3° escluso

$\Gamma \vdash \Delta$

$A_1 \dots A_n \vdash B_1 \dots B_m$

$A_1 \wedge \dots \wedge A_n \vdash B_1 \vee \dots \vee B_m$

$A \vee \neg A$

$\vdash B$

$\vdots$

$$\frac{\vdash A}{\not\vdash A \vee \neg A} \;?$$

in LJ

opp

$\vdots$

$$\frac{\vdash \neg A}{\vdash A \vee \neg A}$$

LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,ax}{\vdash A, \neg A}\,\neg R}{\vdash A \vee \neg A, \neg A}\,\vee R^1}{\vdash A \vee \neg A, A \vee \neg A}\,\vee R^2}{\vdash_{LK} A \vee \neg A}\,CR$$

$\vdash A$ alh $\vdash A \vee \neg A$ $\qquad \vdash \neg A \Rightarrow \vdash A \vee \neg A$,

$A \vdash_{LJ} A \vee \neg A \qquad \neg A \vdash_{LJ} A \vee \neg A$

$\emptyset \vdash A \vee \neg A$ in LJ !

LK

$$\dfrac{}{A \vdash A}\ ax$$

$$\dfrac{\Gamma \vdash \Delta, \overset{\phi}{A} \qquad A, \Gamma' \vdash \overset{B}{\Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ cut$$

Logica Classica (LK)
(calcolo dei sequenti Gentzen 1935)

additiva

$$\dfrac{\Gamma \vdash \cancel{\Delta}, A \qquad \Gamma \vdash \cancel{\Delta}, B}{\Gamma \vdash \cancel{\Delta}, A \wedge B}\ \wedge R$$

$$\dfrac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ \wedge^1_L \qquad \dfrac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ \wedge^2_L$$

$$\dfrac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B}\ \vee^1_R \qquad \dfrac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \vee B}\ \vee^2_R$$

$$\dfrac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ \vee_L$$

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \Rightarrow B, \Delta}\ \Rightarrow R \qquad \dfrac{\Gamma \vdash \Delta, A \qquad \Gamma, B \vdash \Delta}{\Gamma, A \Rightarrow B \vdash \Delta}\ \Rightarrow L$$

$$\dfrac{\Gamma \vdash A, \Delta}{\neg A, \Gamma \vdash \Delta}\ \neg L \qquad \neg R\ \dfrac{\Gamma, A \vdash \Delta}{\Gamma \vdash \Delta, \neg A}$$

STRUTTURALI

$$\dfrac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A}\ WR \qquad \dfrac{\Gamma \vdash \Delta}{A, \Gamma \vdash \Delta}\ WL$$

$$\dfrac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}\ CR \qquad \dfrac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\ C_L$$

$$\dfrac{\Gamma', \Gamma'' \vdash \Delta', \Delta''}{\Gamma'', \Gamma' \vdash \Delta'', \Delta'}\ scambio$$

Logica Intuizionista (LJ)

$$\Gamma \vdash B$$

a destra al massimo 1 formula

Le dimostrazioni in LK/LJ sono alberi i cui nodi sono etichettati da sequenti di LK

