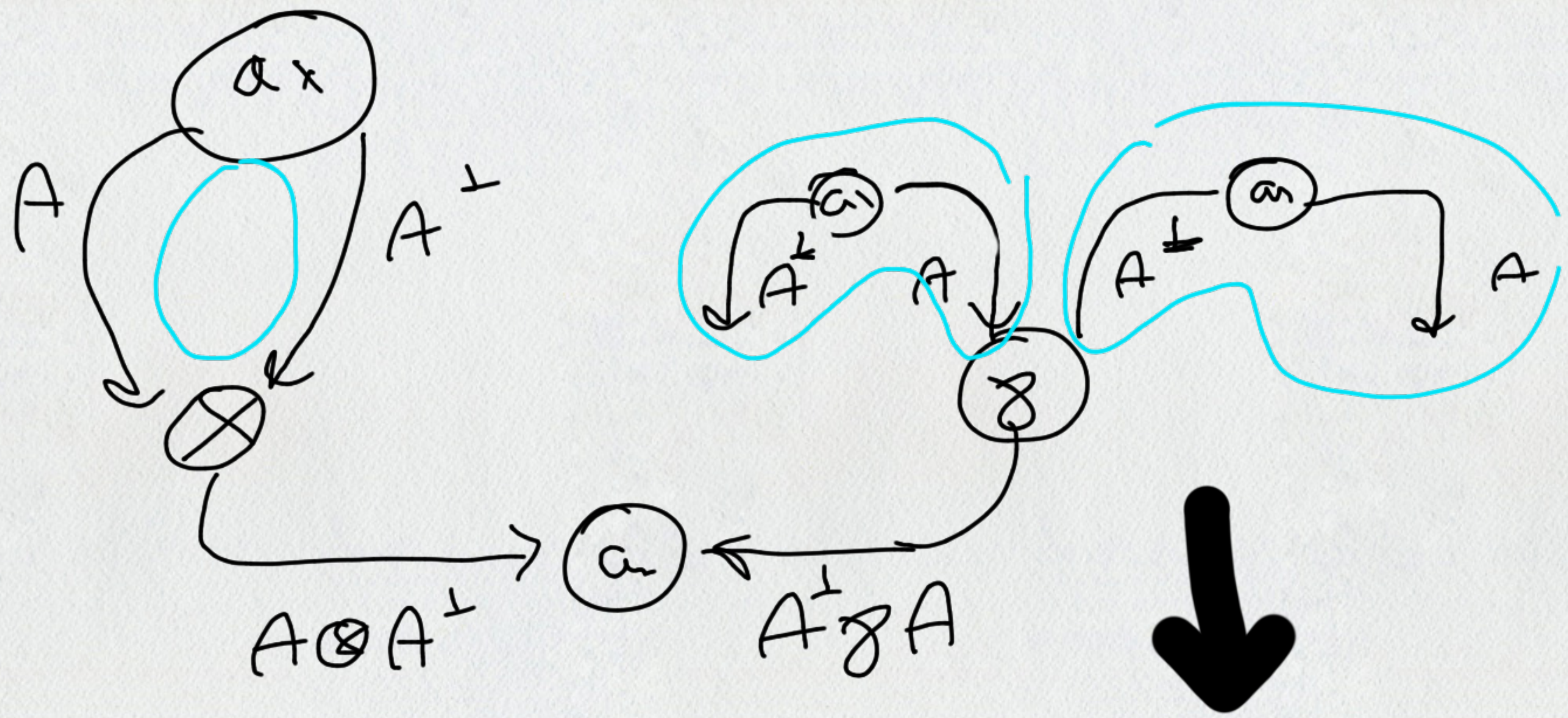
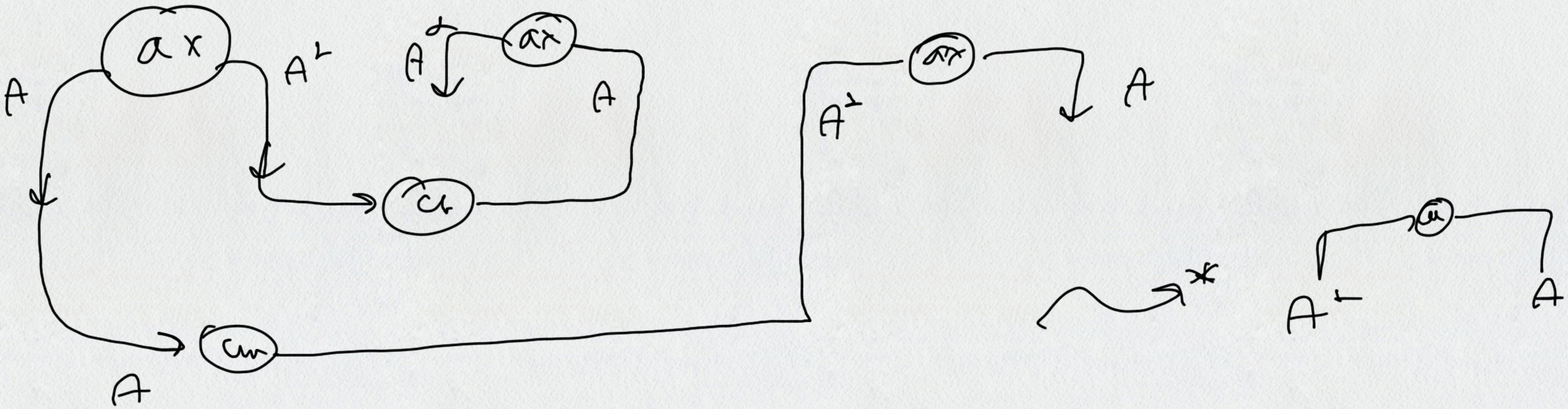
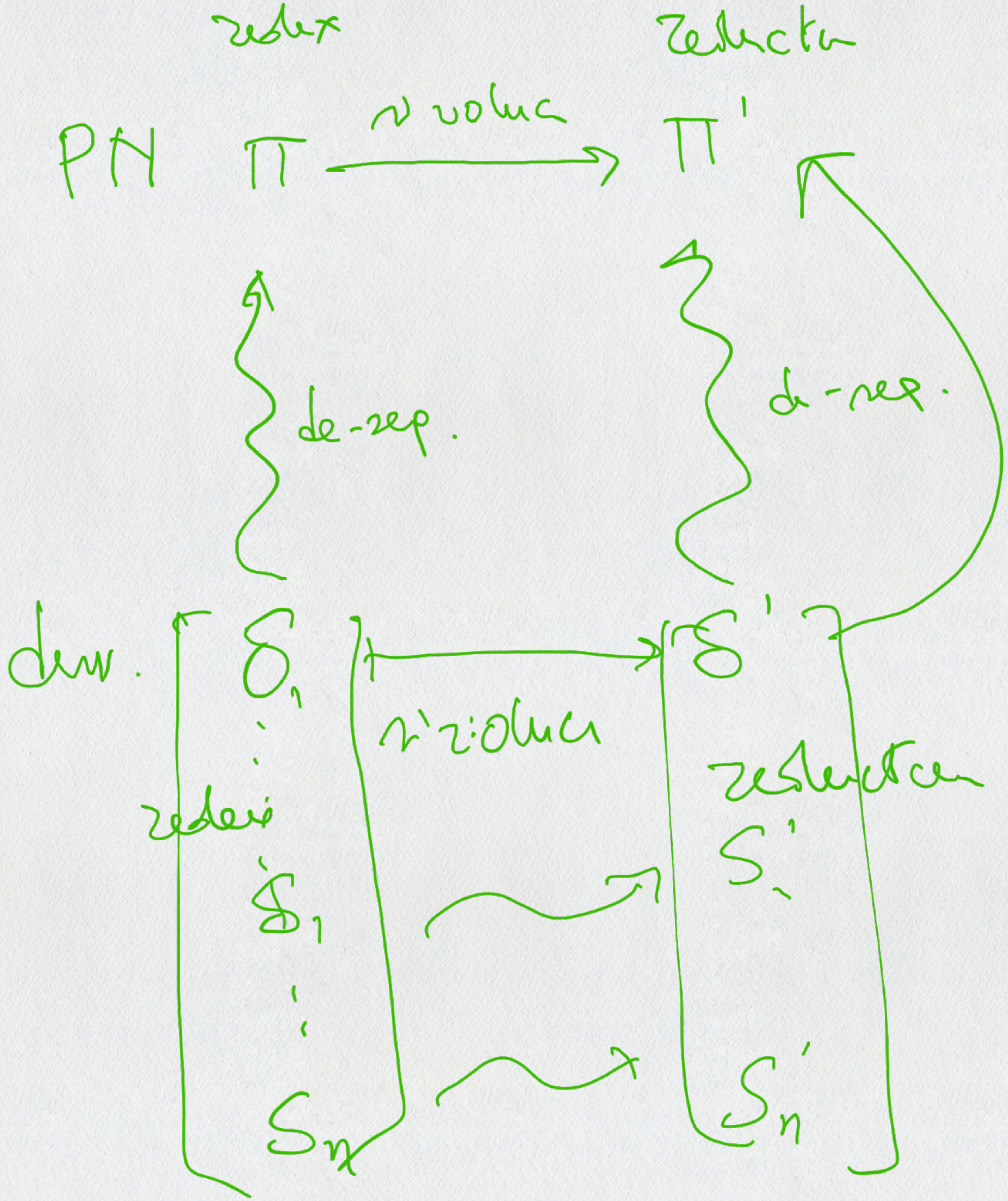
PS non corretto $\forall\ Sq(H)$ è sconnesso e ciclico!



$\underline{MELL} \simeq \lambda\text{calc}$



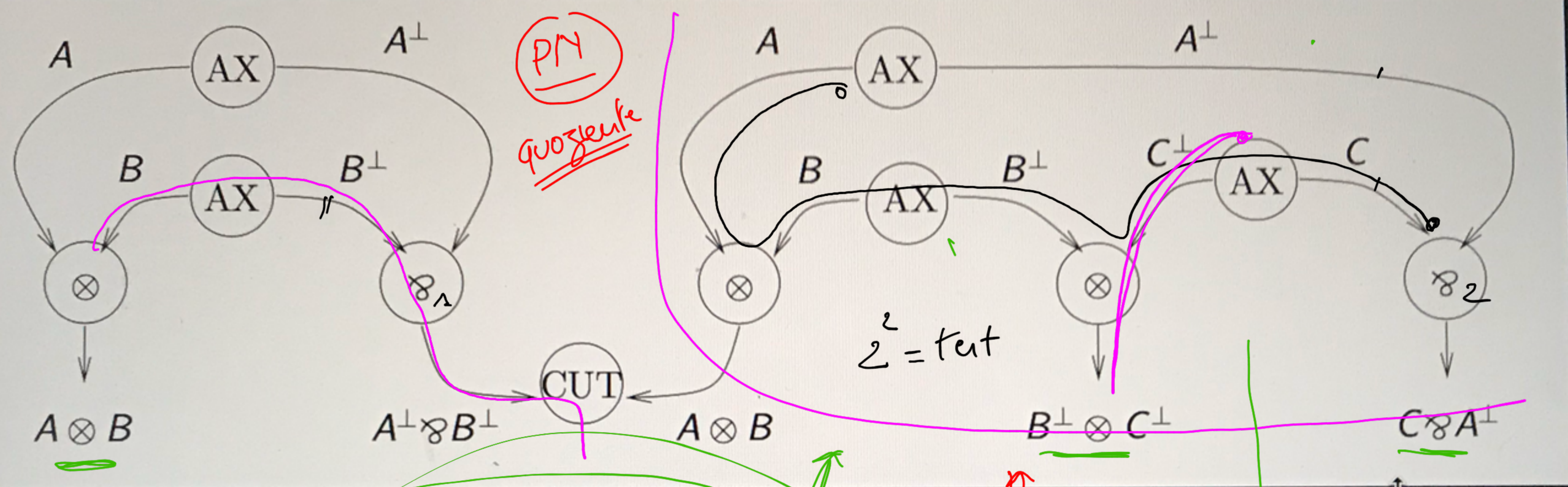


– criterio quadratico $O(n^2)$

– proof search

focusing

focusing



π_1

$$\dfrac{\dfrac{\dfrac{A, A^\perp \qquad B, B^\perp}{\mathbf{A \otimes B}, A^\perp, B^\perp}\;\otimes}{A \otimes B, \mathbf{A^\perp \parr B^\perp}}\;\parr \qquad \dfrac{\dfrac{\dfrac{\dfrac{B, B^\perp \qquad C, C^\perp}{\mathbf{B^\perp \otimes C^\perp}, B, C}\;\otimes \qquad A, A^\perp}{\mathbf{A \otimes B}, B^\perp \otimes C^\perp, A^\perp, C}\;\otimes}{A \otimes B, B^\perp \otimes C^\perp, \mathbf{A^\perp \parr C}}\;\parr}{A \otimes B, B^\perp \otimes C^\perp, A^\perp \parr C}\;\textit{cut}$$

$\Updownarrow$ equivalenza sintattica

π_2

$$\dfrac{\dfrac{\dfrac{A, A^\perp \qquad B, B^\perp}{A \otimes B, A^\perp, B^\perp}\;\otimes}{A \otimes B, A^\perp \parr B^\perp}\;\parr \qquad \dfrac{\dfrac{\dfrac{B, B^\perp \qquad C, C^\perp}{B^\perp \otimes C^\perp, B, C}\;\otimes \qquad A, A^\perp}{A \otimes B, B^\perp \otimes C^\perp, A^\perp, C}\;\otimes}{\dfrac{A \otimes B, B^\perp \otimes C^\perp, A^\perp, C}{A \otimes B, B^\perp \otimes C^\perp, A^\perp \parr C}\;\parr}\;\textit{cut}$$

$\Leftrightarrow$?

