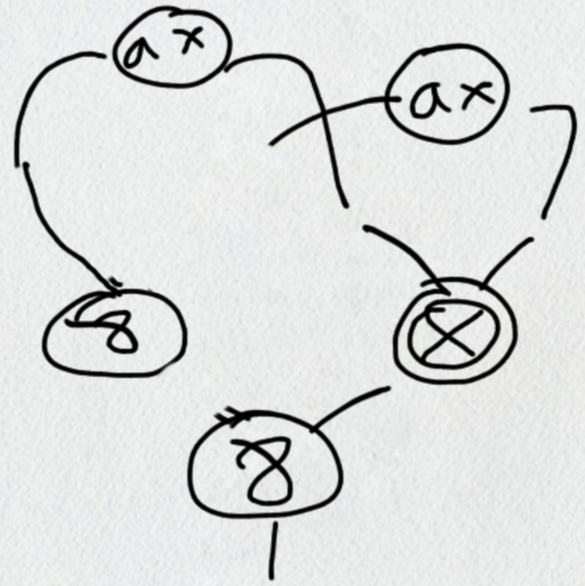
Costi : Complessità in Time — corretta per MLL — ricerca di una dimostrazione $\vdash \Gamma$?

Correttezza

π:



è una pn? — the structure of Multiplicative

criterio Danos-Regnier (1989) ACC

$\forall\, S\varphi(\pi)$ è ACC

2 ⅋ link $\Rightarrow$ 2^2 switch $S\varphi(\pi)$

$O(2^n)$ — $n = \#$ dei ⅋-link

esponenziale!

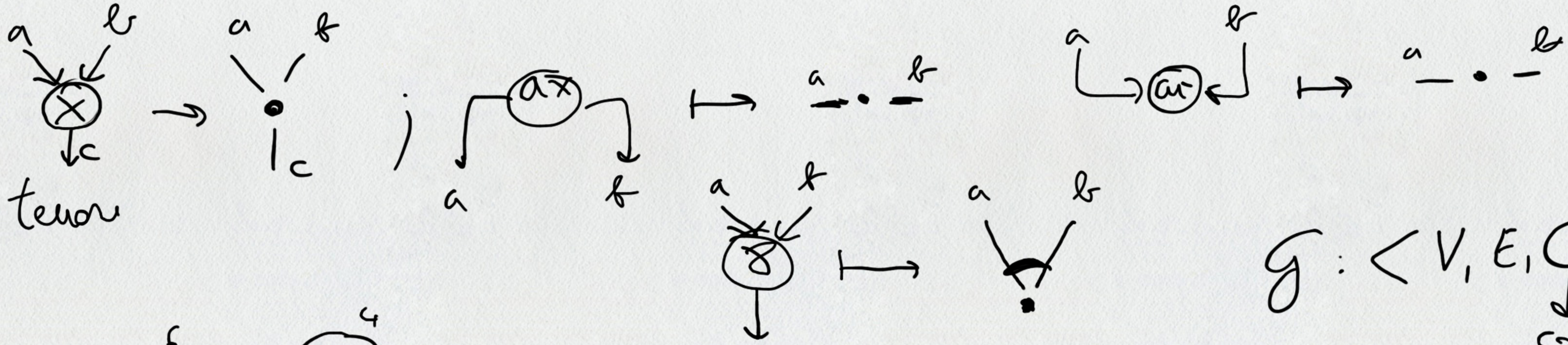
ogni grafo di correttezza (test) switching è acyclic & connected

Criterio "contrattile" — grafo $\rightsquigarrow$ deform — V. Danos phd, 1990

Criterio di correttezza su strutture di prove astratte (grafi con coppie)

PS $\Pi \longrightarrow$ struttura APS un grafo con coppie di archi "appaiati"

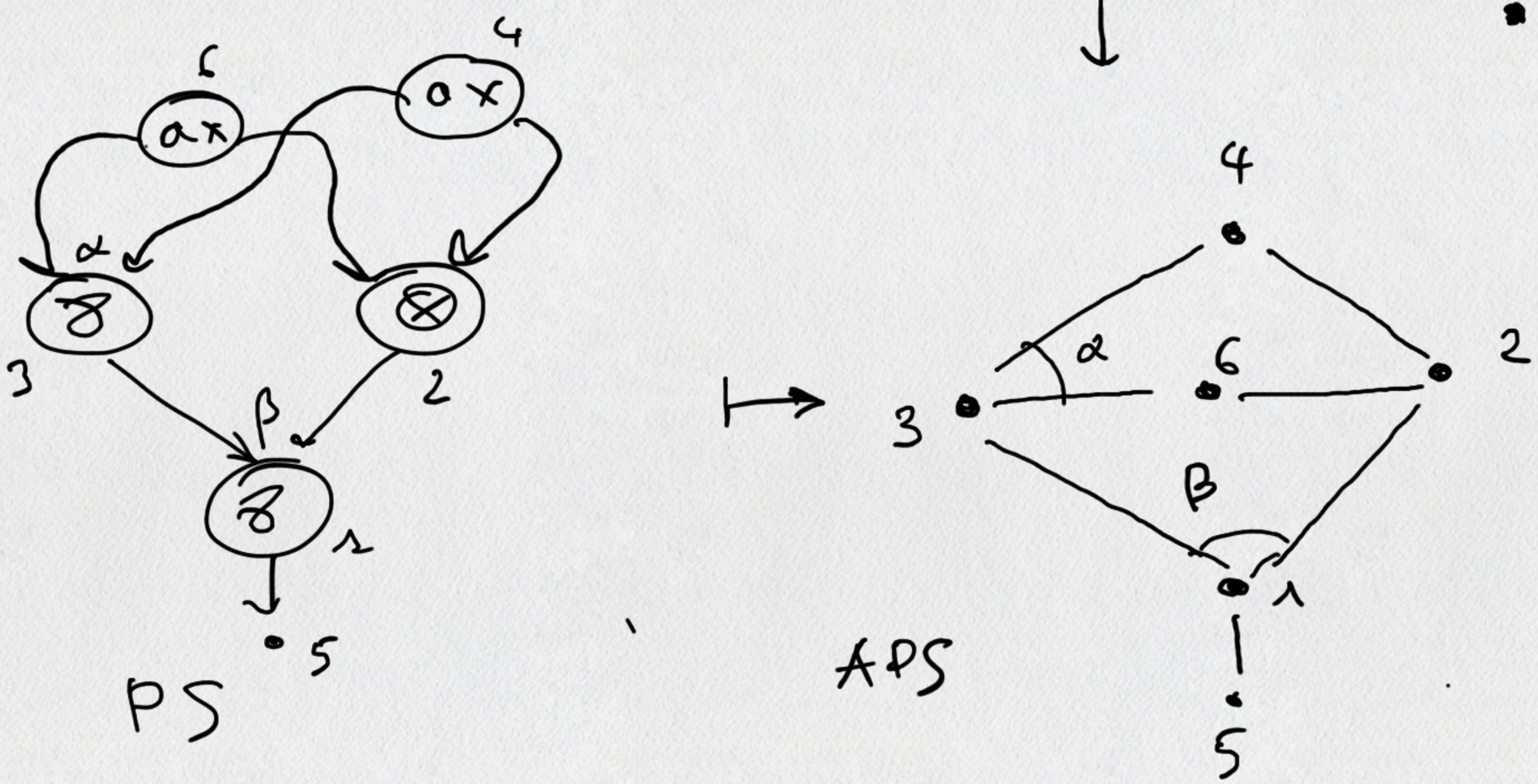
Π :