foglie ○ ... ○ ... ○ assiomi $\overline{A \vdash A}$ regole 0-arie

regole unarie

S_1 ○ $\Gamma \vdash \Delta$ premesse

S ○ $\Gamma' \vdash \Delta'$ conclusione

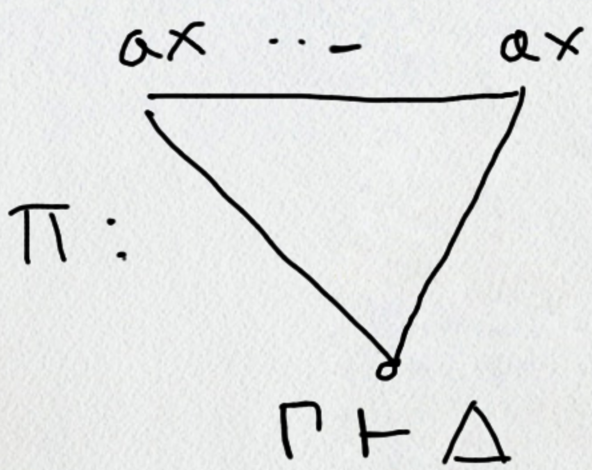
regole binarie

$\Gamma' \vdash \Delta'$ $\Gamma'' \vdash \Delta''$

S_1 ○ ○ S_2 premesse

S ○ $\Gamma \vdash \Delta$ conclusione

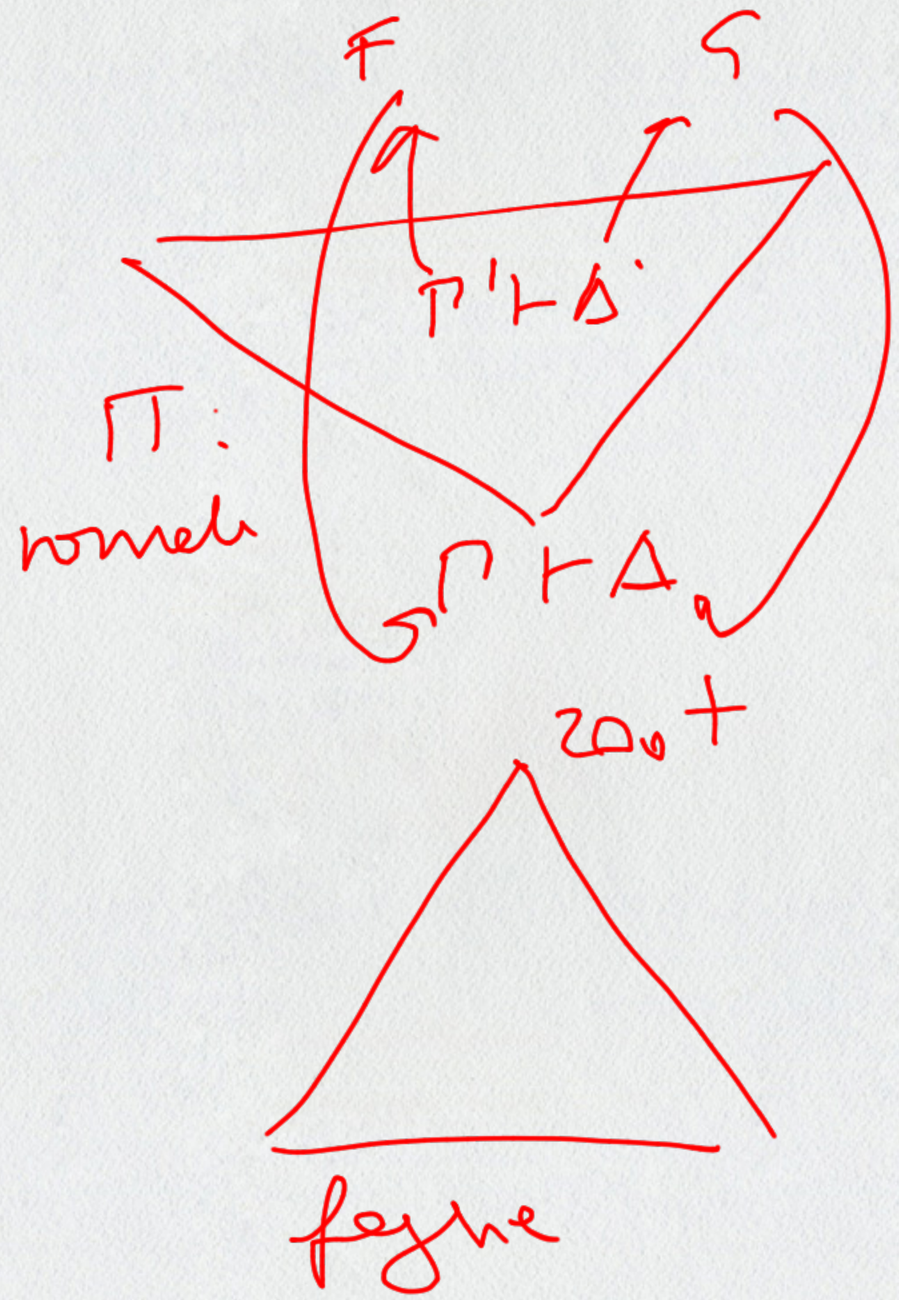
dimostrazioni



$\neg A = B$

$B = A \bullet C$

$\bullet \in \{\wedge, \vee, \Rightarrow\}$



Teorema di Eliminazione del taglio (cut) per LK (Gentzen 1934-35)