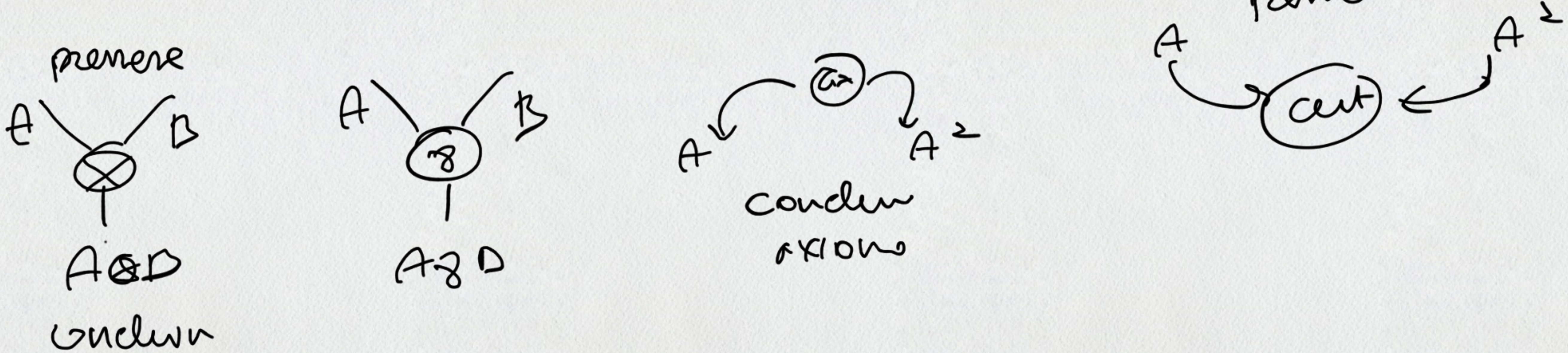
dim. normale π_3

$$\dfrac{\dfrac{\dfrac{A, A^\perp \qquad B, B^\perp}{A \otimes B, A^\perp, B^\perp}\;\otimes \qquad C, C^\perp}{A \otimes B, B^\perp \otimes C^\perp, A^\perp, C}}{A \otimes B, B^\perp \otimes C^\perp, A^\perp \parr C}\;\parr$$

PN: sono PS corrette criterio di correttezza ACC

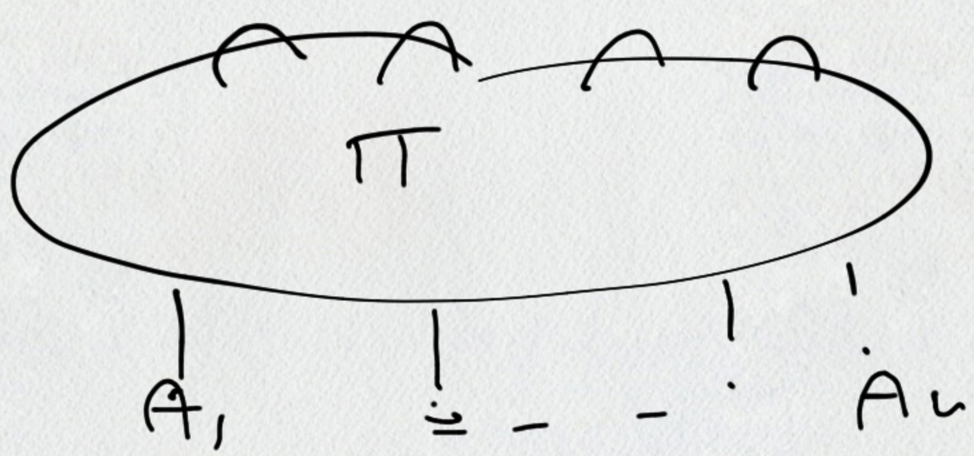
PS

link

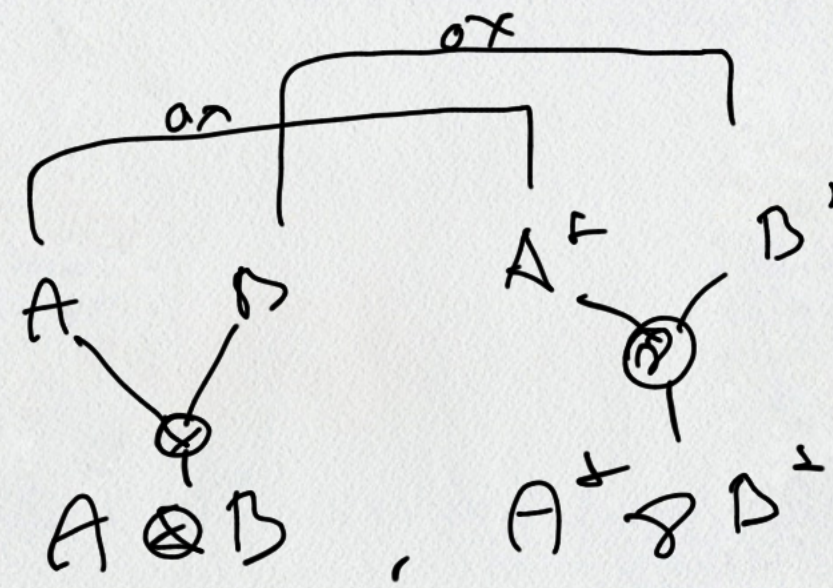


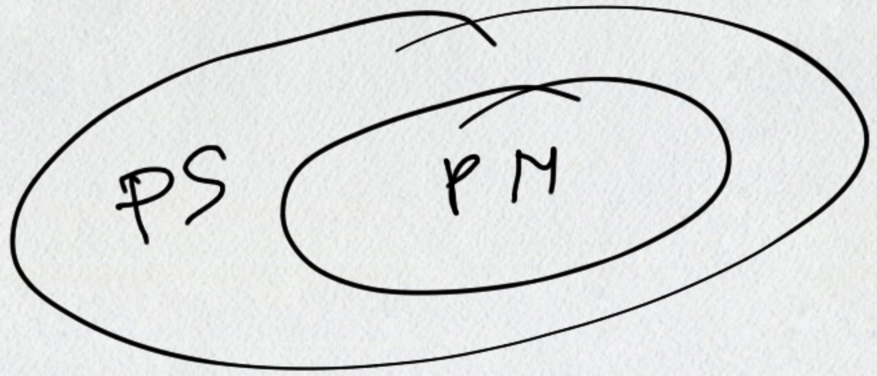
Proof structure è un grafo $\Pi : \langle V, E \rangle$

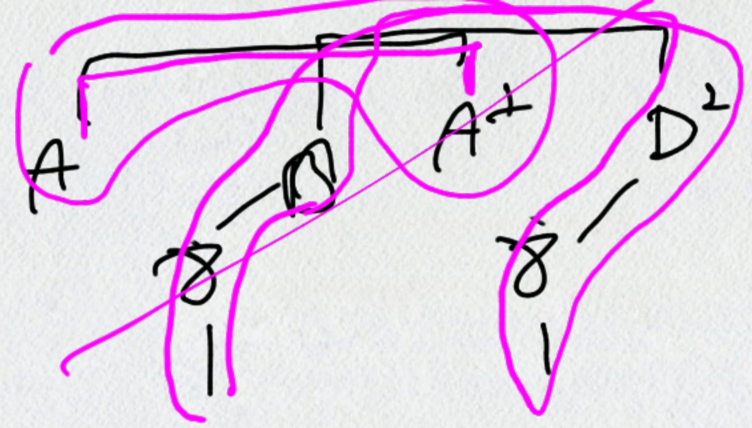
ogni arco deve essere conclusione di esattamente un link e premessa di al massimo un link



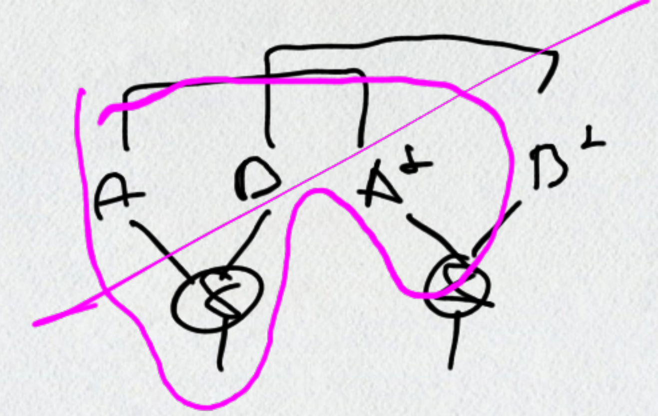
Conclusioni della proof net







$\nvdash_{MLL} A ⅋ B,\ A^{\perp} ⅋ B^{\perp}$



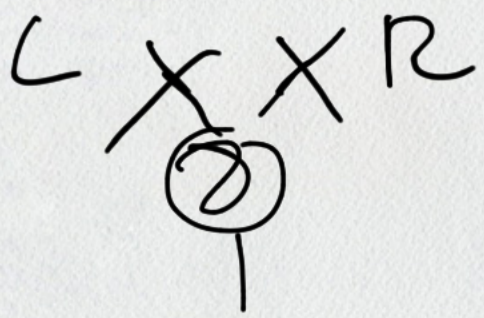
$\nvdash_{MLL} A \otimes B,\ A^{\perp} \otimes B^{\perp}$

Criterio di correttezza

Una PS Π è corretta è una proof net sse

$\forall$ switching (test) $S_{\varphi}(\Pi)$ è aciclico connesso ACC

$\varphi : ⅋\text{-link} \to L/R$



Stabiliamo una corrispondenza PN e S P.

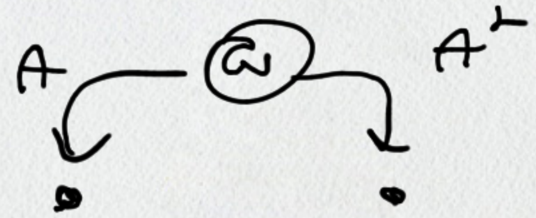
$\vdash_{MLL} \Gamma$ è dim. in MLL sse $\exists$ una PN con concl. $\vdash \Gamma$

$\Longleftrightarrow$

<u>De-sequentialization</u> : $\pi : \vdash \Gamma \longmapsto$ una PN π con concl. Γ

<u>Proof</u> per ind. sull'altezza di π

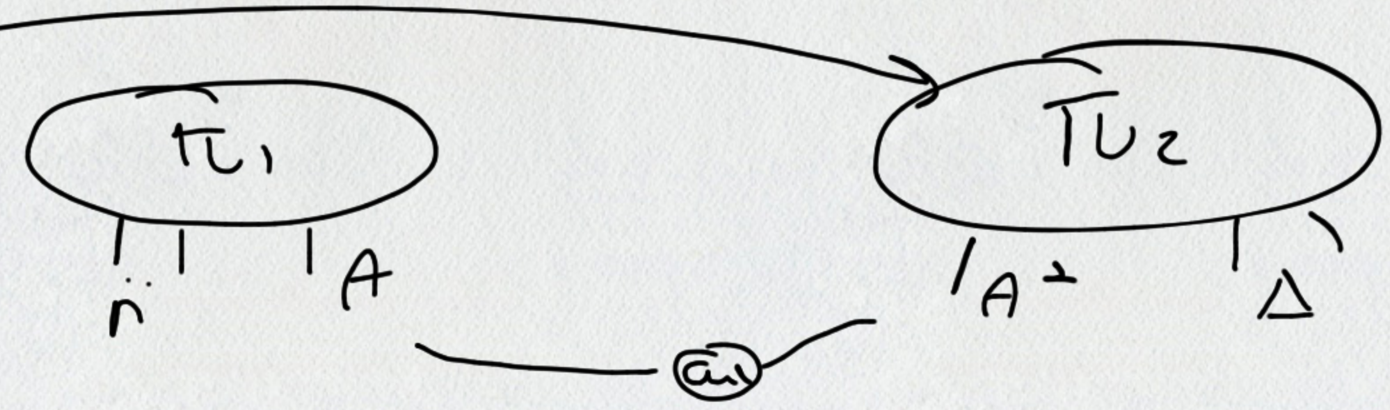
<u>base</u> : $\pi : \overline{A \quad A^\perp} \longmapsto \pi$



handling nodes

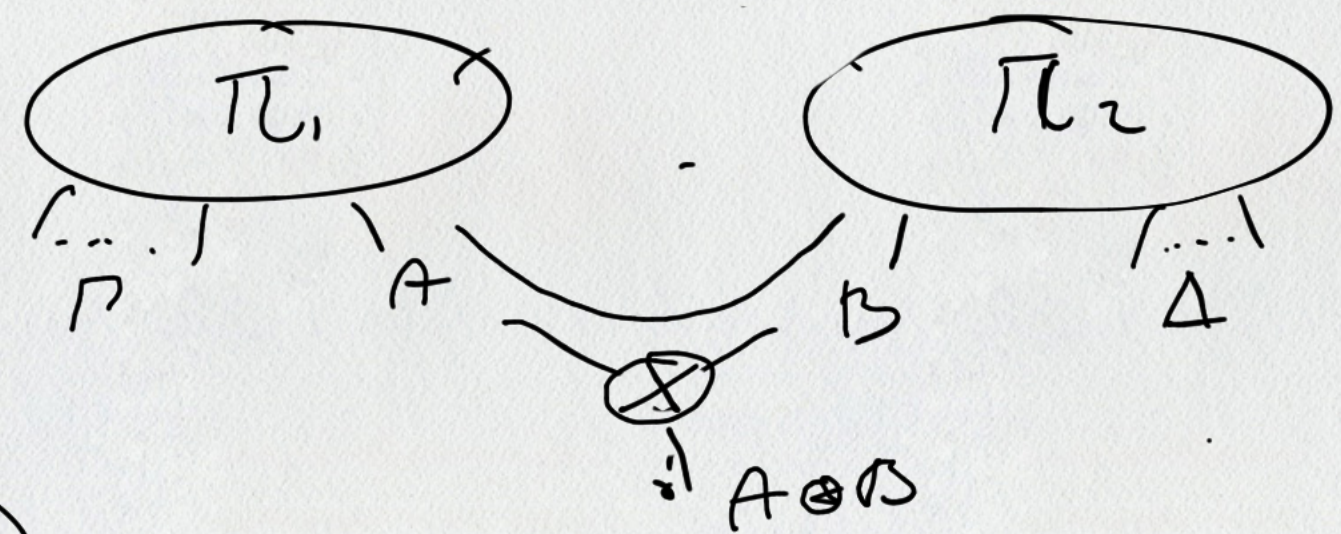
<u>passo induttivo</u>

$$\frac{\pi_1 \; \vdash \Gamma, A \qquad \pi_2 \; \vdash \Delta, A^\perp}{\pi : \vdash \Gamma, \Delta}\text{cut}$$