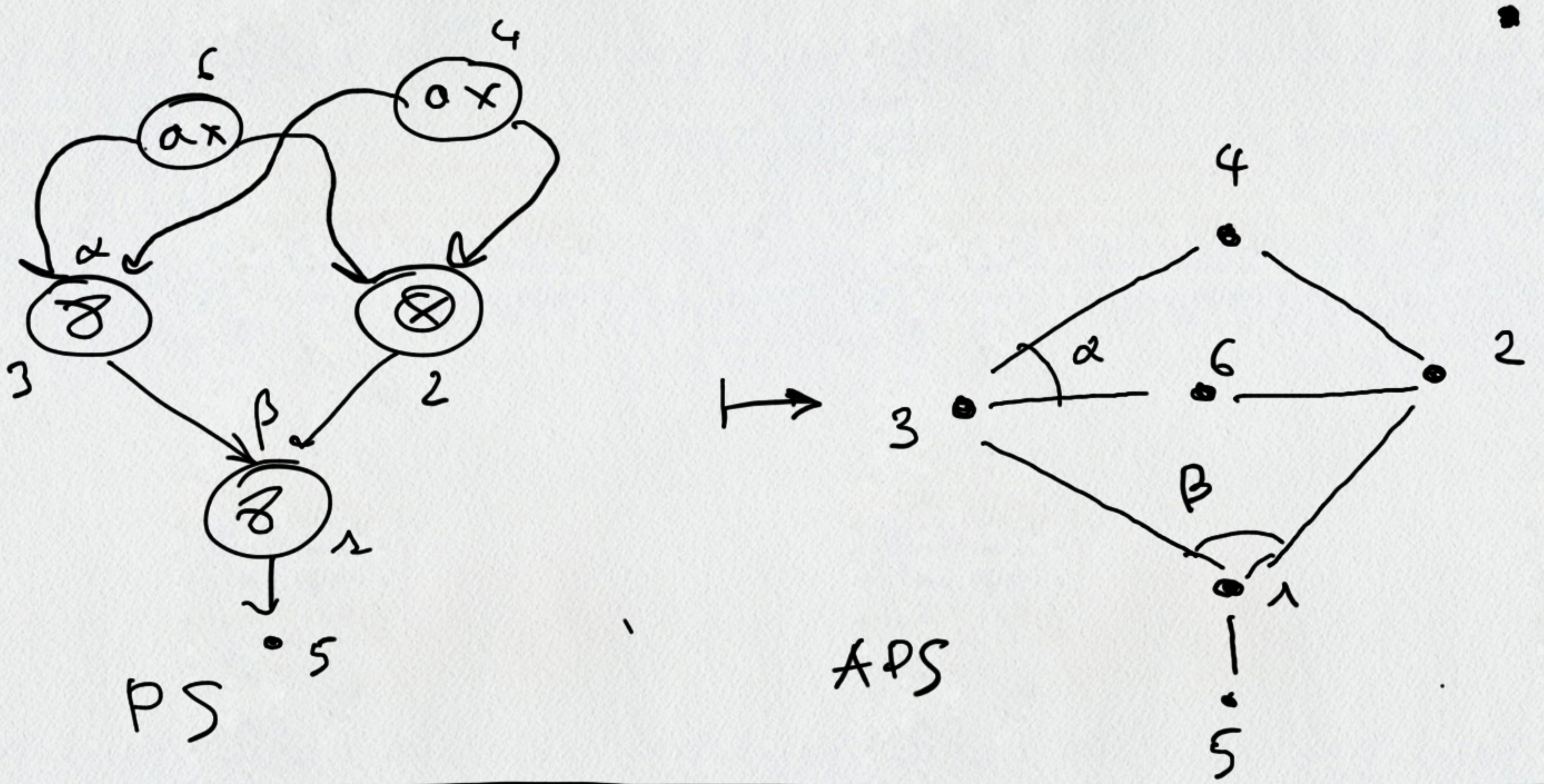
$\mathcal{G} : \langle V, E, C \rangle$

coppie

esempio

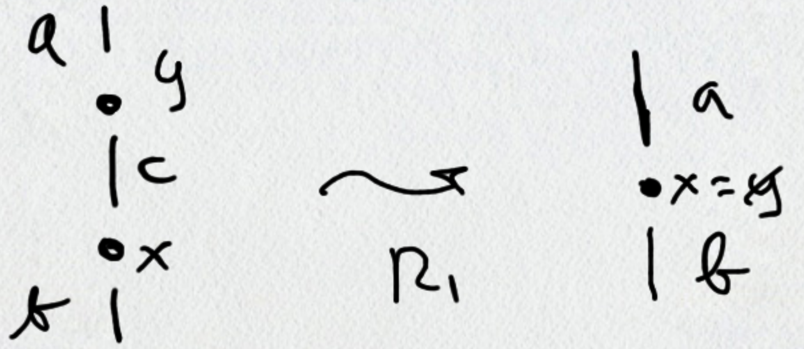


esempio

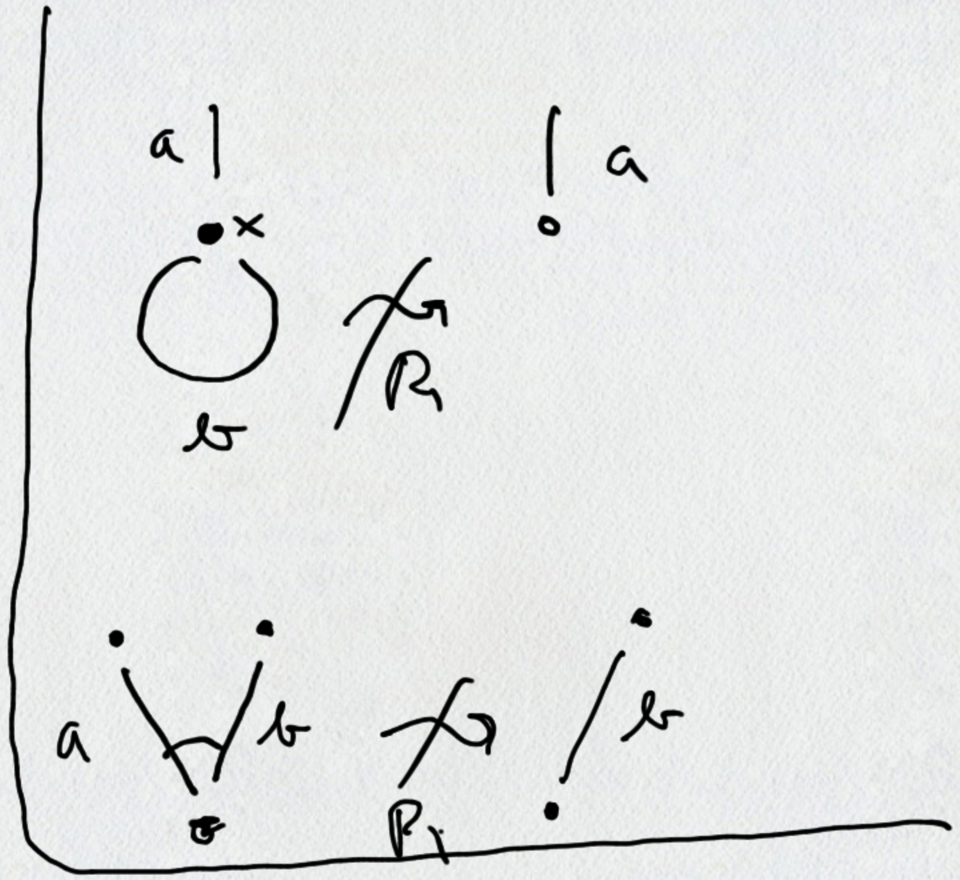


Def regole di riscrittura del graf / o regole di contrazione

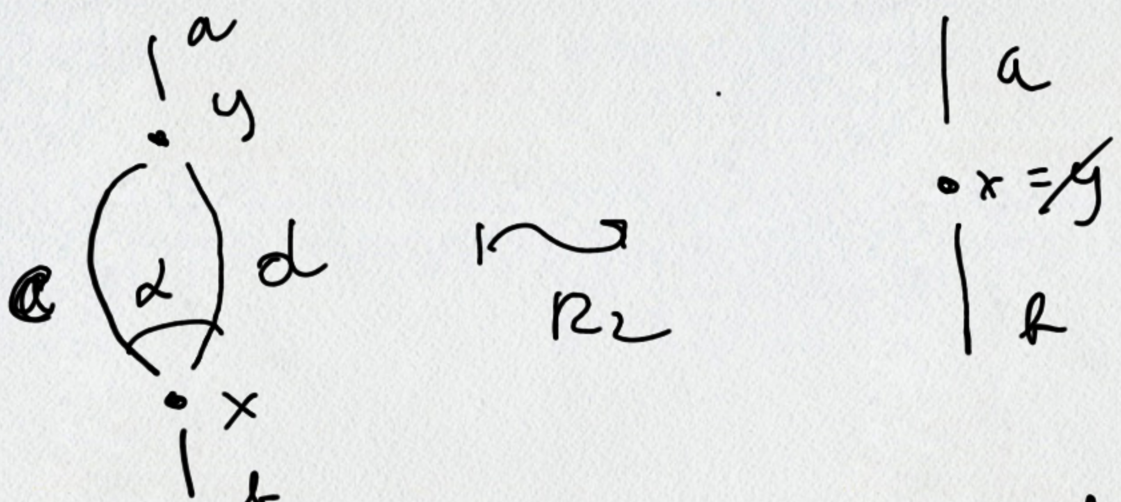
- R_1:



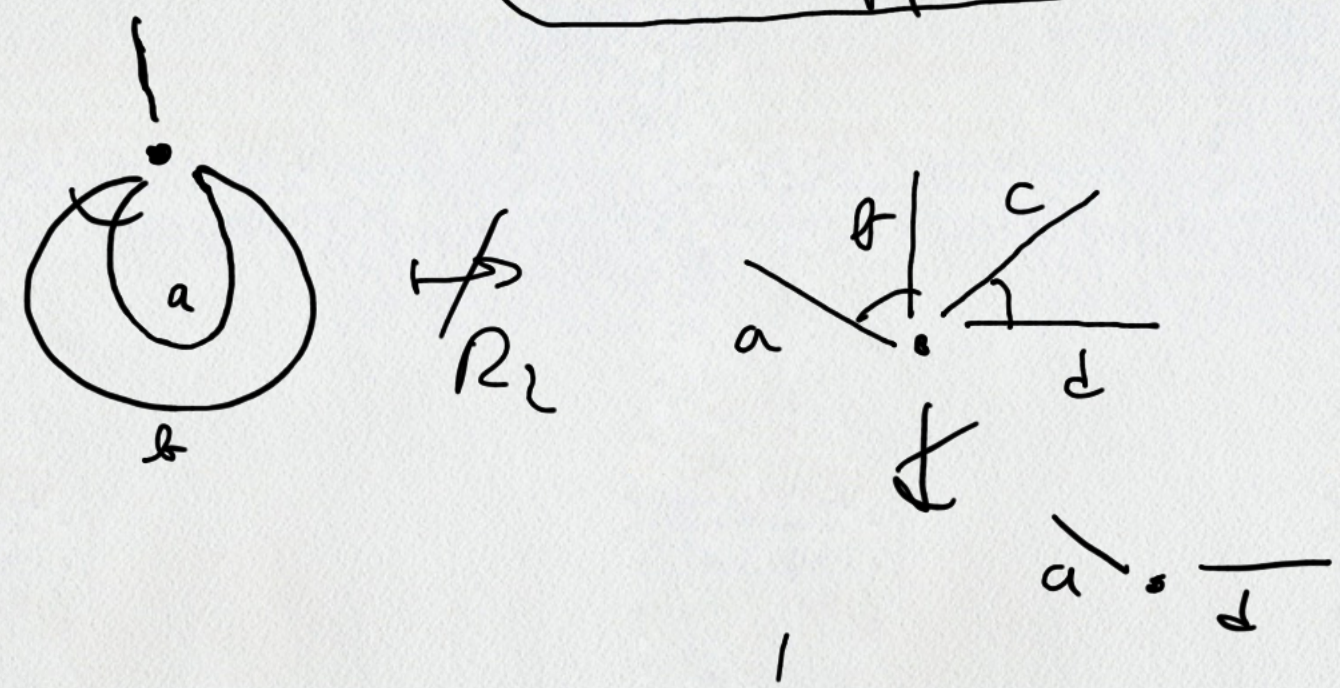
$\boxed{x \neq y}$ e l'arco c non appartiene ad alcuna coppia