La regola del Taglio (cut) è eliminabile dal sistema LK (LJ), ovvero per ogni dimostrazione Π di $\Gamma \vdash \Delta$ in LK è possibile ottenere in modo effettivo (algoritmico) una nuova dimostrazione Π' di $\Gamma \vdash \Delta$ che non contiene alcuna applicazione della regola del Taglio -

$\vdash A \quad \vdash \neg A$

$$\boxed{\dfrac{\vdash A \qquad A \vdash}{\vdash \bot}} \text{cut} \qquad \exists, \forall$$

Conseguenze della Eliminazione del Taglio

- Proprietà delle sottoformule: se Π è una dimostrazione cut-free di $\Gamma \vdash \Delta$ allora in ogni sequente $\Gamma' \vdash \Delta'$ che etichetta un nodo dell'albero della dimostrazione, ogni formula $F \in \Gamma'$ oppure $F \in \Delta'$ è sottoformula di qualche formula della conclusione $\Gamma \vdash \Delta$.
- LK è consistente (non-contraddittorio) perché non è dimostrabile il sequente vuoto "$\vdash \bot$"

$\bot$ assurdo. $A \wedge \neg A$

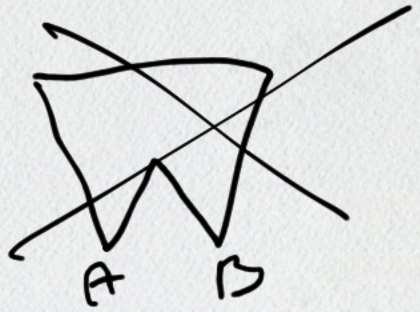
$A \vdash A$

$$\dfrac{\vdash A \qquad \dfrac{\dfrac{\vdash \neg A}{\neg\neg A \vdash}}{A \vdash}\,\mathrm{RAA}}{\vdash \bot}\,\text{cut}$$

cut free ?

$$\overline{\underline{A} \vdash A} \qquad \cdots \qquad \overline{A \vdash \underline{A}}$$

$$\vdash \bot$$



$\vdash A, B$

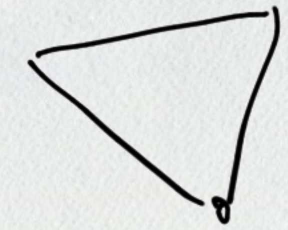
premessi

formule ausiliaria

$$\dfrac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \vee B} \quad R = \vee_R^1$$