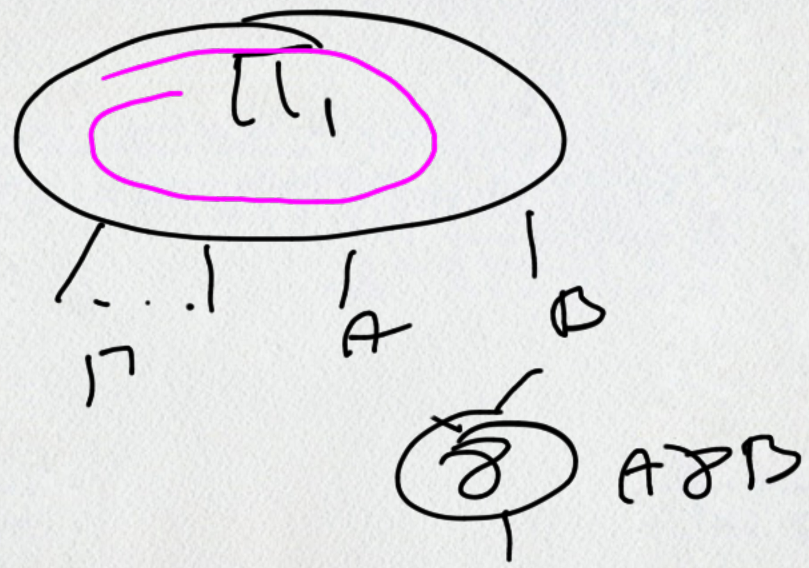


π è ACC

$$\frac{\pi_1 \vdash \Gamma, A \qquad \pi_2 : \vdash \Delta, B}{\pi : \vdash \Gamma, \Delta, A \otimes B}\otimes \longmapsto$$



$$\frac{\pi_1 \; \vdash \Gamma, A, B}{\vdash \Gamma, A \parr B}\parr \longmapsto$$

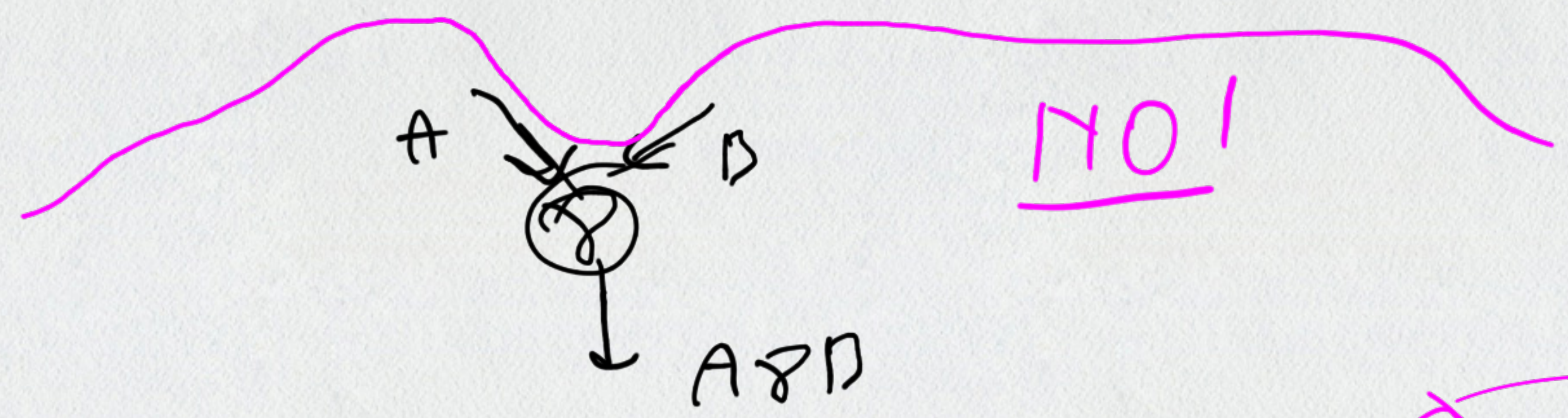


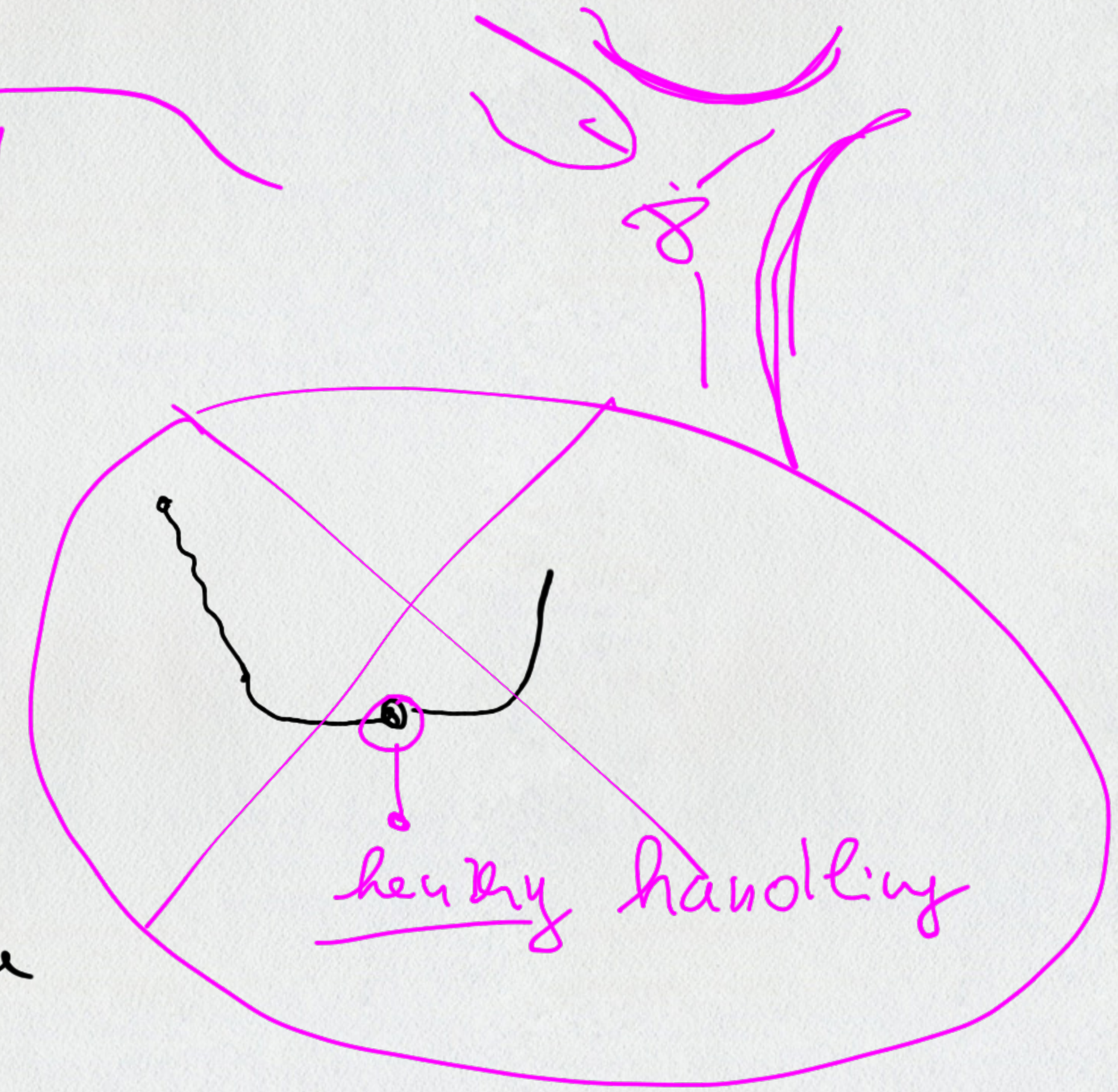
ogni switching è ACC

Sequenzializzazione: ad ogni proof net Π di concl. Γ può essere associata una dim. Π di $\vdash \Gamma$ nel calc. dei seq. di MLL.

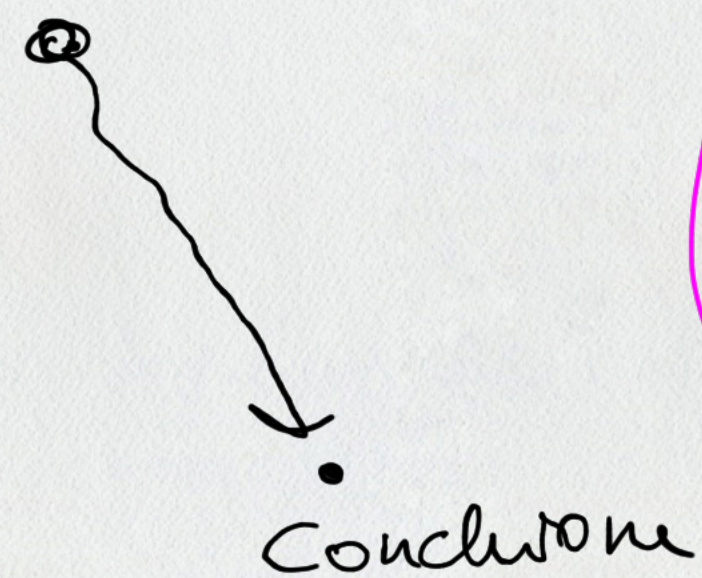
Proof:

definizione: switching path in un PS è un cammino che non passa mai per le due premesse di un link ⅋





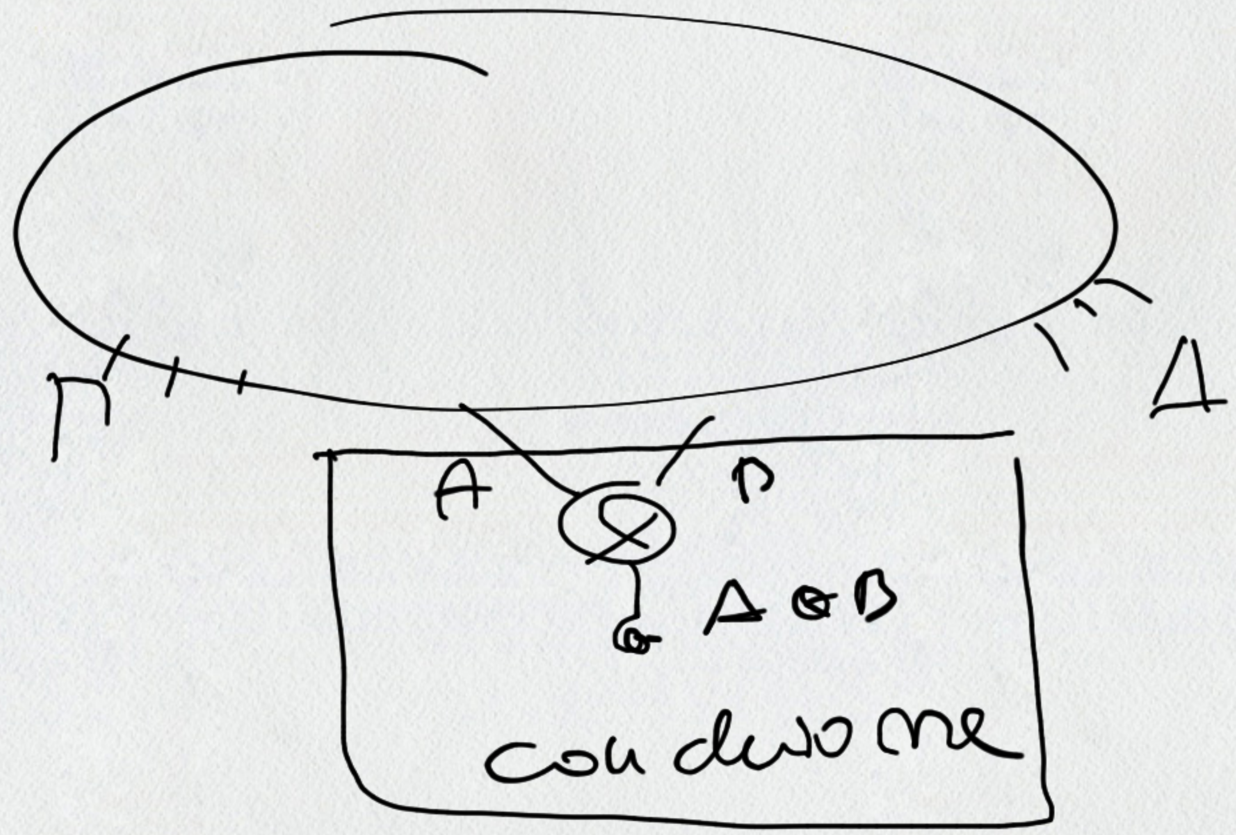
PS cut-free   direct path

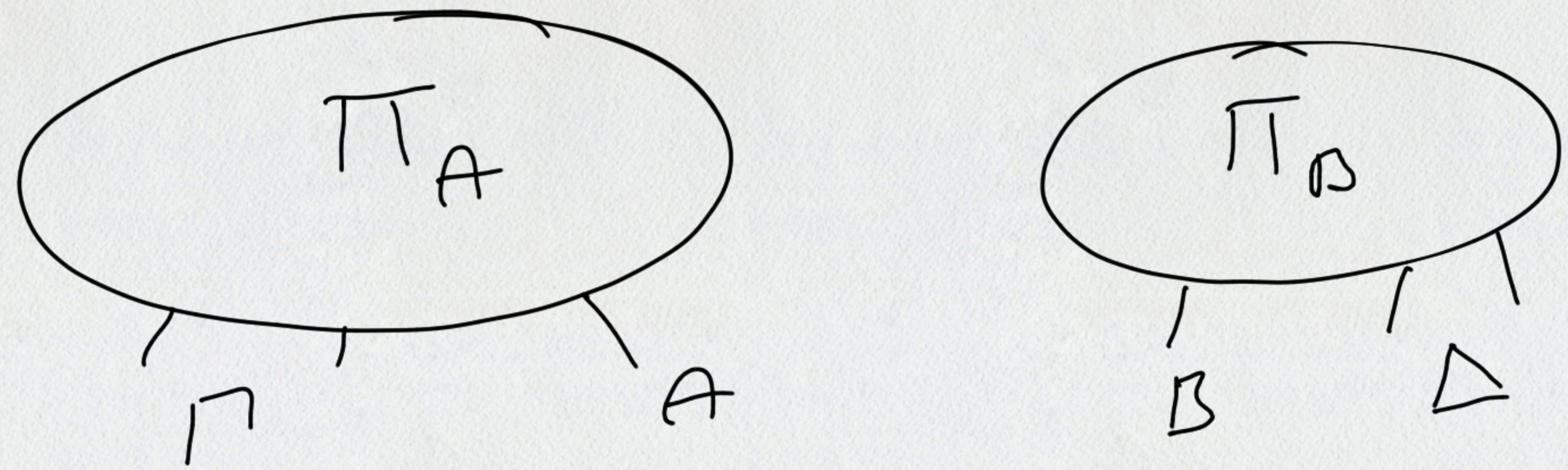


⊗ Link Splitting (scindente)