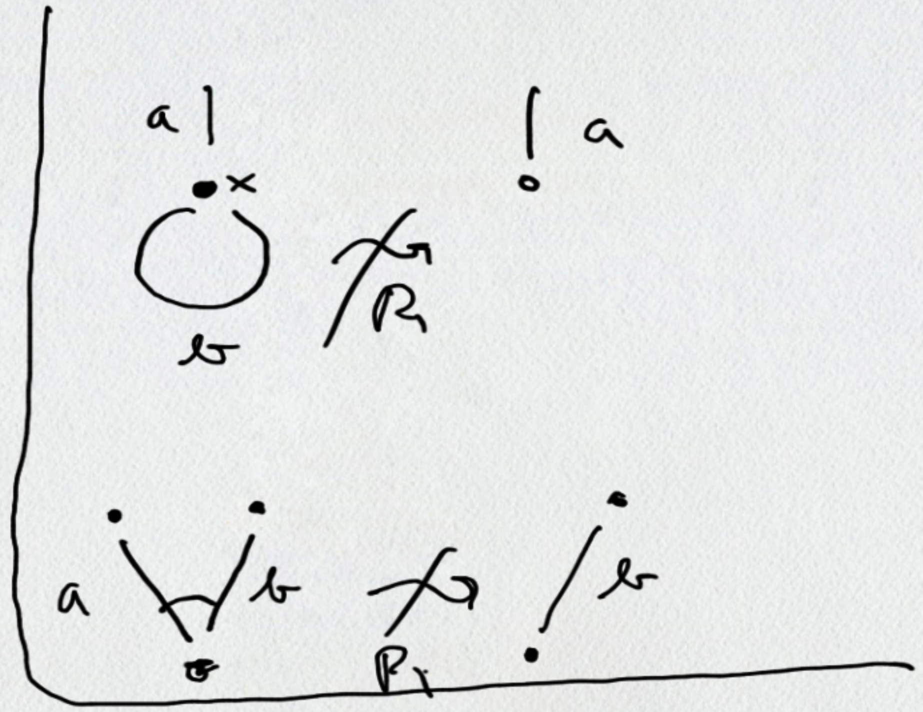
- R_2



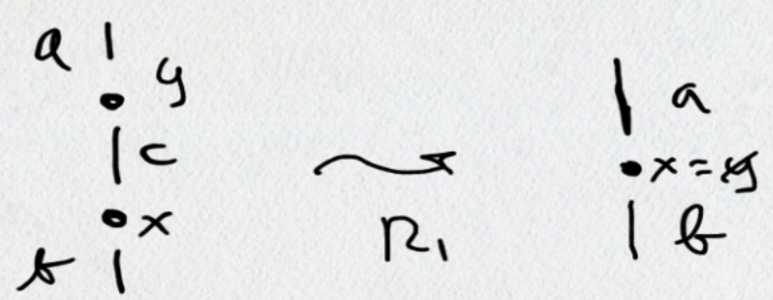
$\underline{x \neq y}$ e c, d appartengono alla stessa coppia



Def. regole di riscrittura del graf / o regole di contrazione

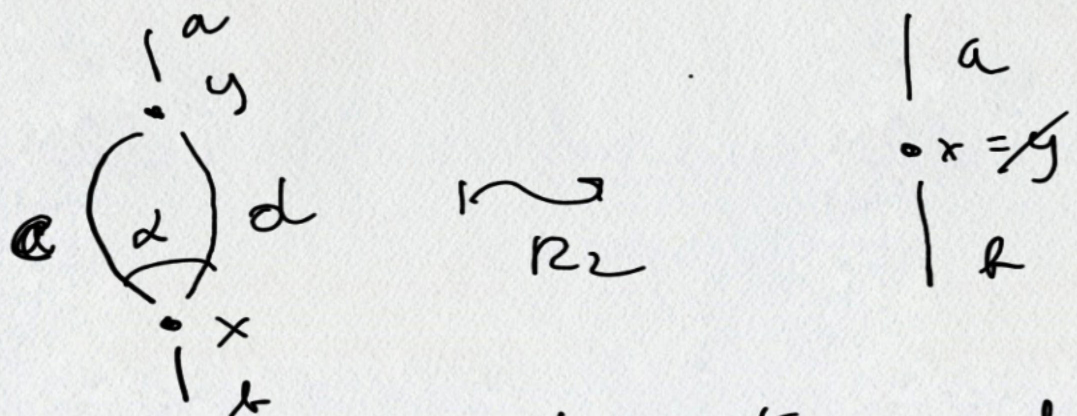


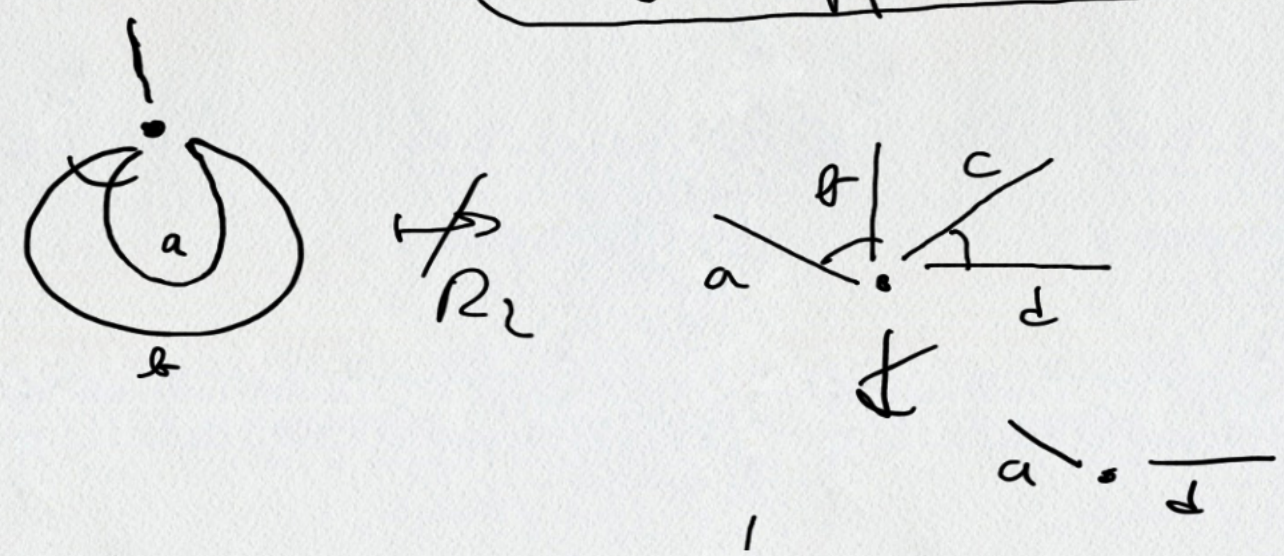
- R_1:



$\boxed{x \neq y}$ e l'arco c non appartiene ad alcuna coppia

- R_2





$x \neq y$ e c, d appartengono alla stessa coppia

Criterio: una ps Π è corretta (pn) sse la corrispondente APS Π^* si contrae (in un numero finito di passi) in un singolo nodo •

ps $\Pi \longmapsto$ APS $\Pi^* \rightsquigarrow \dots \rightsquigarrow \bullet$

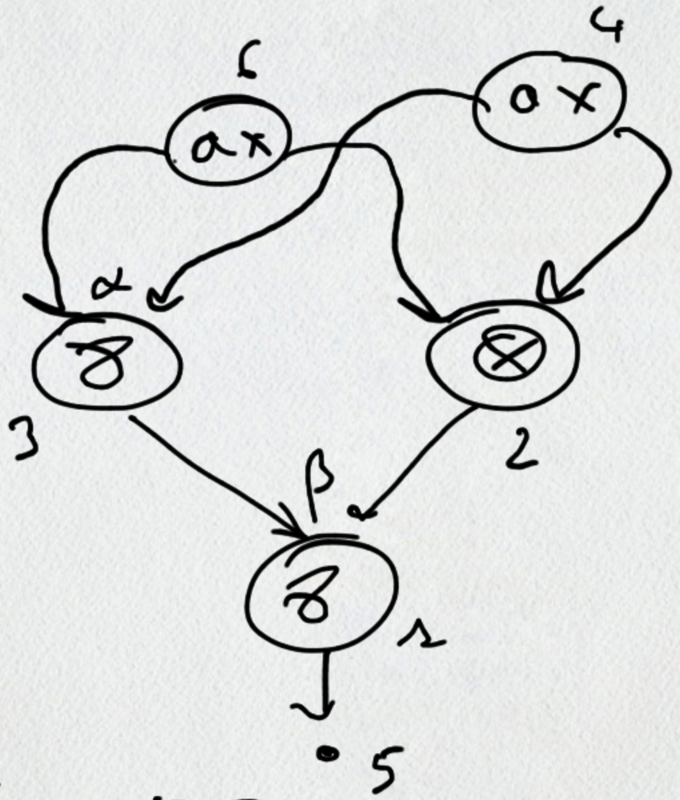
lineare nella taglia del graf

esempio

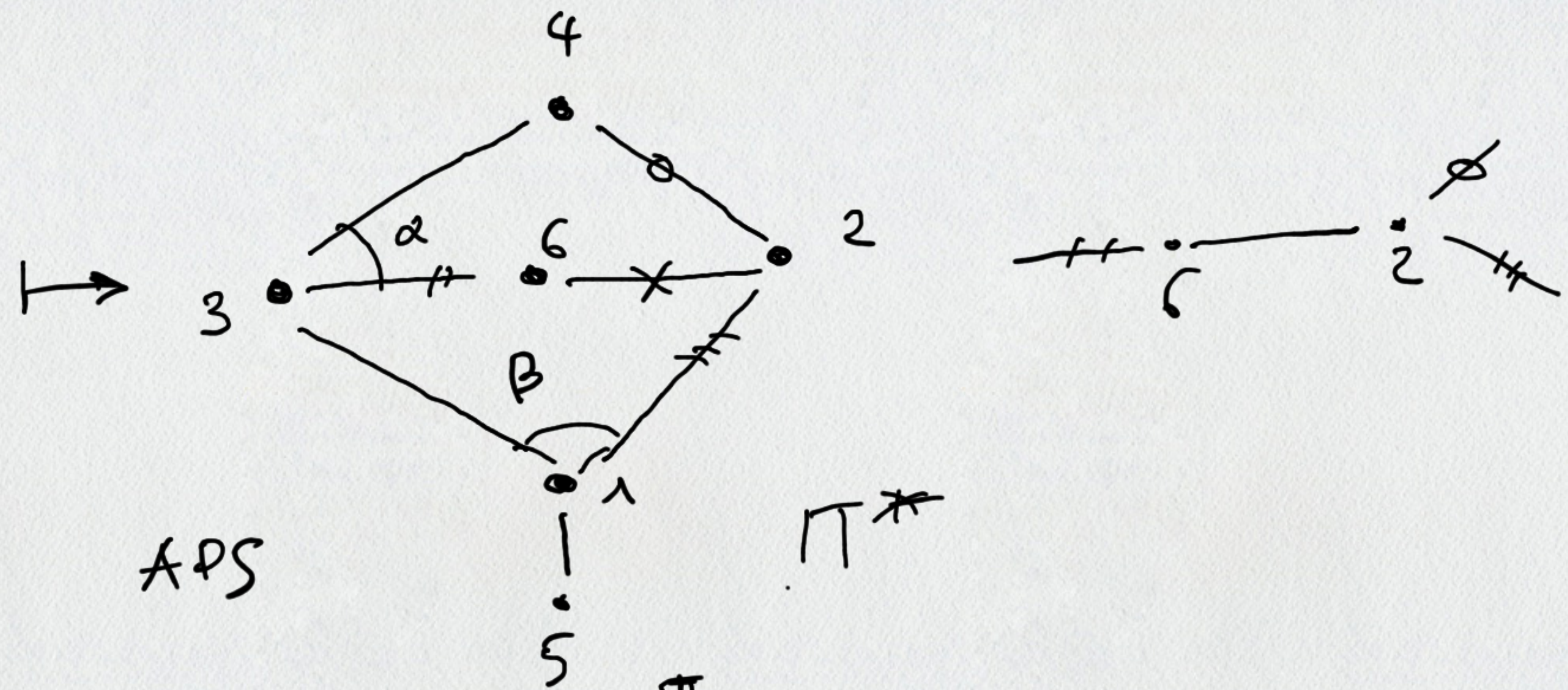
Π

è corretta

?