formule principale dell'inferenza

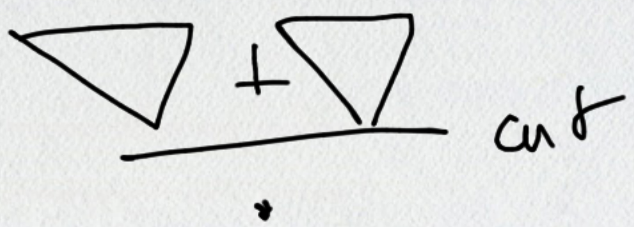
contesto

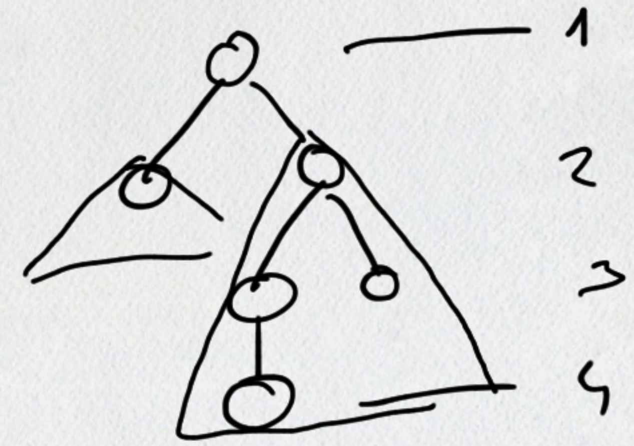


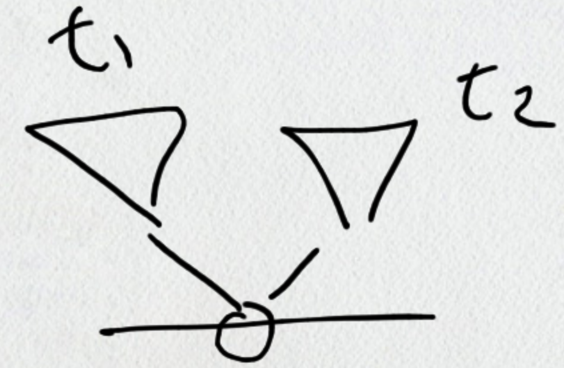
in LK

$\Gamma \vdash \Delta$

$$\dfrac{\overset{\Pi_1}{\Gamma \vdash \Delta, A} \qquad \overset{\Pi_2}{A, \Gamma' \vdash \Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ cut(A)$$







profondità

- livello del taglio cut(A) $= h(\Pi_1) + h(\Pi_2)$

- complessità del cut(A) = complessità logica delle formule sul taglio A

  grado del cut

$\max\{h_{T_1}, h_{T_2}\} + 1$

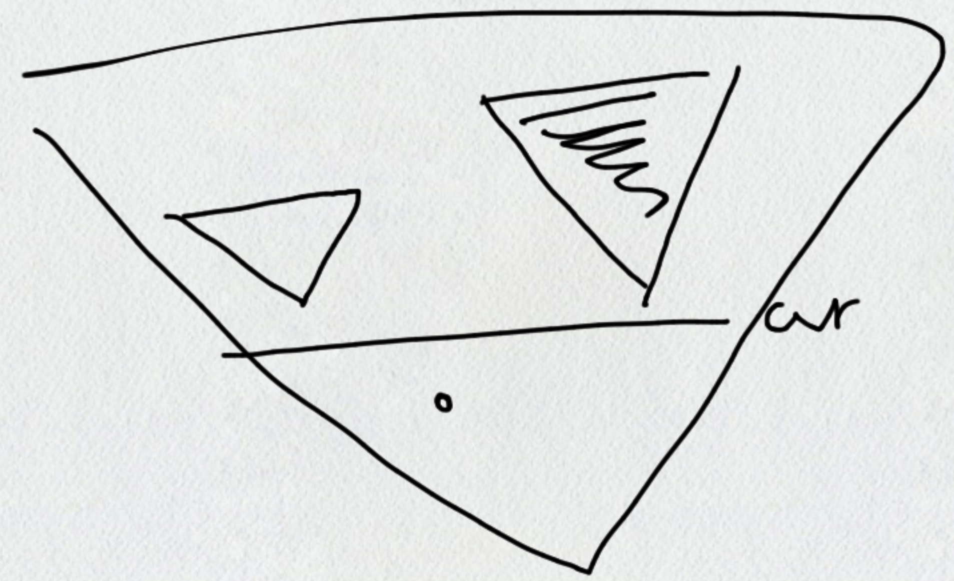
- complessità logica $A = \begin{cases} \text{se } A \text{ è atomica } 1 \\ A = \neg B \Rightarrow \mathbb{C}(A) = \mathbb{C}(B) + 1 \\ A = B \cdot C \Rightarrow \mathbb{C}(A) = \max\{\mathbb{C}(B), \mathbb{C}(C)\} + 1 \end{cases}$

dimostrazione per induzione

- base $\quad 0$
- passo induttivo $\quad n \Rightarrow n+1$

· cut axiome

$$\Pi: \dfrac{\dfrac{}{A \vdash A} \qquad \dfrac{\Pi'}{A, \Gamma \vdash \Delta}}{A, \Gamma \vdash \Delta}\,ax \quad \rightsquigarrow \quad \begin{array}{c}\Pi' \\ \vdots \\ A, \Gamma \vdash \Delta\end{array}$$



· cut-logici

$$\dfrac{\dfrac{\Pi^1}{\Gamma \vdash \Delta, A}\,r \qquad \dfrac{\Pi^2}{A, \Gamma' \vdash \Delta'}\,r}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,cut$$