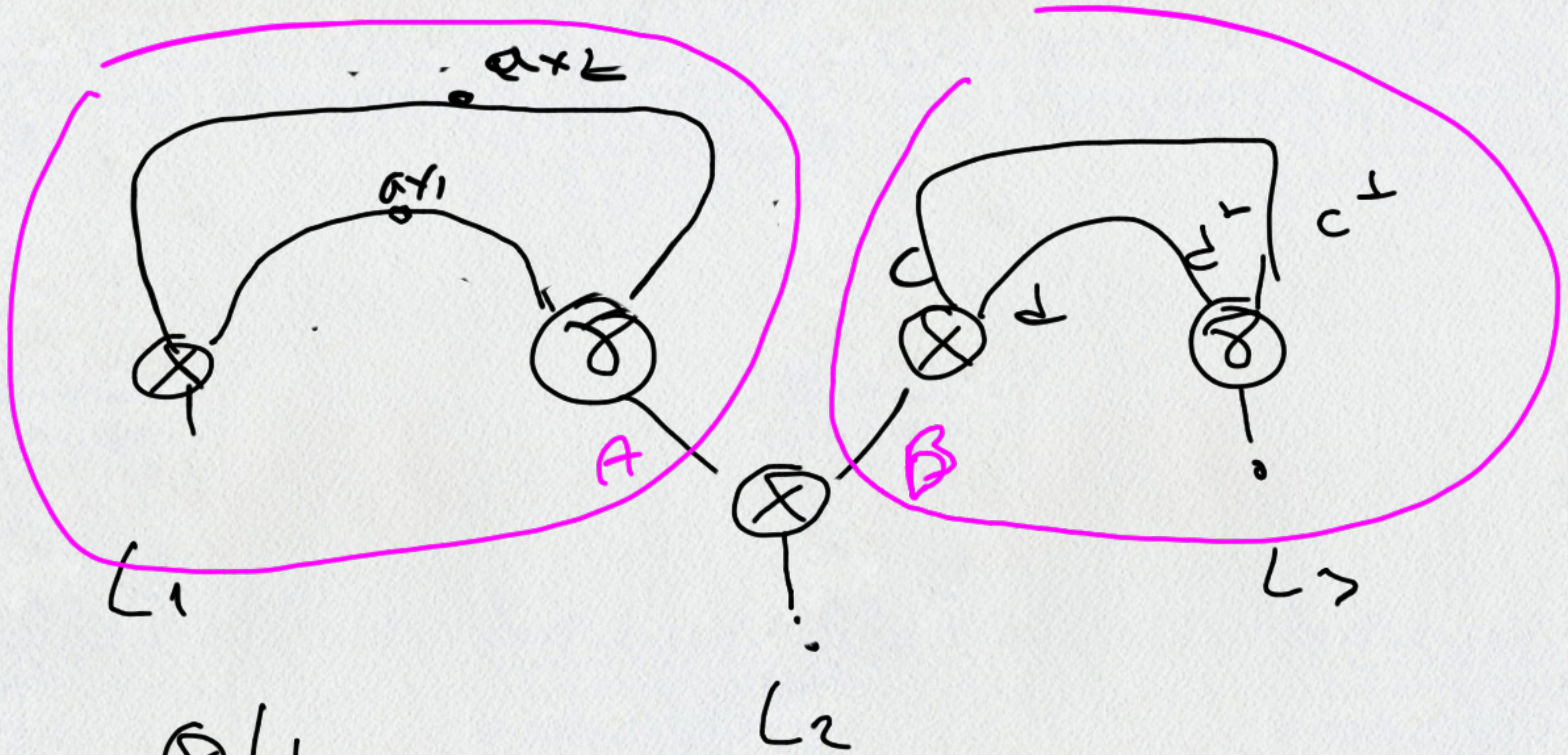
diremo che in un proofnet Π un link conclusione ⊗ è splitting se la rimozione del link (lasciando intatte le premesse) produce/induce due proofnets disgiunte Π_A, Π_B

Π





Esempio di splitting



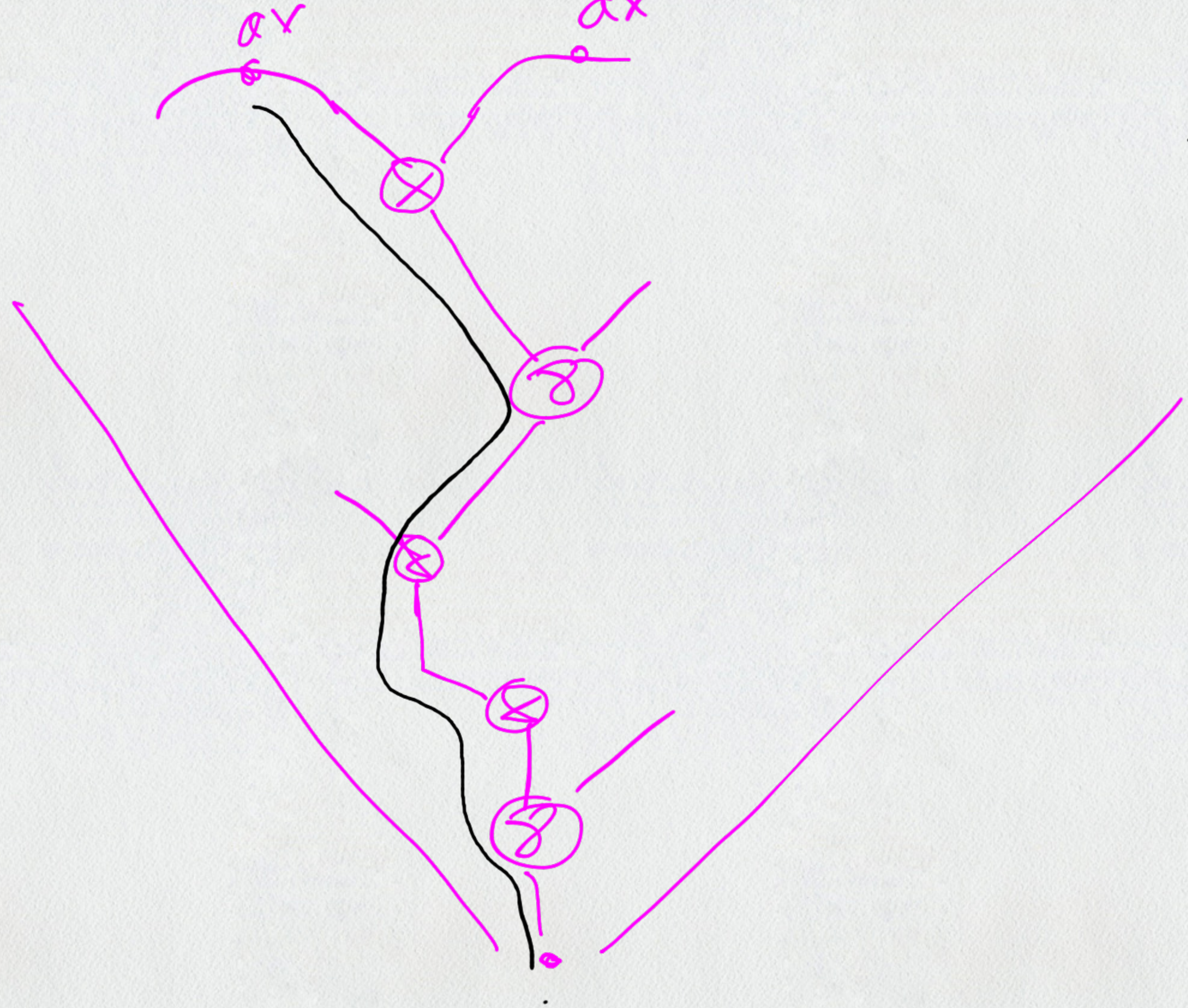
PS

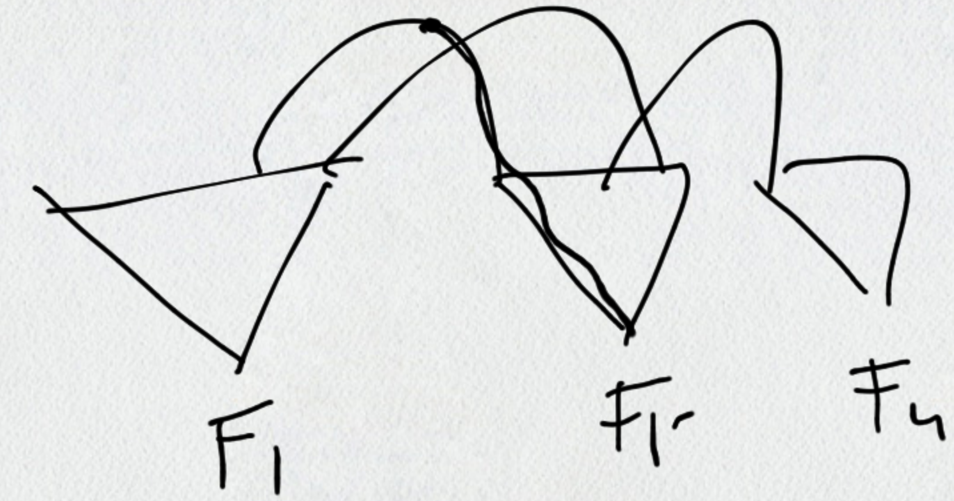
$\otimes L_1$

non e' splitting

$\otimes L_2$

splitting


ax
ax

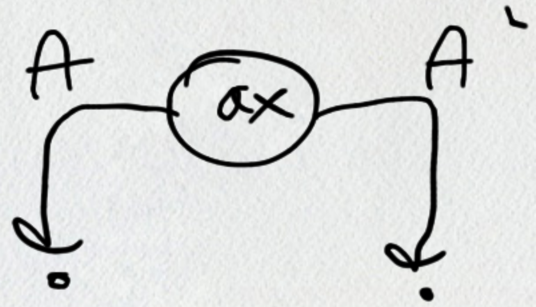

F1
F1r
F4

Dimostrazione della sequenzializzazione Π PN di $\Gamma \longmapsto$ S.P. di $\vdash_{MLL} \Gamma$