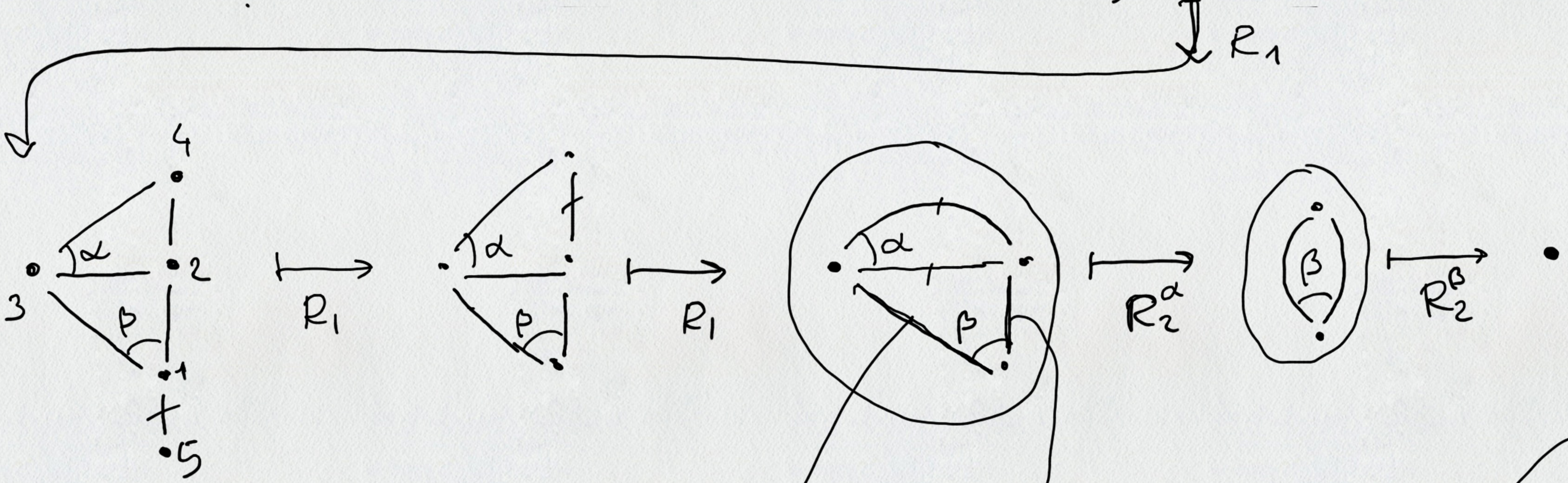


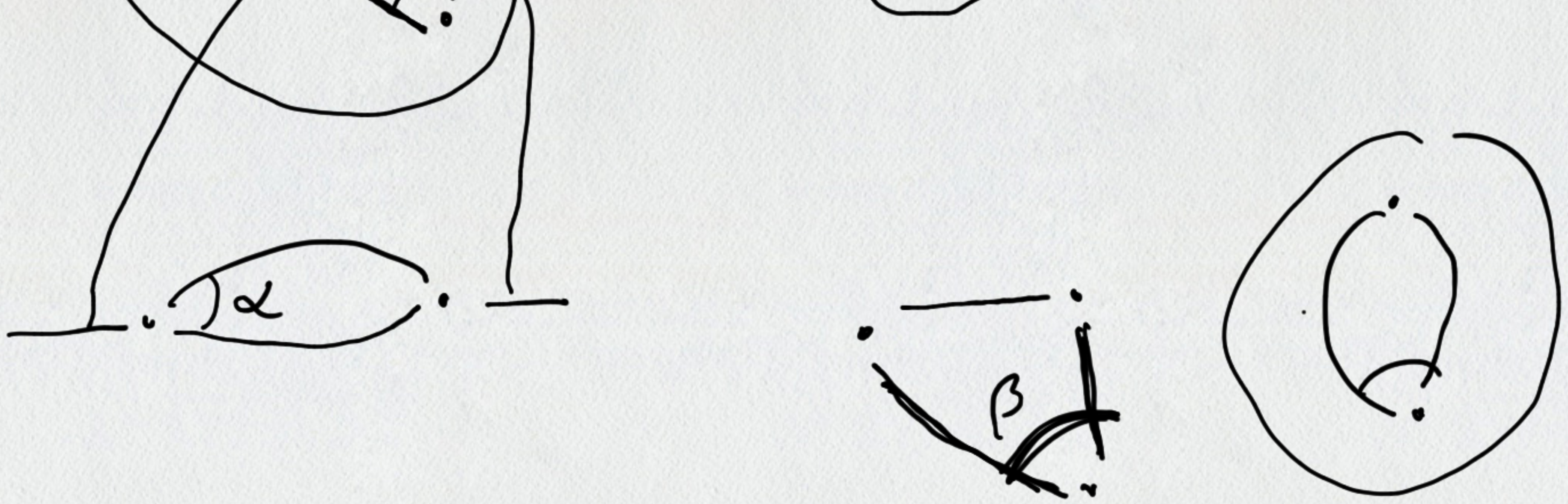
PS



APS

Π^*



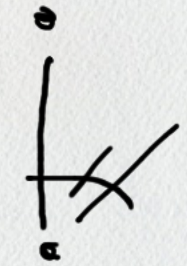


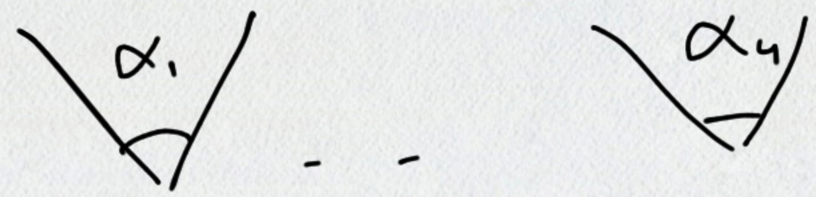
Conto delle contrazioni

$\Pi = \langle V, E \rangle$

Π ps $\longrightarrow$ APS Π^* lineari nella size di Π

$\Pi^* \rightsquigarrow \ldots R_1$





n - controlli R_2 nel peggiore dei casi

$n-1$ controlli

$\vdots$

$n-i$

$\vdots$

1 controllo

$1+2+3+\ldots+n-1+n$ controlli

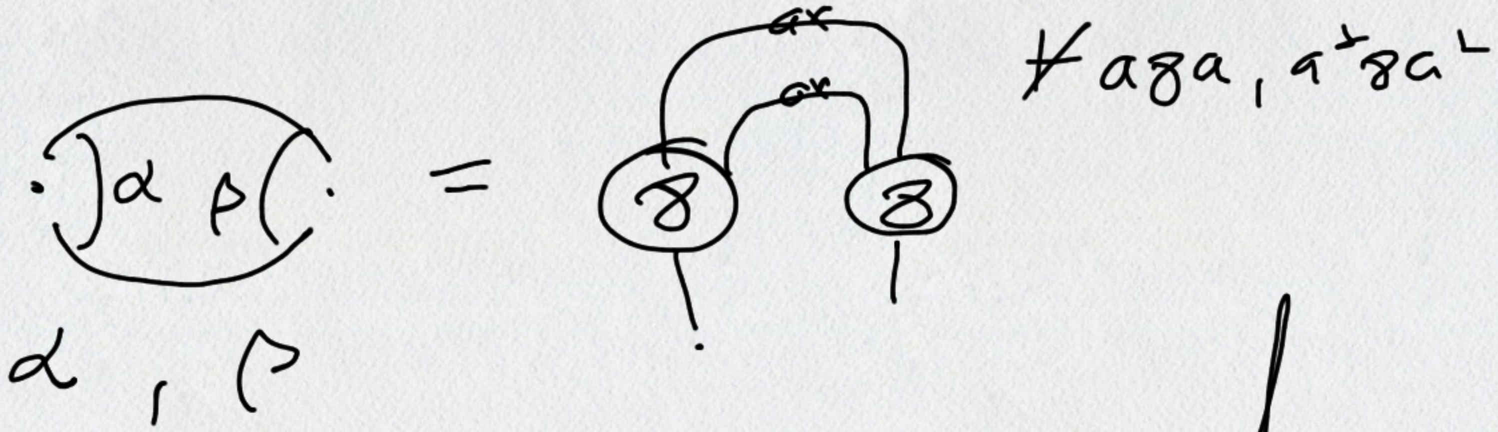
Sommatoria di Gauss

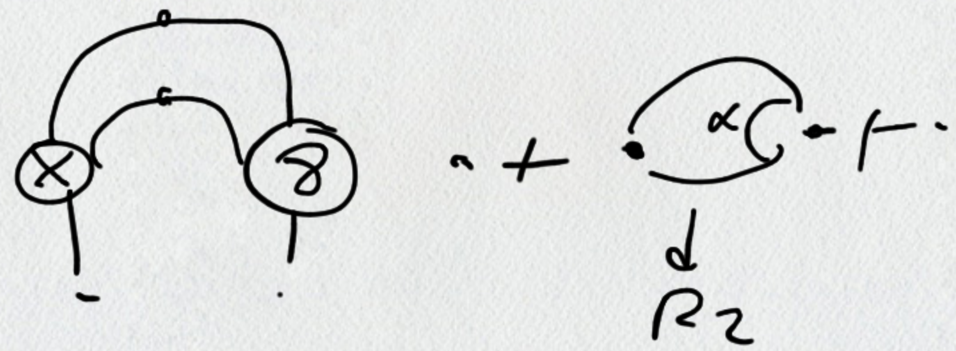
$$\begin{array}{r} 1+2+3+\ldots+n-1+n\; + \\ n+n-1+n-2+\ldots+2+1 \\ \hline \end{array}$$

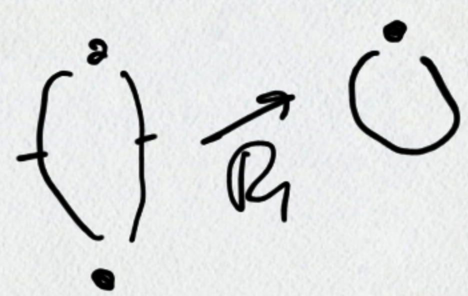
$$= \frac{(n+1)\cdot n}{2} \qquad \sum_{i=1}^{n} = \frac{n\cdot(n+1)}{2} = O(n^2)$$

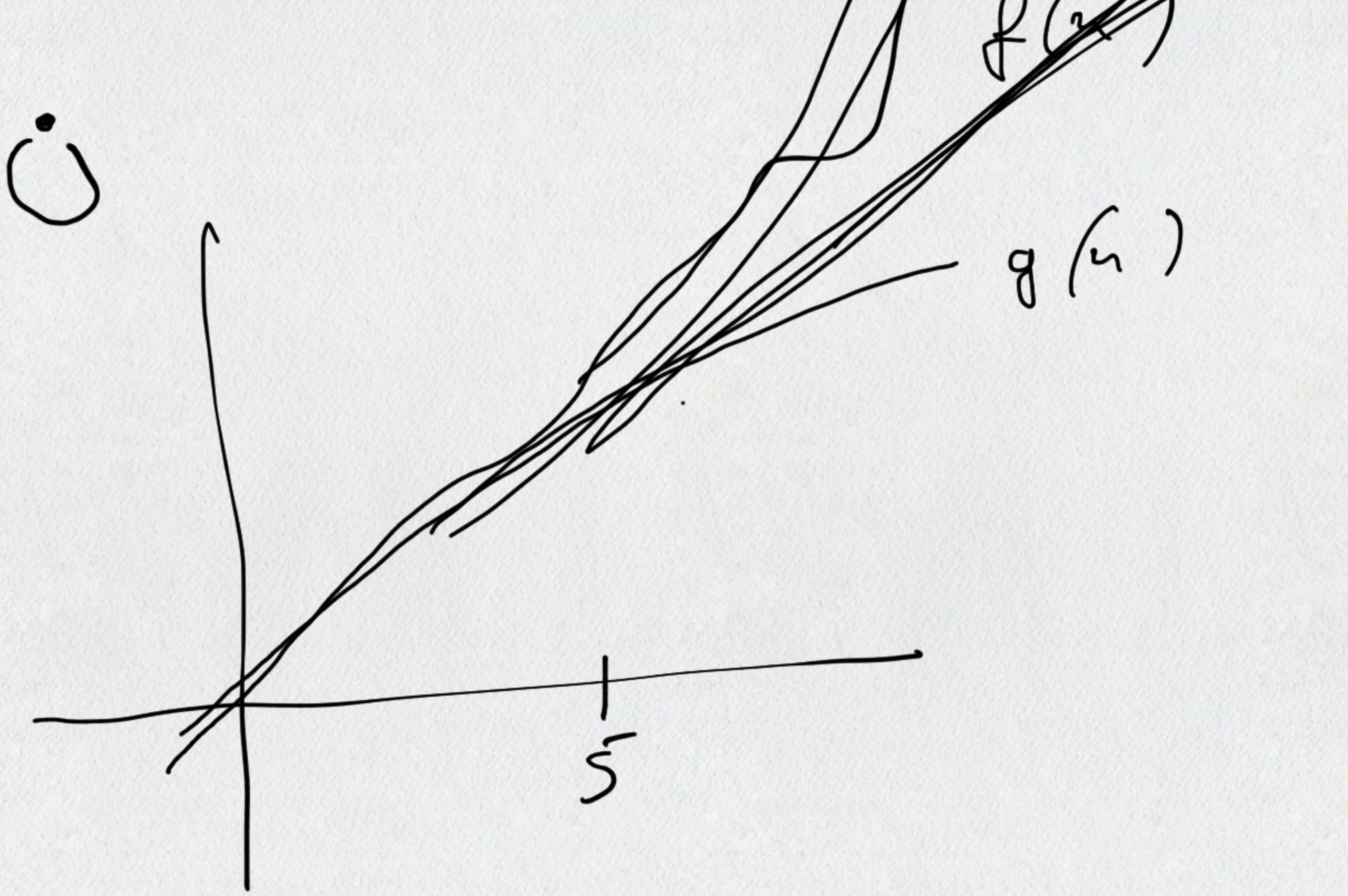
Confronto

ACC	$f: 2^n$	
ee	$g: n^2$	
$- \; n = 2$	$f(n) = 4$	
	$g(n) = 4$	
$- \; n = 3$	$f(3) = 8$	
	$g(n) = 9$	
$- \; n = 4$	$f(n) = 16$	
	$g(n) = 16$	
$- \; n = 5$	$f(n) = 32$	
	$g(n) = 25$	