· cut-strutturale

$$\dfrac{\dfrac{\Pi_1}{\Gamma \vdash \Delta, A} \qquad \dfrac{\Pi_2}{A, \Gamma' \vdash \Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

$$P: \quad \frac{\Gamma \vdash A \qquad A \vdash C}{\Gamma \vdash C}$$

$$P \quad \frac{\Gamma \vdash A \qquad A \vdash C}{}$$

$A \vdash A \quad \cdots \quad B \vdash B$

C

Cut logici   cut $(A \Rightarrow B)$

$$\Pi \quad \dfrac{\dfrac{\overset{\Pi_1}{\Gamma, A \vdash B, \Delta}}{\Gamma \vdash \Delta, \underline{A \Rightarrow B}}\ R\Rightarrow \qquad \dfrac{\overset{\Pi_2}{\Gamma' \vdash A, \Delta'} \qquad \overset{\Pi_3}{\Gamma', B \vdash \Delta'}}{\Gamma', \underline{A \Rightarrow B} \vdash \Delta'}\ L\Rightarrow}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \text{cut}\,(A \Rightarrow B)$$

riduco?

$$d(B) < d(A \Rightarrow B)$$
$$d(A) < d(A \Rightarrow B)$$

$$\Pi' \quad \dfrac{\dfrac{\dfrac{\dfrac{\overset{\Pi_1}{\Gamma, A \vdash \underline{B}, \Delta} \qquad \overset{\Pi_3}{\Gamma', \underline{B} \vdash \Delta'}}{\Gamma, \underline{A}, \Gamma' \vdash \Delta, \Delta'}\ \text{cut}(B) \qquad \overset{\Pi_2}{\Gamma' \vdash \underline{A}, \Delta'}}{\Gamma, \Gamma', \Gamma' \vdash \Delta, \Delta', \Delta'}\ \text{cut}(A)}{\text{contrazioni}}\ c^*}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

$$\langle \underline{\mathbb{C}}\,\text{cut}, \text{livello} \rangle$$

caso cut logico $\vee$

$$\Pi \quad \dfrac{\Pi_1 \; \dfrac{\overset{\Pi_1'}{\Gamma \vdash A, \Delta}}{\Gamma \vdash \underline{A \vee B}, \Delta}\, \vee^1 R \qquad \Pi_2 \; \dfrac{\overset{\Pi_2'}{\Gamma', A \vdash \Delta'} \qquad \overset{\Pi_2''}{\Gamma', B \vdash \Delta'}}{\Gamma', \underline{A \vee B} \vdash \Delta'}\, \vee L}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, cut\,(A \vee B)$$

$$\downarrow$$

$$\Pi' \quad \dfrac{\overset{\Pi_1'}{\Gamma \vdash \underline{A}, \Delta} \qquad \overset{\Pi_2'}{\Gamma', \underline{A} \vdash \Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, cut(A) \qquad d(A) < d(A \vee B)$$

base dell'ind.

$$\Pi \quad \dfrac{\dfrac{}{A \vdash A} \qquad \overset{\overset{\Pi'}{\vdots}}{A, \Gamma \vdash \Delta}}{A, \Gamma \vdash \Delta}\, cut(A) \rightarrow \Pi' \; \overset{\vdots}{A, \Gamma \vdash \Delta}$$

atom

Caso cut logico $\wedge$

$$\dfrac{\dfrac{\overset{\pi_1}{\Gamma\vdash\Delta,A}\qquad \overset{\pi_2}{\Gamma\vdash\Delta,B}}{\Gamma\vdash\Delta,\underline{A\wedge B}}\wedge_R \qquad \dfrac{\overset{\pi_3}{\Gamma',A\vdash\Delta'}}{\Gamma',\underline{A\wedge B}\vdash\Delta'}\wedge_L'}{\Gamma,\Gamma'\vdash\Delta,\Delta'}\,\mathrm{cut}(A\wedge B)$$