Per induzione sulla taglia della PN π

taglia/size di π : $\langle \# V, \# E \rangle$

· base induzione

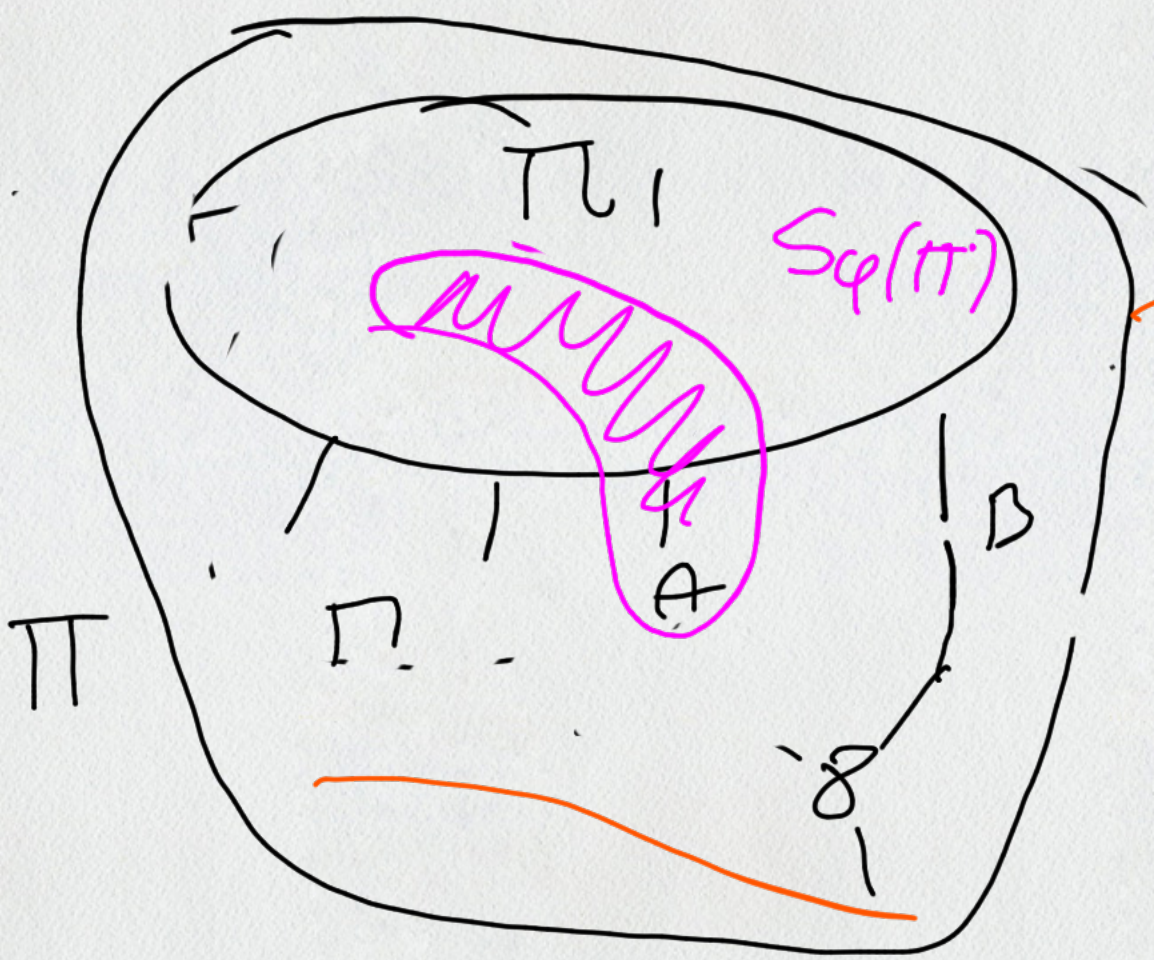


$\longmapsto \quad \overline{\vdash A, A^{\perp}}$ $\quad \# V = 1$ ax

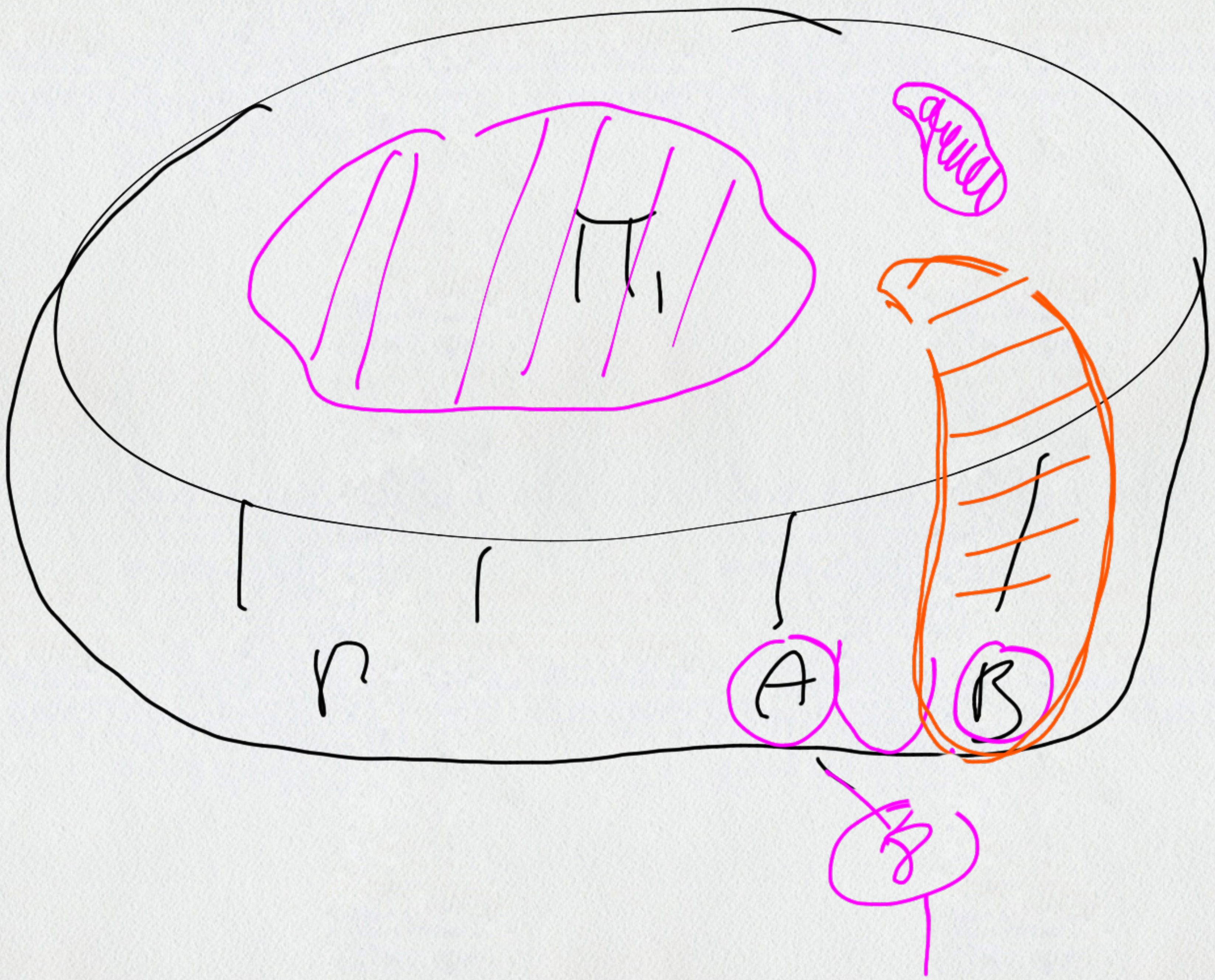
· passo induttivo $\# V > 1$

sottocaso 1 la pn contiene almeno una conclusione ⅋ link

ip. ind su Π_1 $\quad t(\pi_1) < t(\pi)$



$$\dfrac{\Pi_1 \vdash \Gamma, A, B}{\vdash \Gamma, A ⅋ B} ⅋$$



Π

¬(se i pezzi Π è connesso)

⟹ Π_1 è connesso

$\Pi_1 = \Pi - \gamma$-luce $\cap \gamma B$

¬ ACC

∃ $S\varphi(\Pi_1)$ con calq. assenza

allora pro costruire uno

$S\varphi(\Pi)$

$\varphi : \gamma \to R$

$ACC \equiv \forall S\varphi(\Pi) \Rightarrow A.C.$

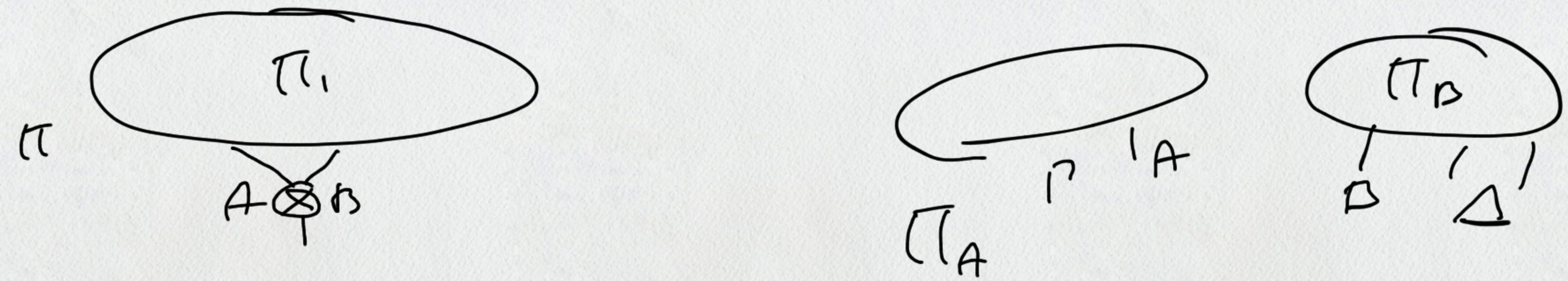
$\neg ACC \equiv \exists S\varphi(\Pi) \Rightarrow \neg A \vee \neg C$

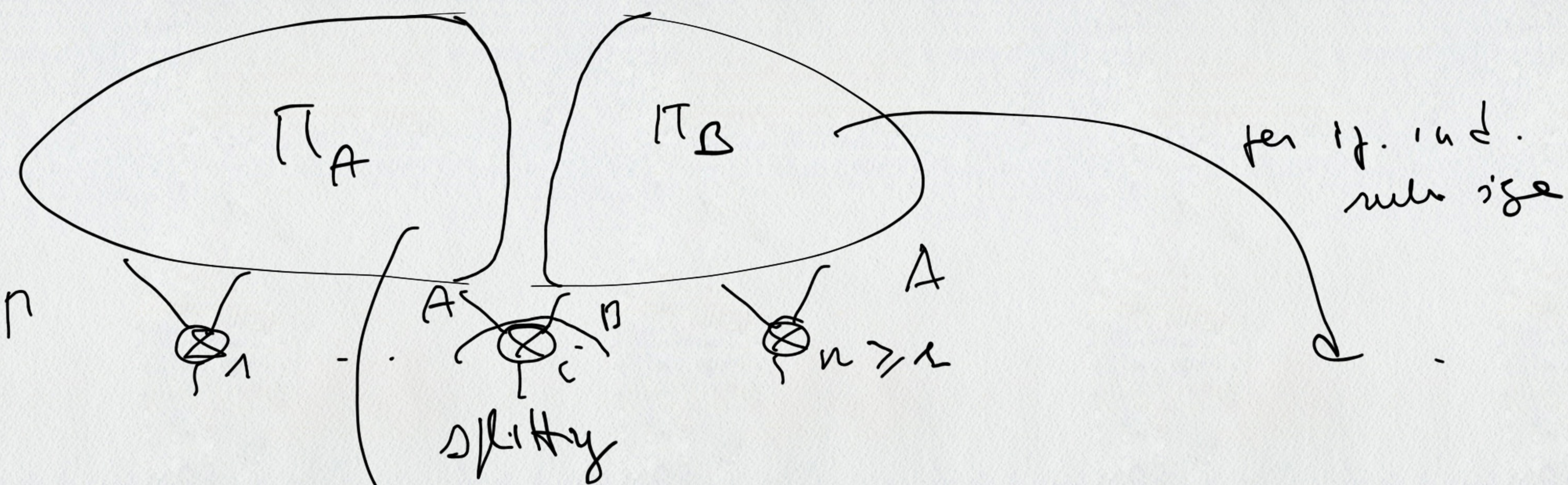
dim di Π_1 è corretta

$$\begin{tikzcd}
\Pi_1 \arrow[r, mapsto, "\text{seq. x ind}"] \arrow[d, "\otimes"'] & \Pi \text{ di } \Gamma, A, B \arrow[d, "\text{regr } \otimes"] \\
\Pi \arrow[r, mapsto] & ? \quad \Gamma, A \otimes B
\end{tikzcd}$$

il grafo commuta

Caso: la rete Π non ha conclusioni di tipo ⅋-link ed ha almeno una conclusione di tipo $\otimes$ link





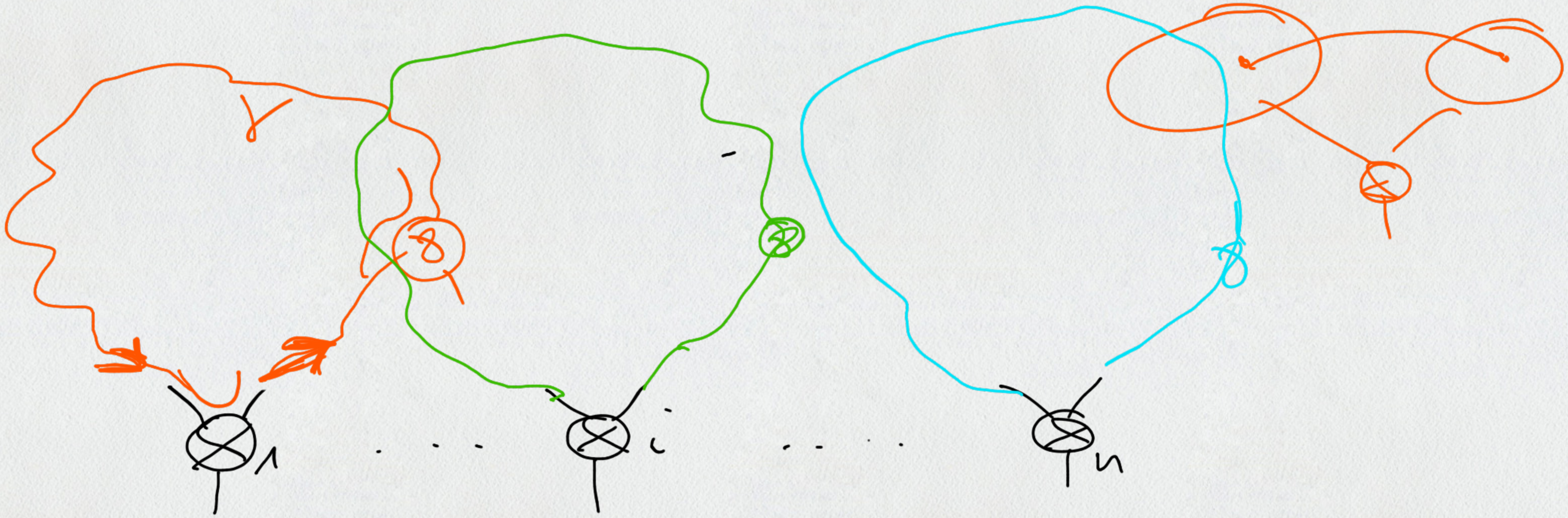
$$\frac{\Pi_1 : \vdash \Gamma, A \qquad \Pi_2 : \vdash \Delta, B}{\Pi : \vdash \Gamma, \Delta, A \otimes B}\ \otimes$$

se $\exists$ un $\otimes_i$ splittante

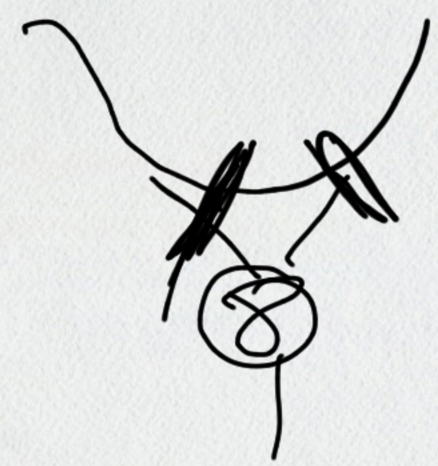
Splitting Lemma

Lemma di splitting: se Π è una PN di conclusioni Γ non contenente alcune conclusioni $\wp$-link ed almeno un $\otimes$-link allora $\exists$ un $\otimes_i$-link terminal (conclusion) splittante. (Π_A, Π_B)

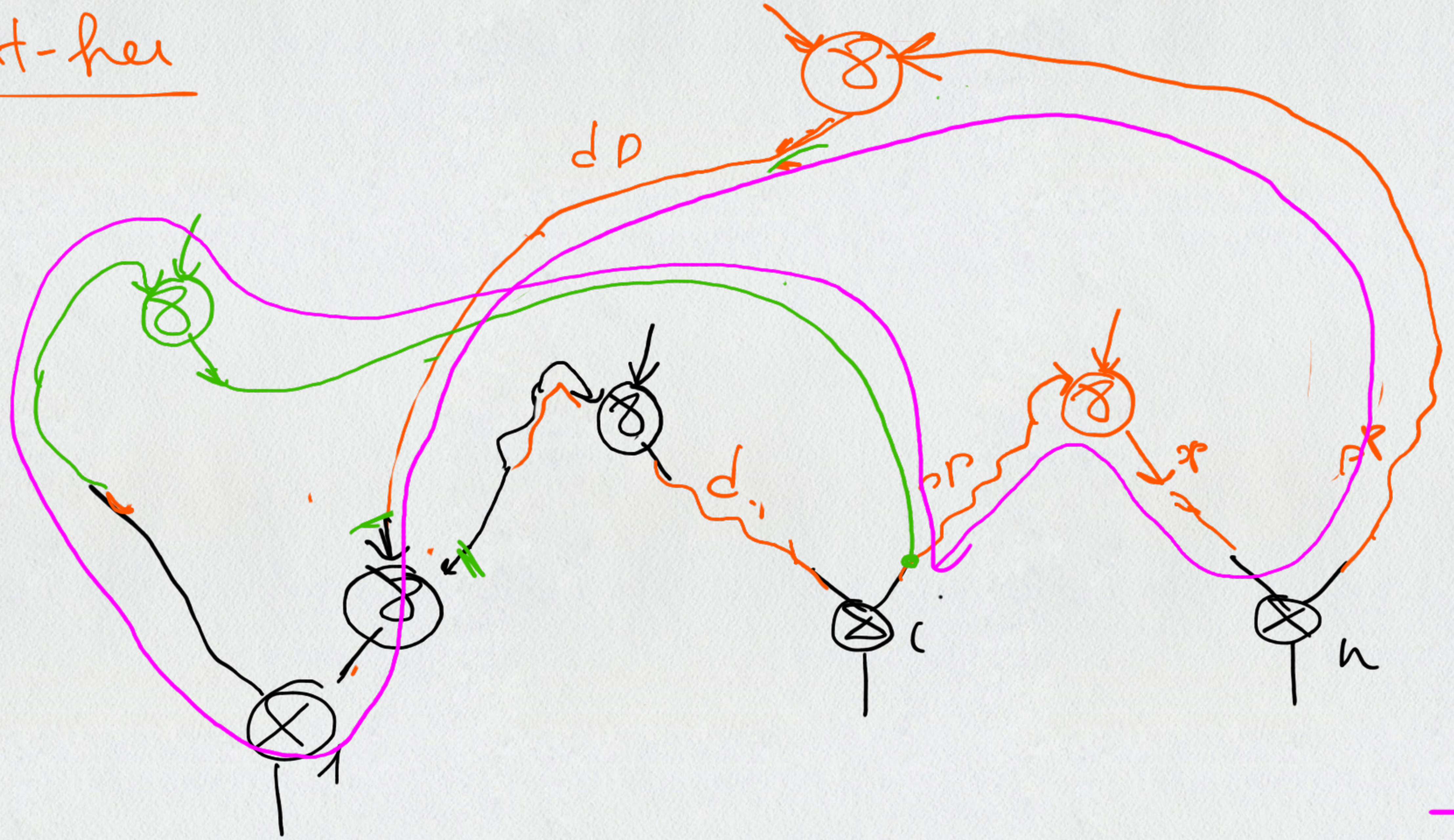
Proof: RAA: nessun $\otimes_i$ è splittante, Π è connettiva