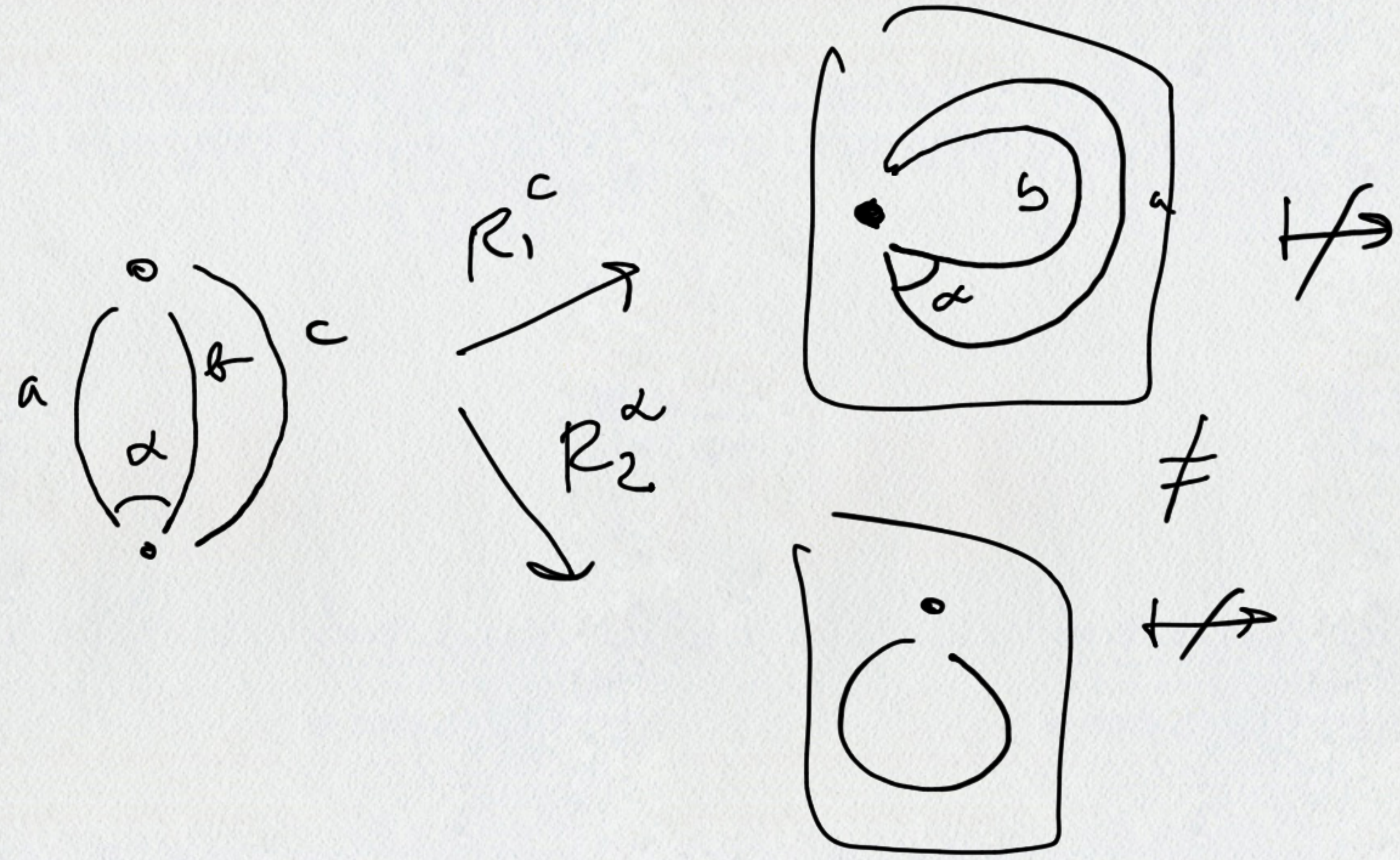






Guerrini S. 2000, usando alg. di unificazione. lineare $O(n)$

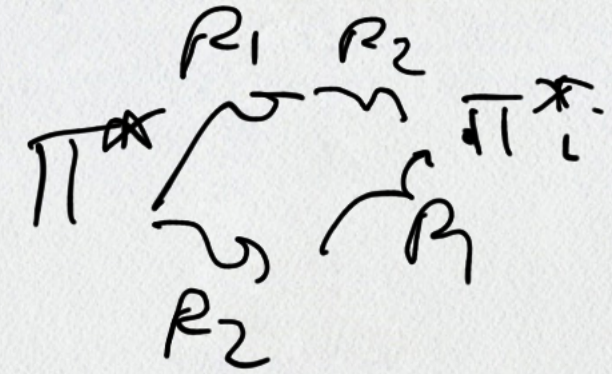
se Π^* non è contraibile $\Longrightarrow$ non è detto di essere confluente



<u>non è confluente</u>

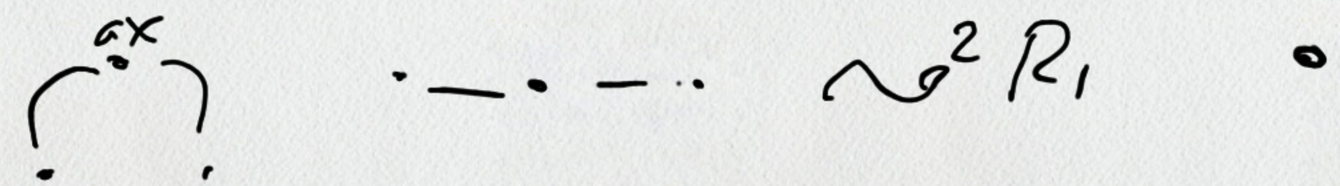
se $\exists$ una strategia $\sigma : \Pi^* \rightsquigarrow \Pi^*_1 \dots \rightsquigarrow^* \Pi^*_n = \bullet$ che termina in forma normale $\bullet$, allora tutte le strategie di contrazione terminano con la medesima forma normale $\bullet$.