$\downarrow$ riduco

$$\dfrac{\overset{\pi_1}{\Gamma\vdash\Delta,A}\qquad \overset{\pi_3}{\Gamma',A\vdash\Delta'}}{\Gamma,\Gamma'\vdash\Delta,\Delta'}\,\mathrm{cut}(A) \qquad d(A)<d(A\wedge B)$$

ip. ind

$$\dfrac{\dfrac{\Gamma,A\vdash\Delta}{\Gamma\vdash\Delta,\neg A}\qquad \dfrac{\Gamma'\vdash\Delta',A}{\Gamma',\neg A\vdash\Delta'}}{\Gamma,\Gamma'\vdash\Delta,\Delta'}\,\mathrm{cut}(A) \Longrightarrow \dfrac{\dfrac{\Gamma'\vdash\Delta',A\qquad \Gamma,A\vdash\Delta}{\Gamma',\Gamma\vdash\Delta',\Delta}}{\Gamma,\Gamma'\vdash\Delta,\Delta'}\,\text{ex-scambio}$$

Caso tagli strutturali

$$\Pi: \quad \dfrac{\Pi_1 \quad \dfrac{\begin{array}{c}\Pi_1' \\ \Gamma, B \vdash C, \Delta, A\end{array}}{\Gamma \vdash B \Rightarrow C, \Delta, A} \Rightarrow R \qquad \dfrac{\vdots\ \Pi_2}{A, \Gamma' \vdash \Delta'}\ ?}{\Gamma, \Gamma' \vdash B \Rightarrow C, \Delta, \Delta'} \ cut(A)$$

riduzione ?

agisco sulla premessa del taglio.

$$\Pi' \quad \dfrac{\dfrac{\begin{array}{c}\Pi_1' \\ \Gamma, B \vdash C, \Delta, A\end{array} \qquad \begin{array}{c}\Pi_2 \\ A, \Gamma' \vdash \Delta'\end{array}}{\Gamma, \Gamma', B \vdash C, \Delta, \Delta'} \ cut(A)}{\Gamma, \Gamma' \vdash B \Rightarrow C, \Delta, \Delta'} \Rightarrow R$$

$h(\Pi_1') + h(\Pi_2) <$

$h(\Pi_1) + h(\Pi_2)$

ip. ind.

$$\text{tagli strutturali} \quad w, c$$

$$\pi_1 \; \dfrac{\dfrac{\begin{matrix}\pi'_1 \\ \Gamma \vdash \Delta\end{matrix}}{\Gamma \vdash \Delta, \underline{A}}\, w \qquad \begin{matrix}\vdots\ \pi_2 \\ \underline{A}, \Gamma' \vdash \Delta'\end{matrix}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, \text{cut}(A) \; \pi'$$

$$\downarrow$$

$$\dfrac{\begin{matrix}\pi'_1 \\ \Gamma \vdash \Delta\end{matrix}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, \text{weakening}$$

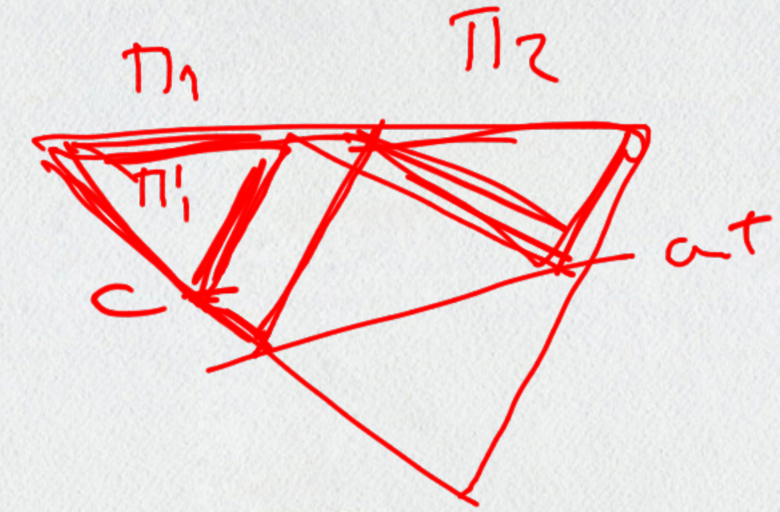
$$\dfrac{\dfrac{\begin{matrix}\pi_1 \\ \Gamma \vdash \Delta\end{matrix}}{\Gamma \vdash \Delta, A}\, w_R \qquad \dfrac{\begin{matrix}\pi_2 \\ \Gamma' \vdash \Delta'\end{matrix}}{A, \Gamma' \vdash \Delta'}\, w_L}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, \text{cut}$$