$\Rightarrow \exists\ Sw(\Pi)$ con un ciclo



Prova cut-free



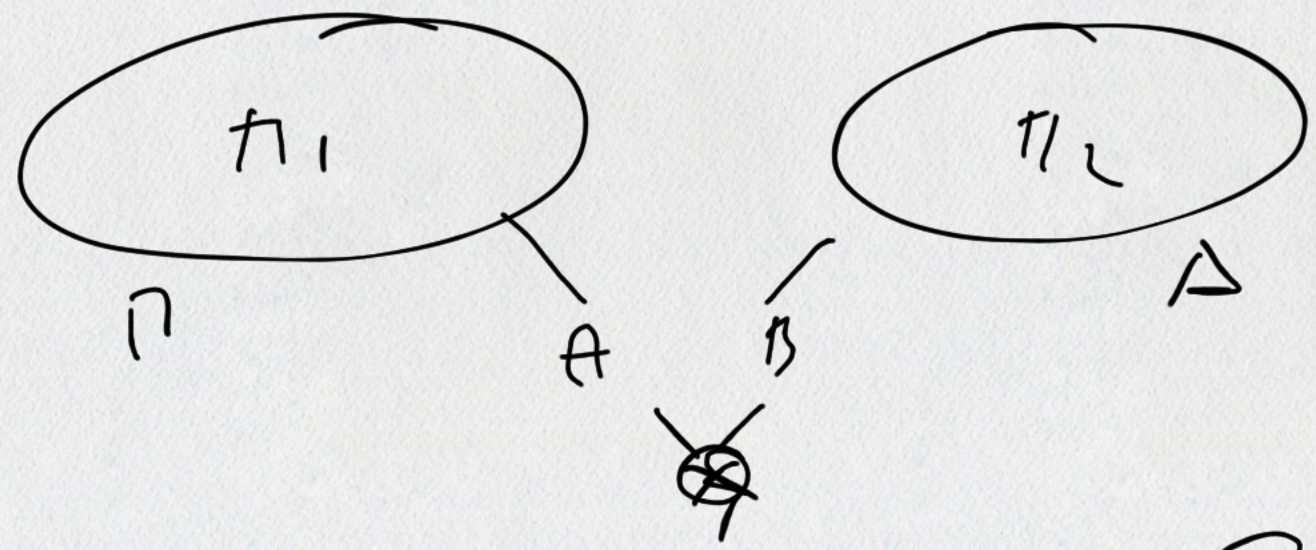
S.p. $\Rightarrow$ $\exists$ un $S_\varphi(\Pi)$ con un ciclo $\Rightarrow$ PN non era corretta $\perp$

Ciclo!

ho creato un γ switching path $\Rightarrow$ $\exists$ un $S_\varphi(\Pi)$ in cui γ può essere eseguito. contraddicendo l'ipotesi che Π è corretta!

Sequentializzazione

Se una rete non contiene ⅋ link conclusione e contiene almeno un $\otimes$ link concl. $\Longrightarrow$ via Splitting Lemma ne selezioniamo uno splittante



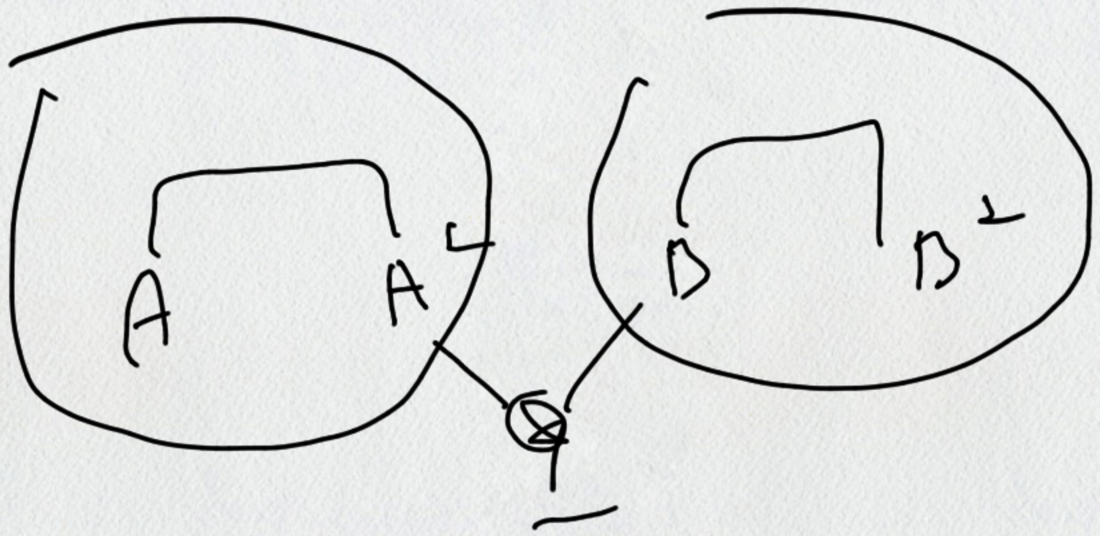
splitting $\in$ Split(π)

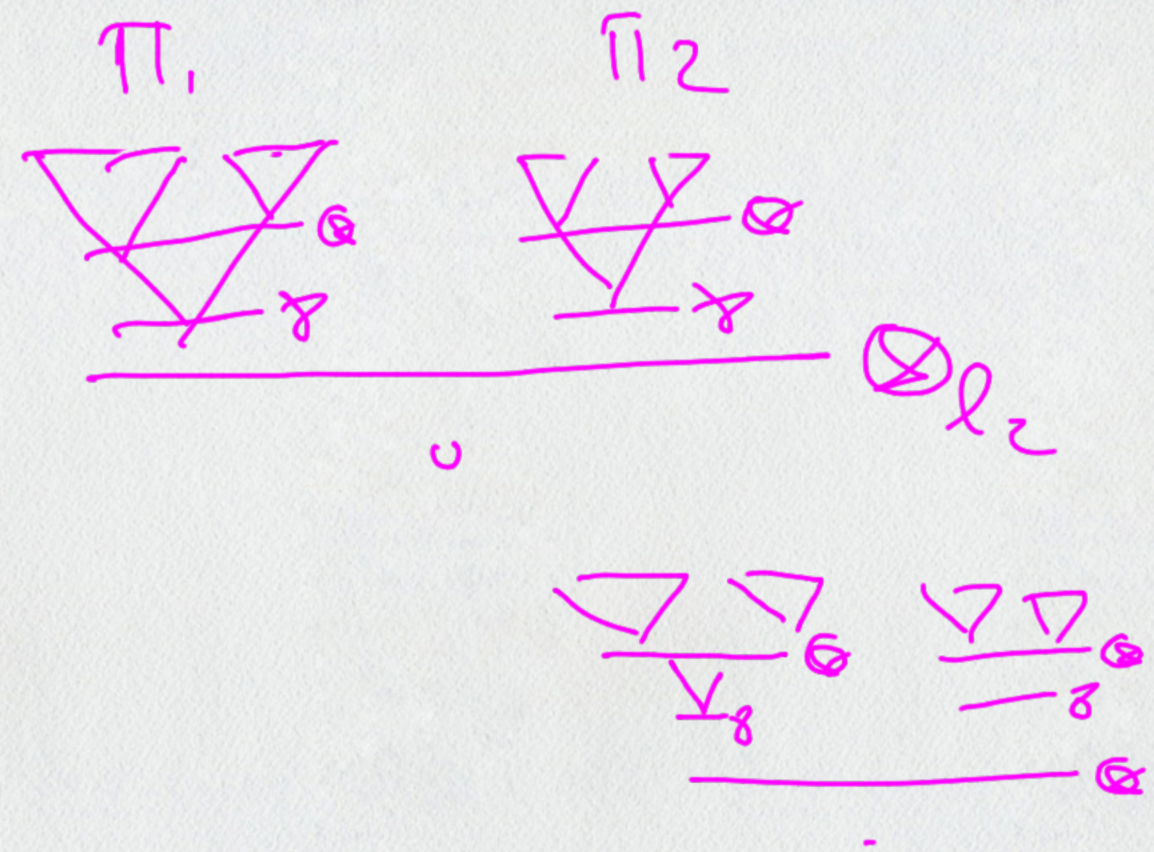
per ip. ind.

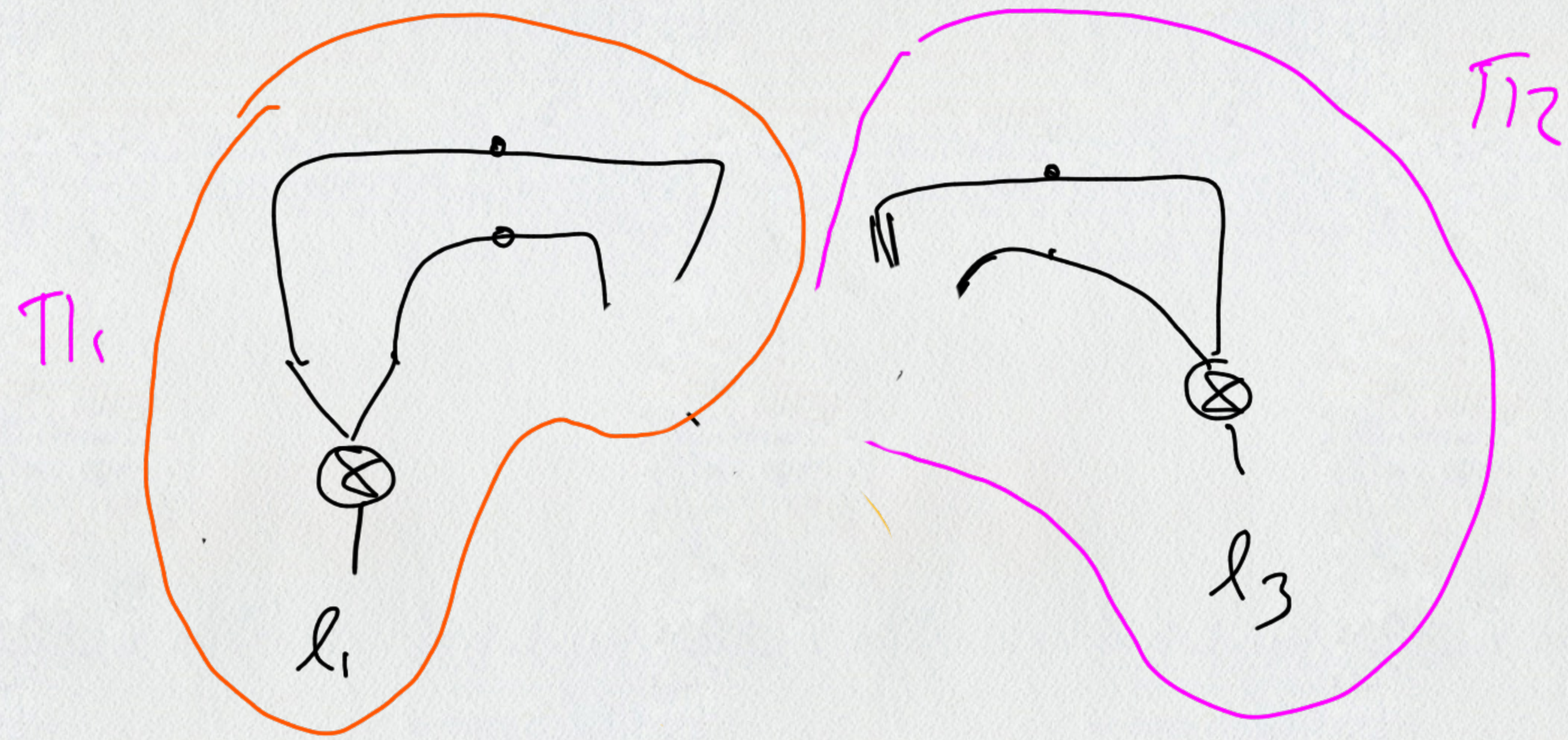
$$\frac{\vdash \Gamma, A \qquad \vdash \Delta, B}{\vdash \Gamma, \Delta, A \otimes B}\ \otimes$$

$\square$

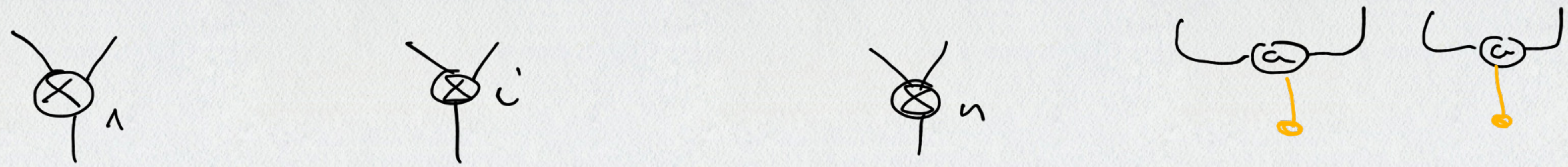
Rimane il caso delle reti con i cut link