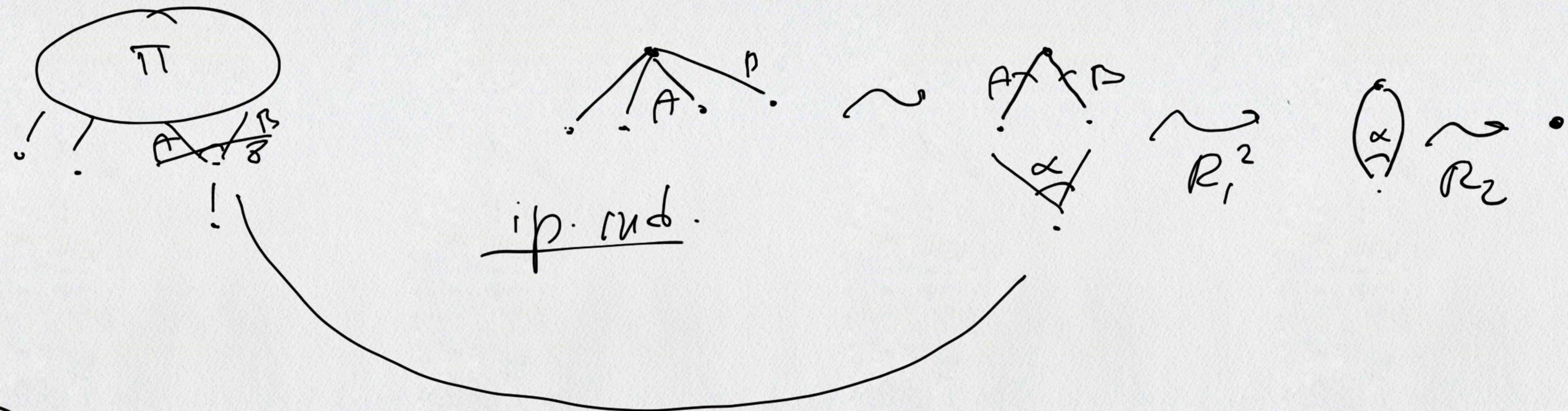
$CC \equiv ACC$

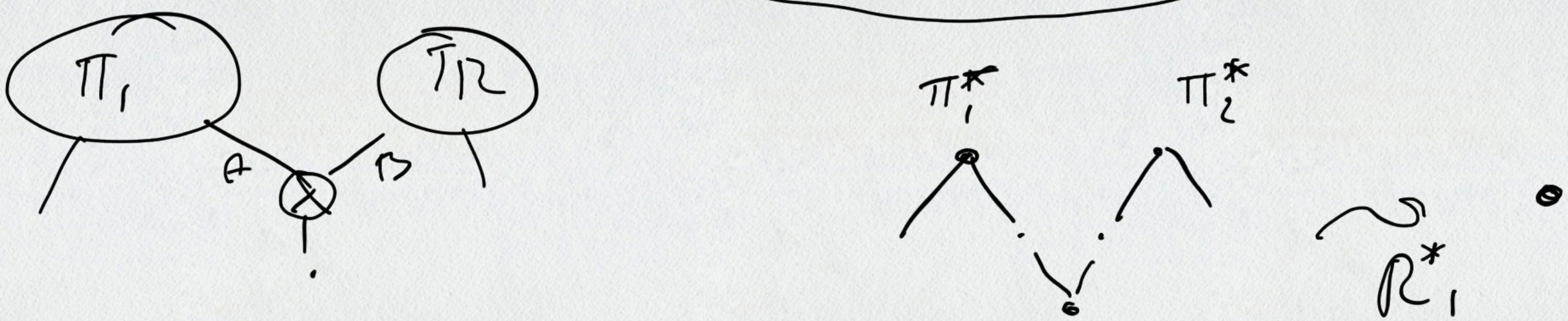


th Π è corretta ACC $\Longrightarrow$ Π è conv con CC

proof: ind sulla Taglia di Π

ax 

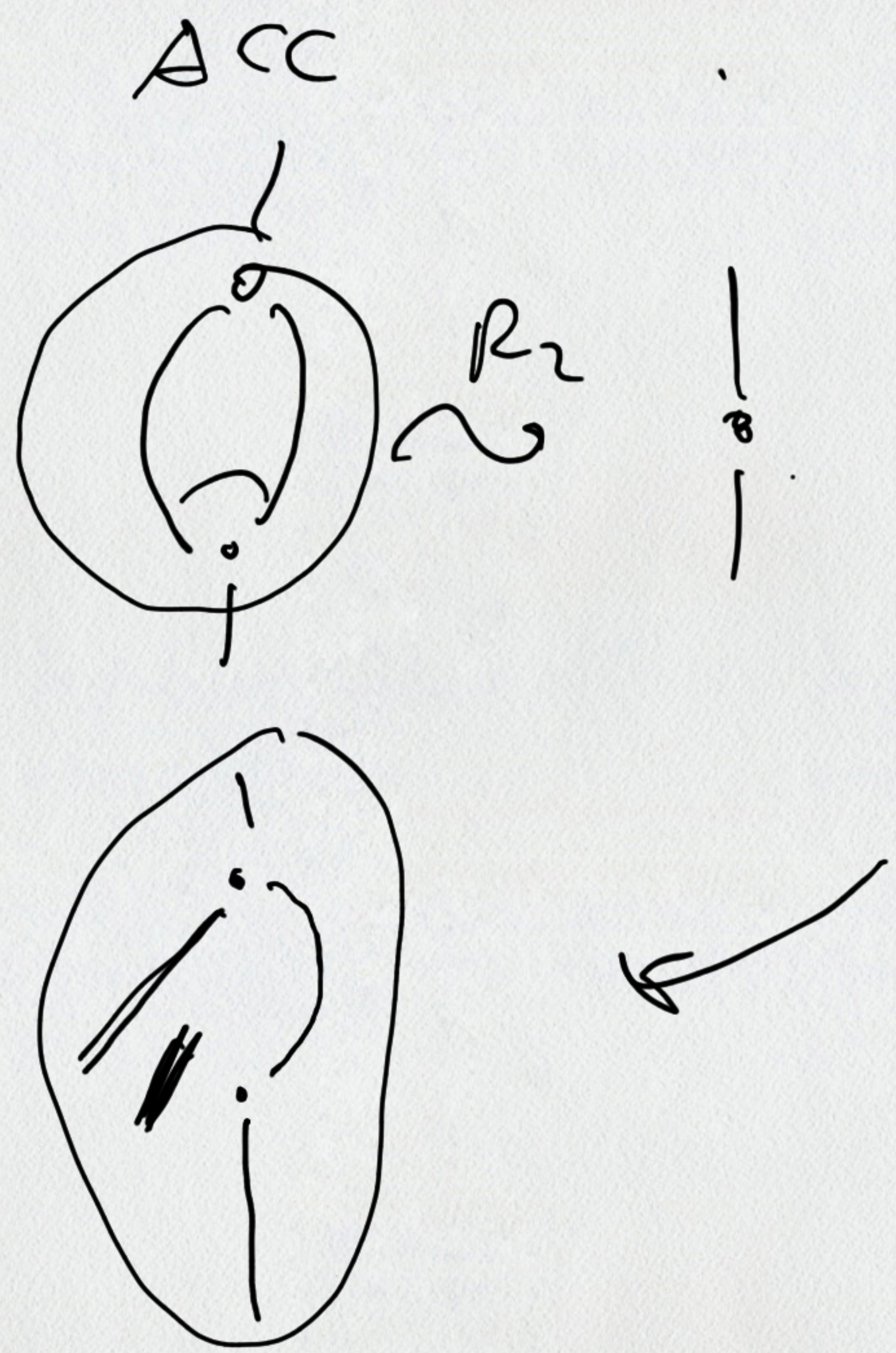


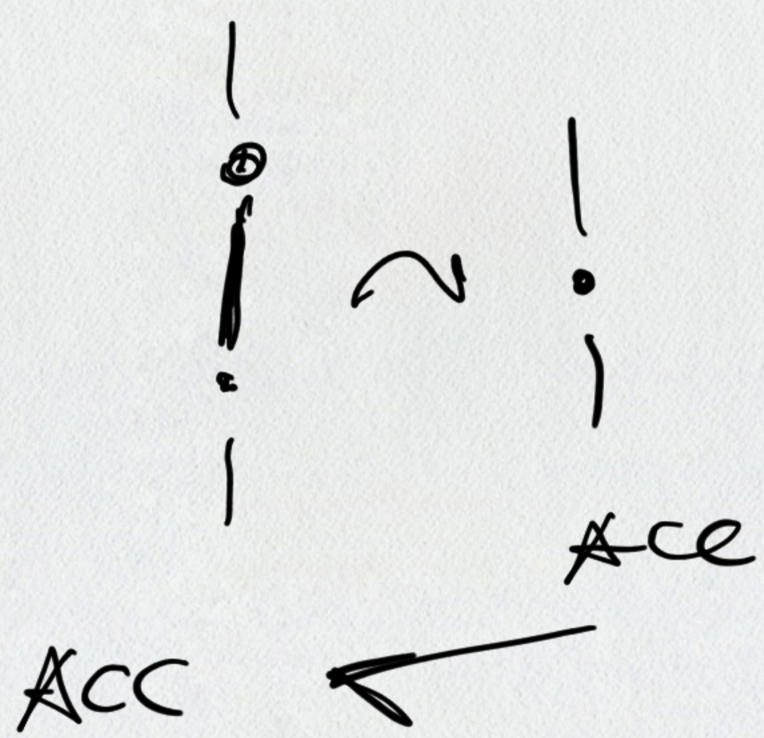


se $\Pi_1^* \overset{R_i}{\rightsquigarrow} \Pi_2^*$ $i = 1, 2$

allora Π_1^* è ACC se Π_2^* è ACC

$\boxed{\#V - \#E = 1}$ $\underline{\text{ACC}}$





Ricerca delle dimostrazioni <u>Proof-search</u> (automatica)

$$\vdash \underline{d^\perp}, \underline{(d \otimes b)}, a^\perp, \underline{(a \otimes b) \otimes c}, c^\perp$$

? dimostrabile?