~~riduzione 1~~ oppure riduzione 2.

$$\dfrac{\Gamma \vdash \Delta}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, \text{weakening} \qquad\qquad \dfrac{\Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta', \Delta}\, \text{weakening}$$

caso taglio strutturale con C

$$
\Pi \;\; \dfrac{\Pi_1 \;\; \dfrac{\dfrac{\Pi_1''}{\Gamma \vdash \Delta, A, A}}{\Gamma \vdash \Delta, A}\, CR \qquad \dfrac{\Pi_2}{A, \Gamma' \vdash \Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\, cut\, A
$$



$$
\dfrac{\dfrac{\dfrac{\dfrac{\Pi_1'}{\Gamma \vdash \Delta, A, A} \quad + \quad \dfrac{\Pi_2}{A, \Gamma' \vdash \Delta'}}{\Gamma, \Gamma' \vdash \Delta, \Delta', A}\, cut(A)_1 \qquad + \quad \dfrac{\Pi_2}{A, \Gamma' \vdash \Delta'}}{\Gamma, \Gamma', \Gamma' \vdash \Delta, \Delta', \Delta'}\, cut(A)_2}{\dfrac{\text{contrazioni in } \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}}
$$

$$h(\Pi_1') + h(\Pi_2) < h(\Pi_1) + h(\Pi_2)$$

$$h(\Pi_1') + h(\Pi_2) + h(\Pi_2)$$

multicut

$$\dfrac{\dfrac{\begin{array}{c}\Pi_1\\ \Gamma \vdash \Delta, A, A\end{array}}{\Gamma \vdash \Delta, A}\,c_R \qquad \dfrac{\begin{array}{c}\Pi_2\\ A, A, \Gamma' \vdash \Delta'\end{array}}{A, \Gamma' \vdash \Delta'}\,c_L}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

$\nabla \quad \nabla$

multicut

$$\dfrac{\Gamma \vdash \Delta, \overbrace{A_1 \ldots A_n}^{A^n} \qquad \overbrace{A_1 \ldots A_m}^{A^m}, \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

Key step

cut logico.

volu?

Commutation step

cut-strutturale

$$\dfrac{\begin{array}{c}\Pi_1\\ \Gamma \vdash \Delta, A, A\end{array} \qquad \begin{array}{c}\Pi_2\\ A, A, \Gamma' \vdash \Delta'\end{array}}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,\text{multicut}$$

ip. ind

$n \neq m$

$$\frac{\Gamma \vdash \Delta, A^{n \geq 0} \qquad \Gamma', A^{m \geq 0} \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \text{ multi-cut}$$

Case $n = 0$ $\quad m > 0$

$$\frac{\begin{array}{c}\Pi_1 \\ \Gamma \vdash \Delta\end{array} \qquad \begin{array}{c}\Pi_2 \\ \Gamma', A^m \vdash \Delta'\end{array}}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \text{ multi cut}$$

$$\frac{\Pi_1}{\Gamma', \Gamma \vdash \Delta', \Delta} \text{ W L/R}$$

$$\text{cut} \frac{c\dfrac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A} \qquad \dfrac{A, A, \Gamma' \vdash \Delta'}{A, \Gamma' \vdash \Delta'} c}{}$$

$$W\ \frac{\vdash B}{\vdash B, A} \qquad \frac{\vdash A}{\vdash A, B}\ W$$

$$\begin{array}{cc} \vdash A_1 \ \ldots & \vdash A_n \ \ldots \\ u & u \end{array}$$

$$\vdash \Omega$$

W

C.

F, T

Regole strutturali W, C e regole logiche $\boxed{\wedge, \vee}$ $A \Rightarrow B = \neg A \vee B$

- versione additiva (quella vista fin ora) si richiede il medesimo contesto Γ, Δ nelle due premesse
- versione moltiplicativa delle regole logiche **contesto libero**

$\Gamma \neq \Gamma'$?