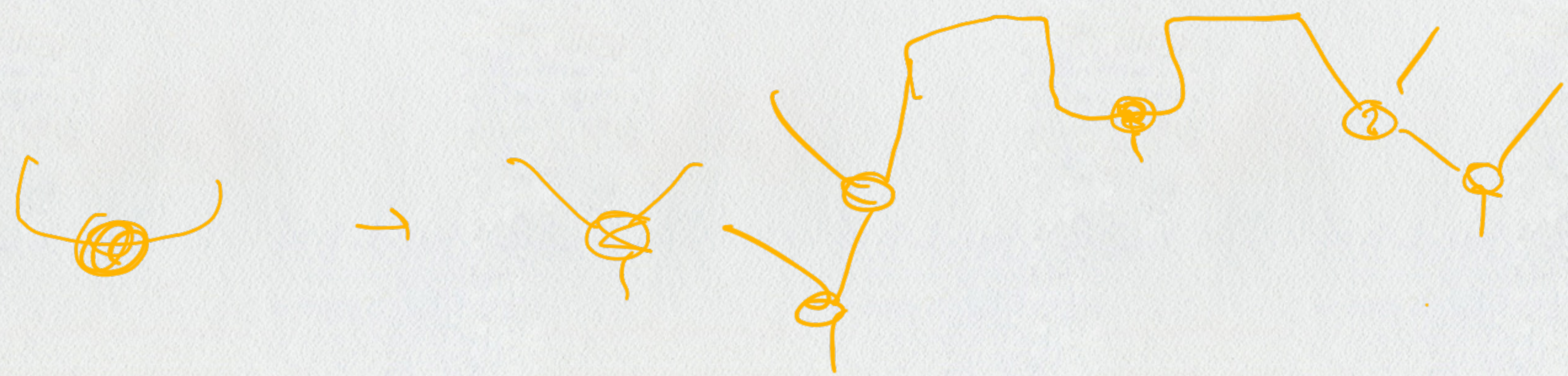

A
A⊥
B
B⊥
⊗

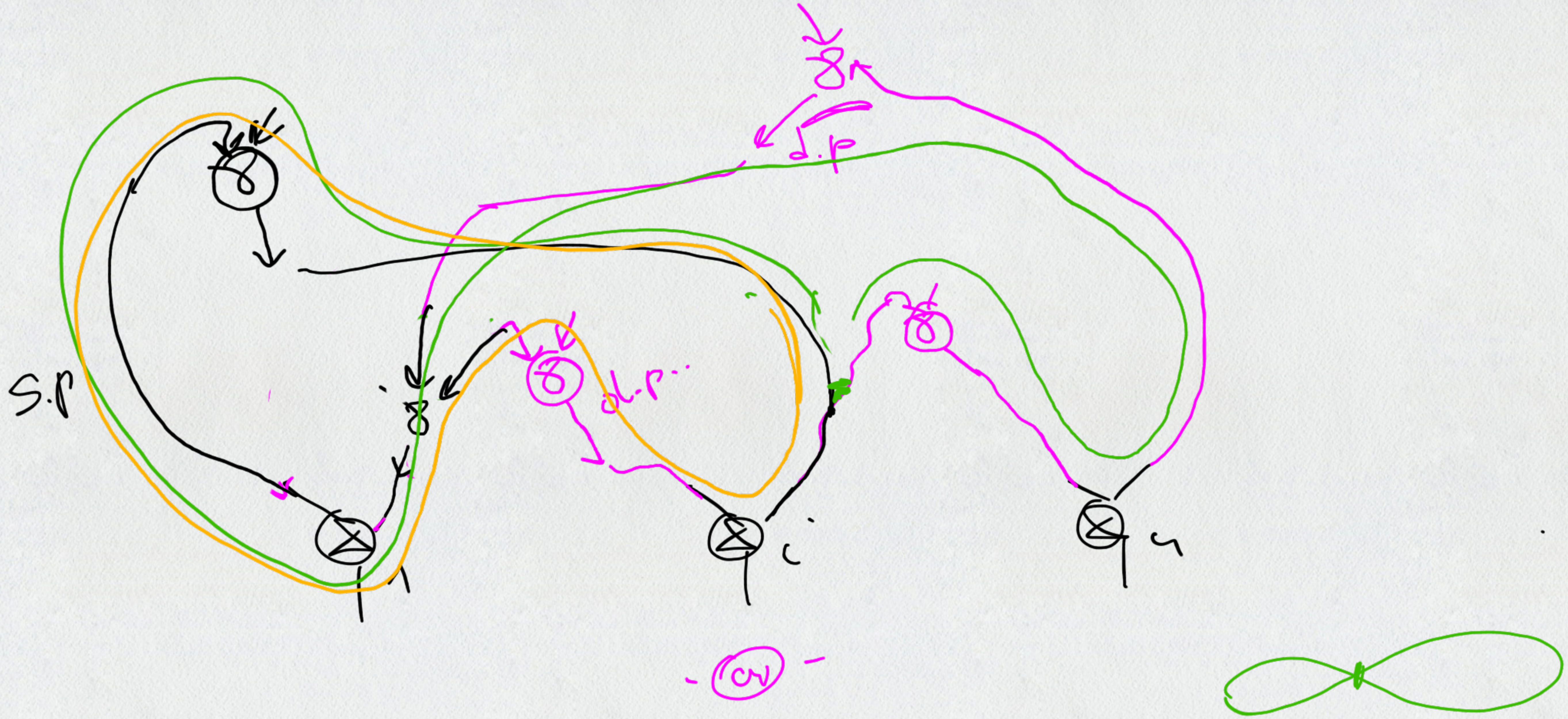

Π1
Π2
⊗
⅋
⊗ℓ2


Π1
Π2
⊗
ℓ1
ℓ3

Splitting Lemma (cont'd)



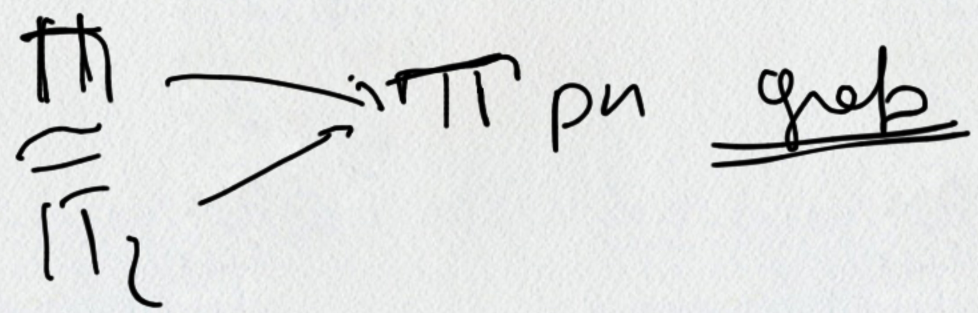




S.p
d.p
d.p

cut-elimination

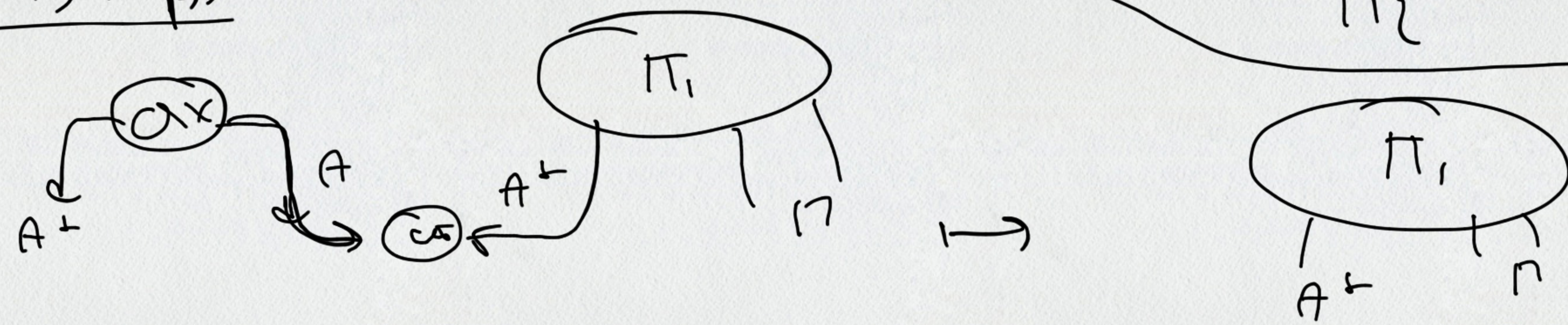
Π_1, Π_2 sono equivalenti (forte) se si sequenzializzano nello stesso proof net.



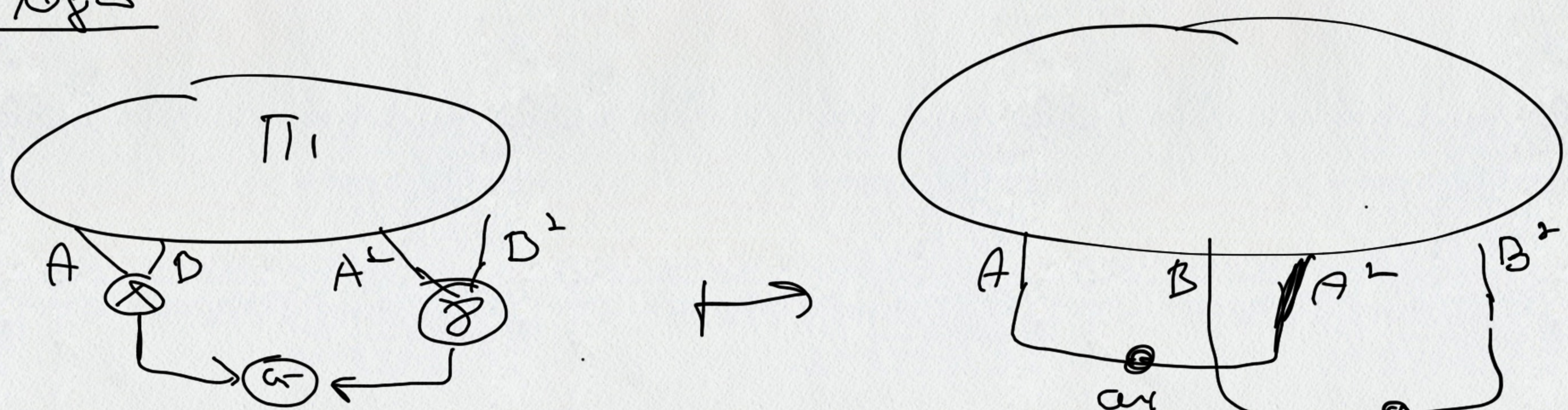
PS

reduction steps.

axiom



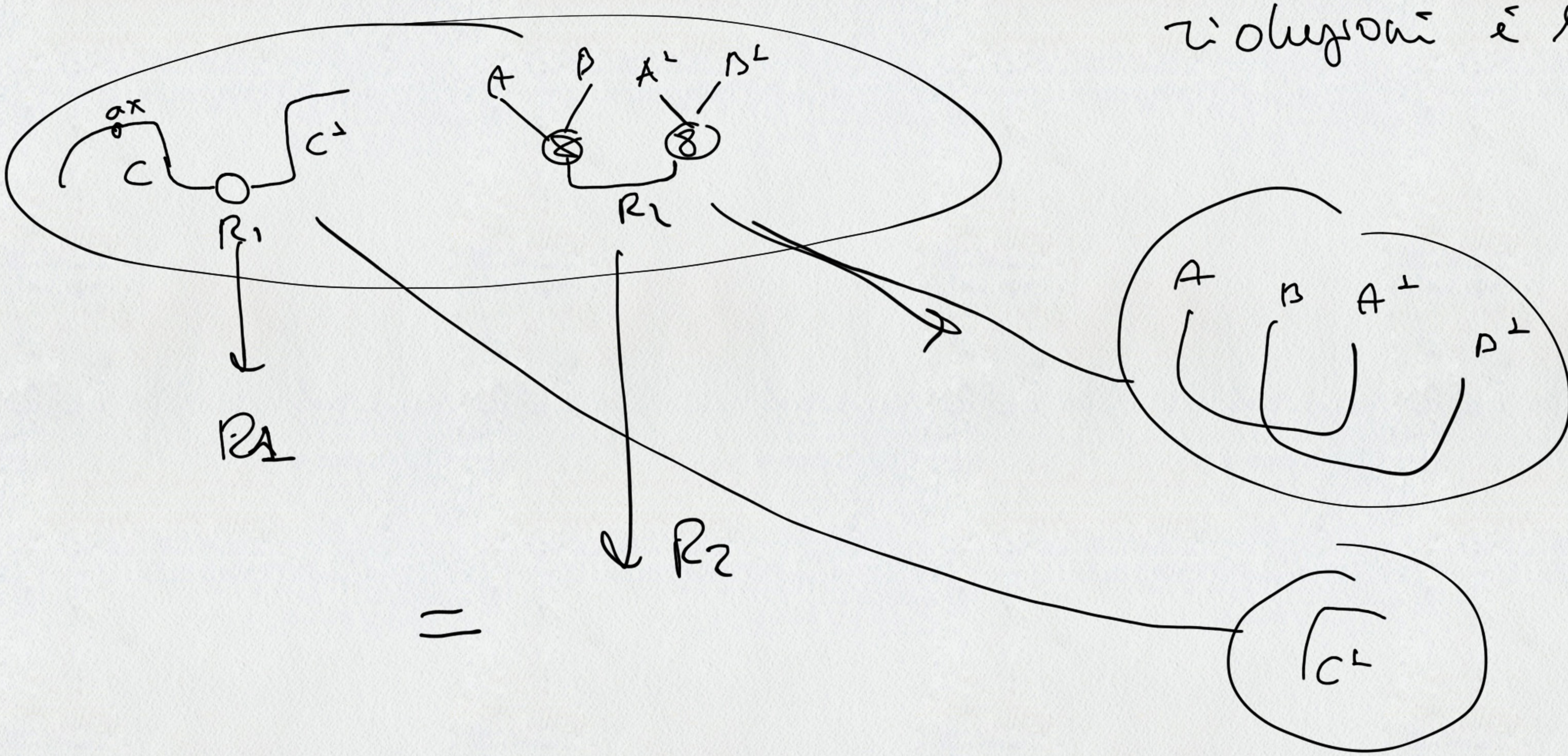
taglio logico



terminazione per induzione sulla taglia cut sulle strutture di forma

Confluenza locale è immediata.

riduzioni è locale



$R_2 \to R_1$

R_1, R_2
$R_2 - R_1$ >.

