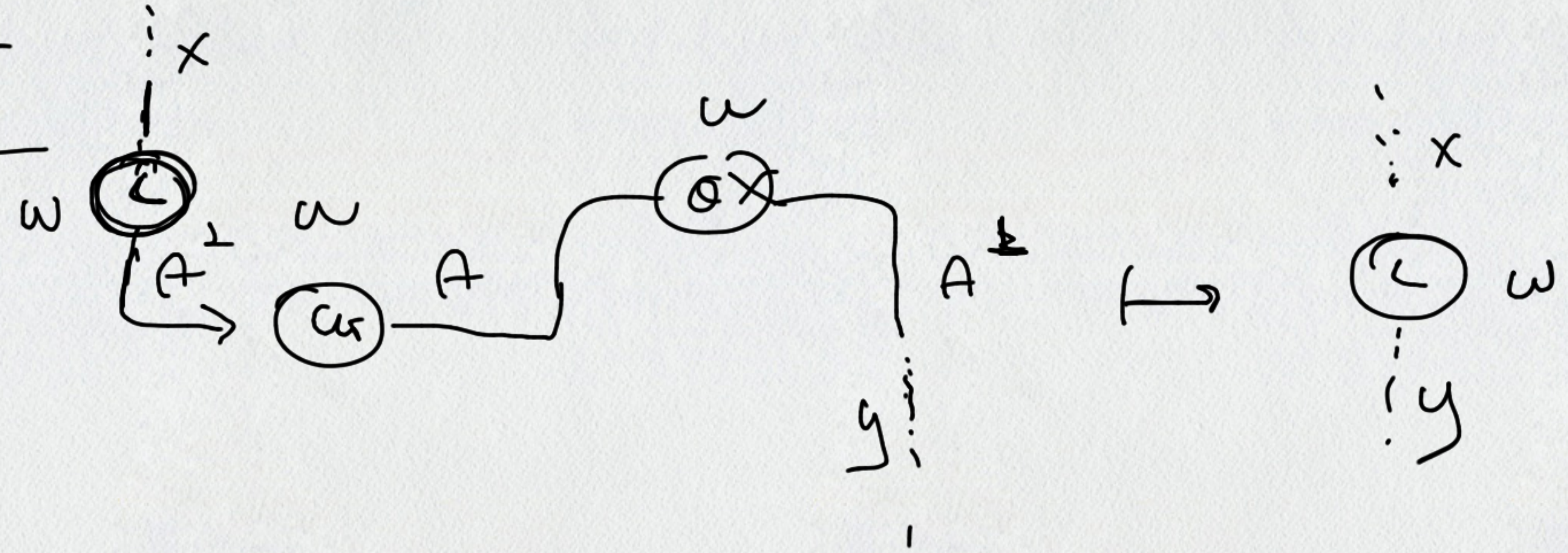


$\Gamma = B_1 \ldots B_n$



Riduzione del cut

cut axiome



taglio logico