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ \wedge_m L \qquad \frac{\Gamma \vdash \Delta, A \qquad \Gamma' \vdash \Delta', B}{\Gamma, \Gamma' \vdash \Delta, \Delta', A \wedge B}\ \wedge_m R$$

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \vee B \vdash \Delta, \Delta'}\ \vee_m L \qquad \frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \vee B}\ \vee_m R$$

"$\vdash$" = "$\Longrightarrow$"

A	B	$A \Rightarrow B$
0	0	1
0	1	1
1	0	0
1	1	1

la versione additiva $\equiv$ alla versione moltiplicativa delle regole

ogni regola additiva è derivabile da quelle moltiplicative e viceversa.

diciamo che una regola $R : \dfrac{S_1 \quad ... \quad S_n}{S}$ premesse / conclusione

è derivabile da un sistema di regole $\mathcal{J}$ (che non contiene R)

sse

$$\mathcal{J} \; \dfrac{S_1 \quad ... \quad S_n}{S}$$

posso derivare usando solo le regole di $\mathcal{J}$

derivianno la $\wedge_m R$ da quelle $\wedge_a R$

$$\dfrac{\Gamma \vdash \Delta, A \qquad \Gamma' \vdash \Delta', B}{\Gamma, \Gamma' \vdash \Delta, \Delta', A \wedge B}\ \wedge_m R$$

$$\dfrac{\dfrac{\dfrac{\Gamma \vdash \Delta, A}{}\ w_{L/R}}{\Gamma, \Gamma' \vdash \Delta, \Delta', A} \qquad \dfrac{\dfrac{\Gamma' \vdash \Delta', B}{}\ w_{L/R}}{\Gamma, \Gamma' \vdash \Delta, \Delta', B}}{\Gamma, \Gamma' \vdash \Delta, \Delta', A \wedge B}\ \wedge_a R$$

deriviamo la $\wedge_a R$ dalle $\wedge_m R$

$$\dfrac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B}\ \wedge_a R$$

$$\dfrac{\dfrac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma, \Gamma \vdash \Delta, \Delta, A \wedge B}\ \wedge_m R}{\Gamma \vdash \Delta, A \wedge B}\ c_{L/R}^{+}$$

Esercizio derivare la $\wedge_m L$ dalle $\wedge_a L$

$\wedge_a^{1/2} L$ dalle $\wedge_m L$

Taglio logico

$$\dfrac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash \Delta, A \wedge B}\ \wedge_a R \qquad \dfrac{\Gamma', A \vdash \Delta'}{\Gamma', A \wedge B \vdash \Delta'}\ \wedge^1_a L$$

$$\dfrac{\Gamma \vdash \Delta, A \wedge B \qquad \Gamma', A \wedge B \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \text{cut } (A \wedge B)$$

riduco.

$$\dfrac{\Gamma \vdash A, \Delta \qquad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \text{cut } A$$

$$\dfrac{\overset{\Pi_1}{\Gamma \vdash A, \Delta} \qquad \overset{\Pi_2}{\Gamma \vdash B, \Delta}}{\Gamma \vdash \Delta, \underline{A \wedge B}}\ \wedge_a R \qquad \dfrac{\overset{\Pi_3}{\Gamma', A, B \vdash \Delta'}}{\Gamma', \underline{A \wedge B} \vdash \Delta'}\ \wedge_m L$$

$$\dfrac{\Gamma \vdash \Delta, A \wedge B \qquad \Gamma', A \wedge B \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \text{cut}(A \wedge B)$$

riduco. il taglio logico.

$$\dfrac{\dfrac{\overset{\Pi_1}{\Gamma \vdash \underline{A}, \Delta} \qquad \overset{\Pi_3}{\Gamma', \underline{A}, B \vdash \Delta'}}{\Gamma, \Gamma', \underline{B} \vdash \Delta, \Delta'}\ \text{cut}(A) \qquad \overset{\Pi_2}{\Gamma \vdash \underline{B}, \Delta}}{\Gamma, \Gamma, \Gamma' \vdash \Delta, \Delta, \Delta'}\ \text{cut}(B)$$

contraction L/R su Γ, Δ

$$\overline{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

Logica Lineare (GIRARD, 1987)

- controllo "ottimale" delle strutturali W, C
- modalità ?, ! op. unari
- reti dimostrative Proof nets

dimostrazioni = grafi

eliminazione del taglio = deformazione del grafo