Strong normalization

Tutte le strategie di riduzione terminano con la stessa forma normale, se sono profonde.

La riduzione preserva la correttezza

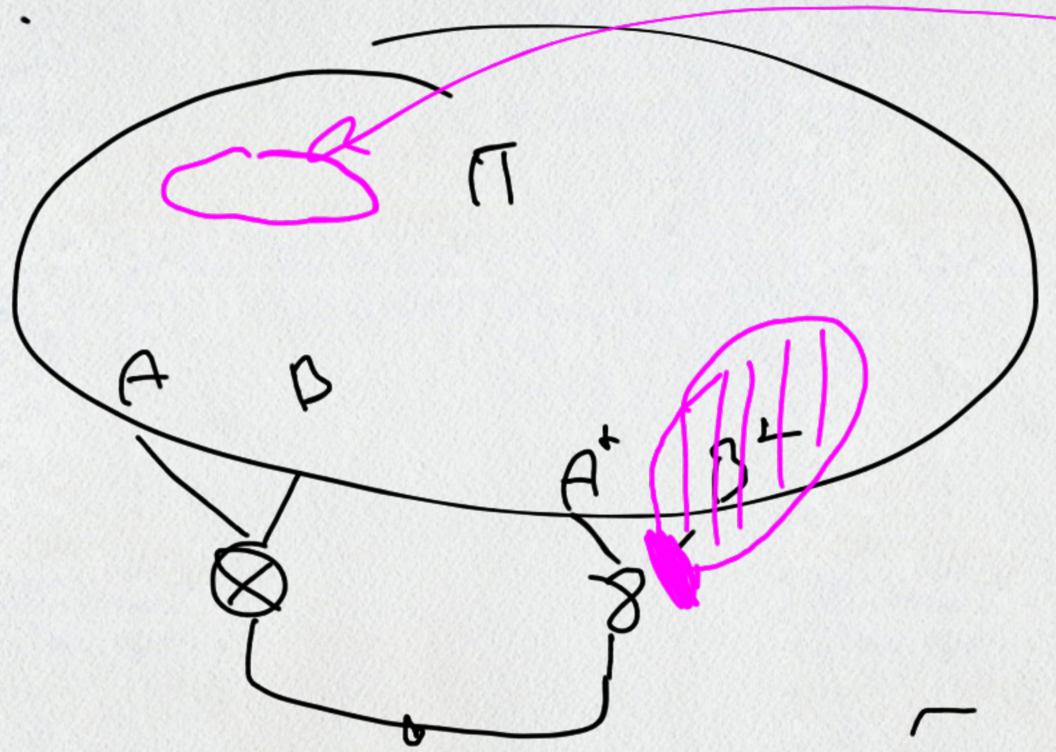
se Π è una PN che si riduce a Π' allora anche Π' è una PN.

[stabilità della correttezza (ACC) sotto la riduzione]

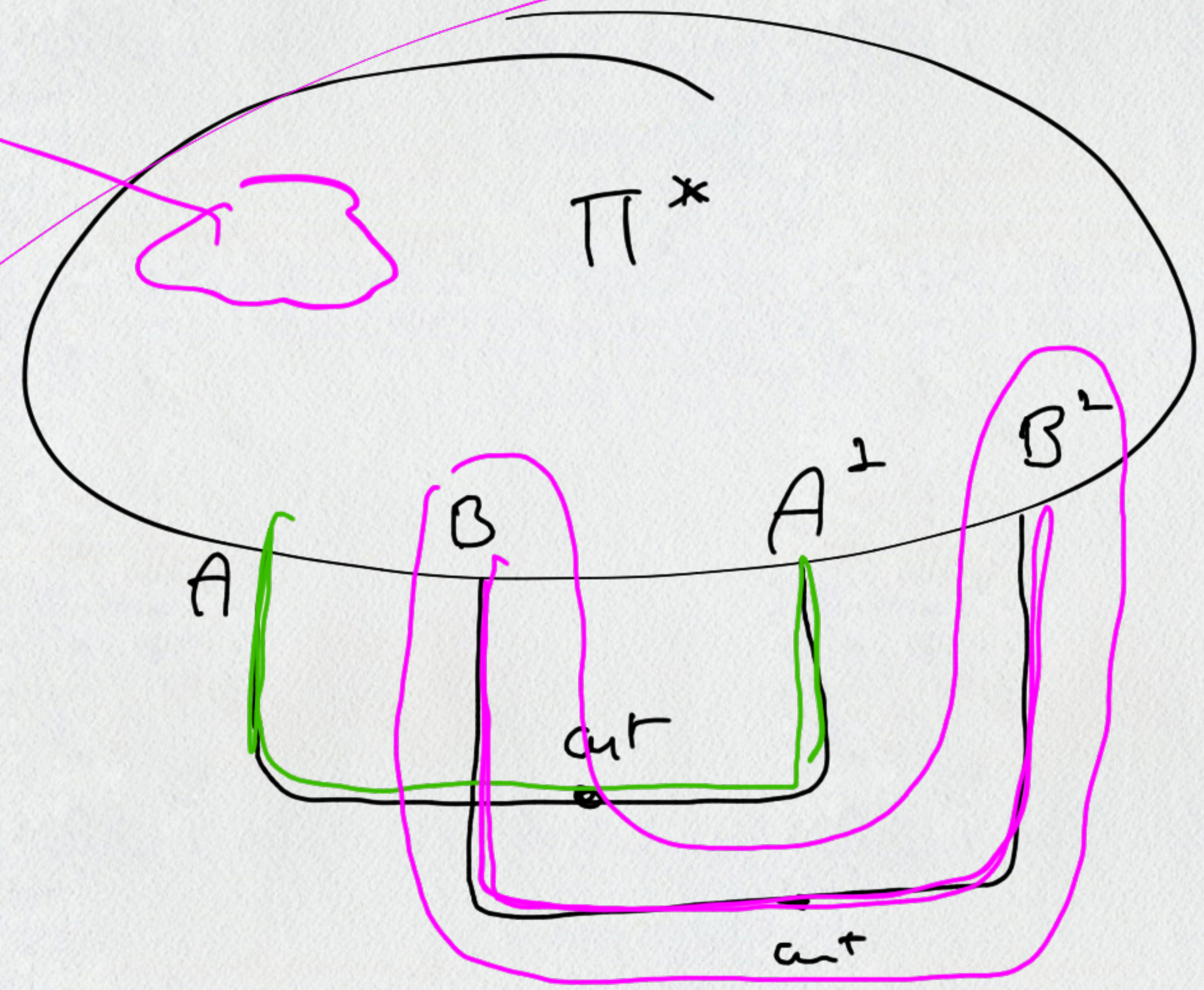
ip. x assurdo

$\exists$ un $S\varphi(\Pi')$ sconness

es.

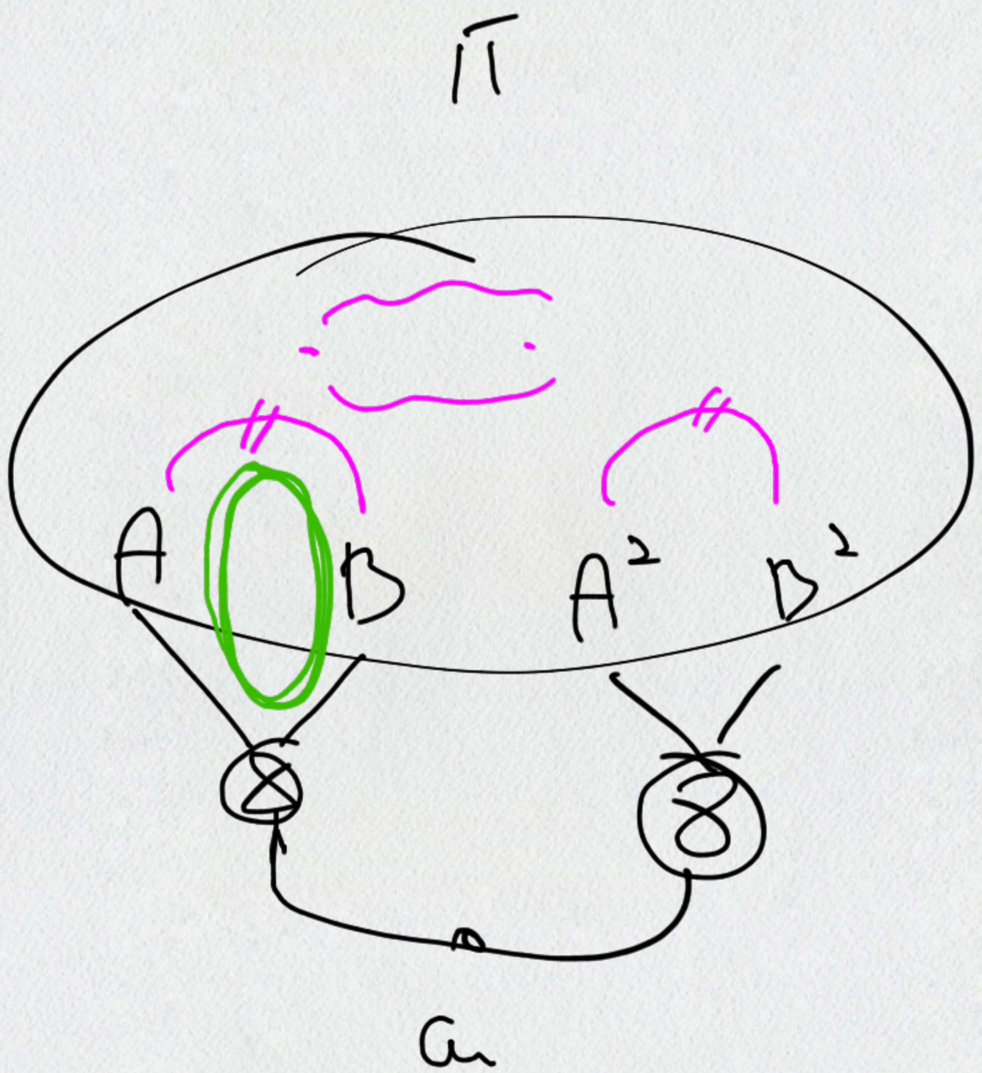


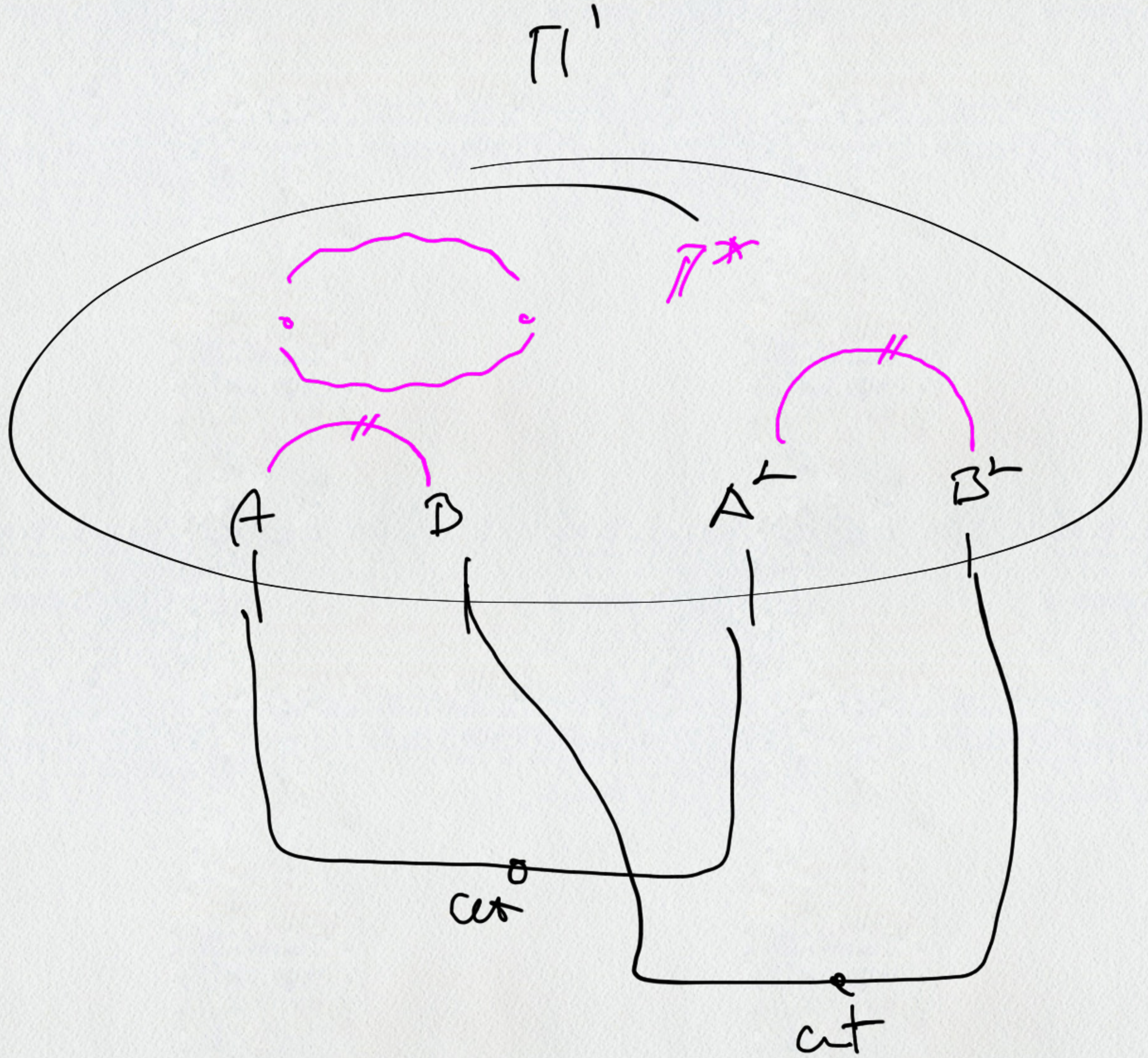
Redex



Reductum è corretto.

pro costruire un $S\varphi(\Pi)$





posso ottenere un $Sq(\Pi)$ con lo stesso ciclo.

per assurdo ipotesi

$\exists\, Sq(\Pi')$ con un ciclo

La complessità è lineare $O(n)$