<u>cut-free</u>

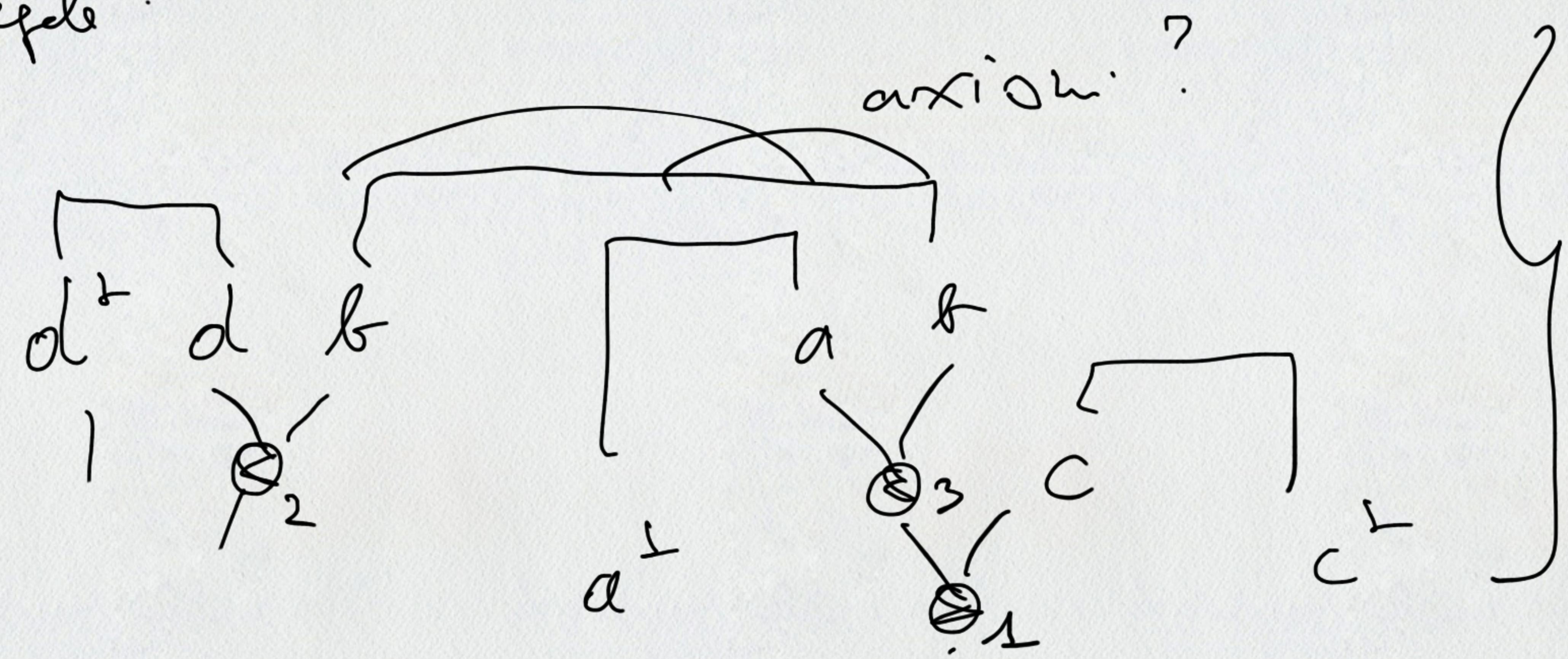
calcolo dei sequenti

$⅋ / \otimes$?

qual è ultima regola?

axiomi logic

bottom-up



regola del ⅋ è negativa
regola del $\otimes$ è positiva

$$\left\{\begin{array}{l} 1-3-2 \\ 1-2-3 \\ 2-1-3 \end{array}\right. \Big\} \to \text{proibite}$$

Ridurre il non-determinismo (il costo)

- don't care non-determinism (parallelismo) $\left\{\begin{array}{l}\text{formule} \\ \text{asynchrone } \parr\end{array}\right.$
- true non-determinism (sincronicità/scelte) formule synchrone

$\otimes$ vero non-det.

splitting

$$\dfrac{\vdash \Gamma, A \parr B \qquad \dfrac{\dfrac{}{A^\perp, A} \qquad \dfrac{}{B^\perp, B}}{A^\perp \otimes B^\perp, A, B}\,\otimes}{\vdash \Gamma, A, B}\,\text{cut}$$

$$\vdash \Gamma, A \parr B \iff \vdash \Gamma, A, B$$

$\parr$ è invertibile. asynchrone

1° strategia : 1-2-3

non f.c.

$$\cfrac{\overline{d^\perp, d} \qquad \cfrac{\overline{b, b^\perp} \qquad \overline{a, a^\perp}}{b, a^\perp, a\otimes_3 b^\perp}\otimes_3}{\cfrac{d^\perp, d\otimes_2 b, a^\perp, a\otimes b^\perp \qquad \overline{c, c^\perp}}{d^\perp, d\otimes_2 b, a^\perp, (a\otimes_3 b^\perp)\otimes_1 c, c^\perp}\otimes_1}\otimes_2$$

2° strategia 1-3-2

f o c

$$\cfrac{\cfrac{\cfrac{\overline{d^\perp, d} \qquad \overline{b, b^\perp}}{d^\perp, d\otimes b, b^\perp}\otimes_2 \qquad \overline{a, a^\perp}}{d^\perp, d\otimes b, a^\perp, (a\otimes b^\perp)}\otimes_1 \qquad \overline{c, c^\perp}}{d^\perp, d\otimes_2 b, a^\perp, (a\otimes_3 b^\perp)\otimes_1 c, c^\perp}\otimes_1$$

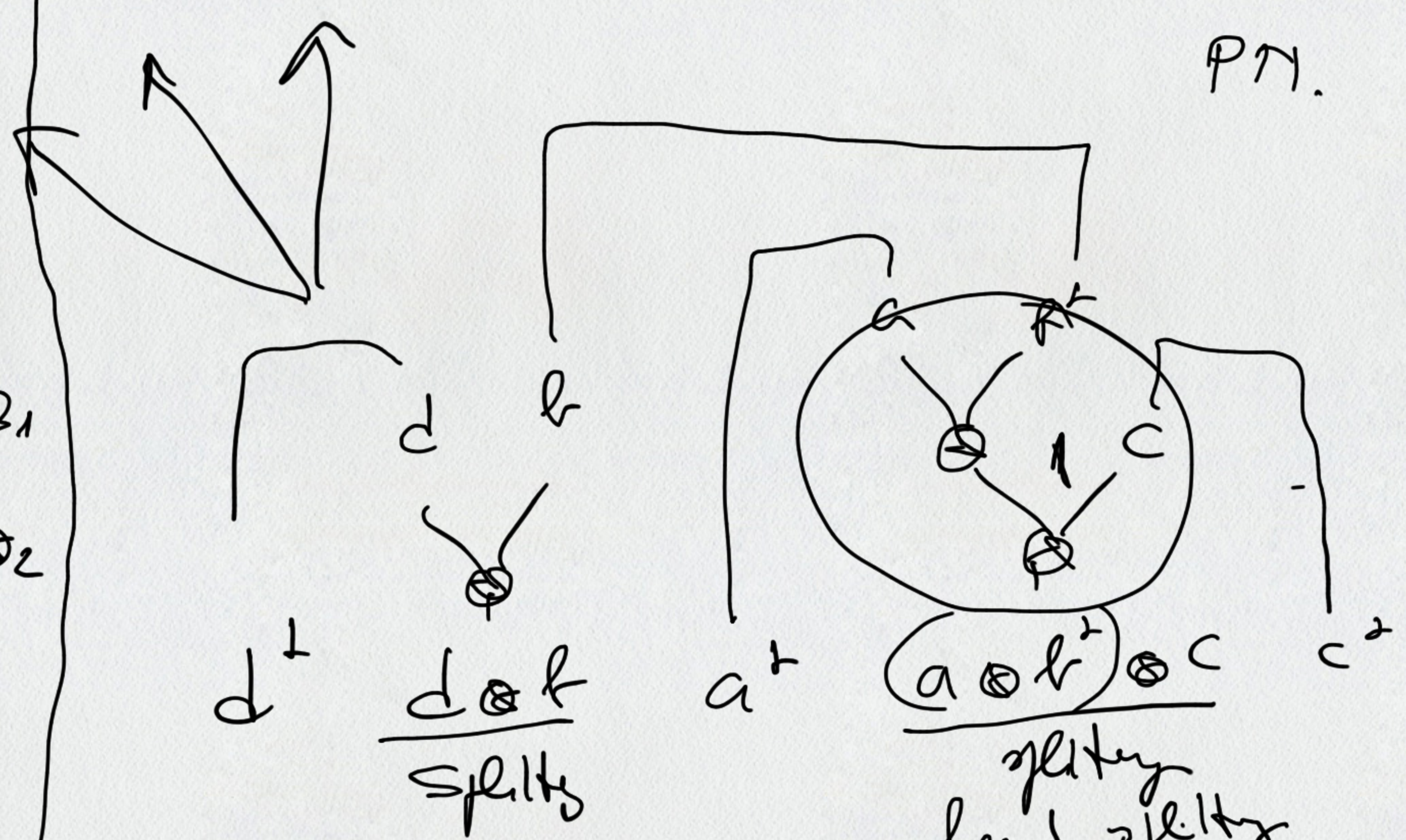
3ª strategia 2-1-3

f.o.c.

$$\cfrac{\overline{d^\perp, d} \qquad \cfrac{\cfrac{\overline{b, b^\perp} \qquad \overline{a^\perp, a}}{b, a^\perp, a\otimes_3 b}\otimes_3 \qquad \overline{c, c^\perp}}{b, a^\perp, (a\otimes b^\perp)\otimes c, c^\perp}\otimes_1}{d^\perp, d\otimes_2 b, a^\perp, (a\otimes_3 b^\perp)\otimes_2 c, c^\perp}\otimes_2$$

punti separazione

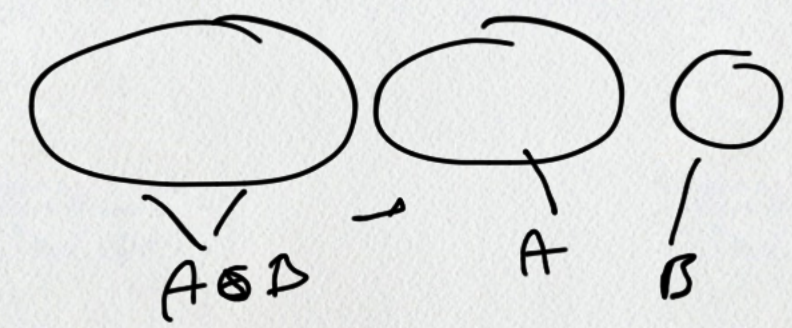
PN.



Def: formula/condizione focusing (focalizzazione) $F \in \Gamma$

data una rete Π di Γ corretta diremo che $F = A \otimes B$ è focusing

$$F \in foc(\Pi) \quad \text{se}$$



- $F = A \otimes B$ è splittante in Π_A Π_B e

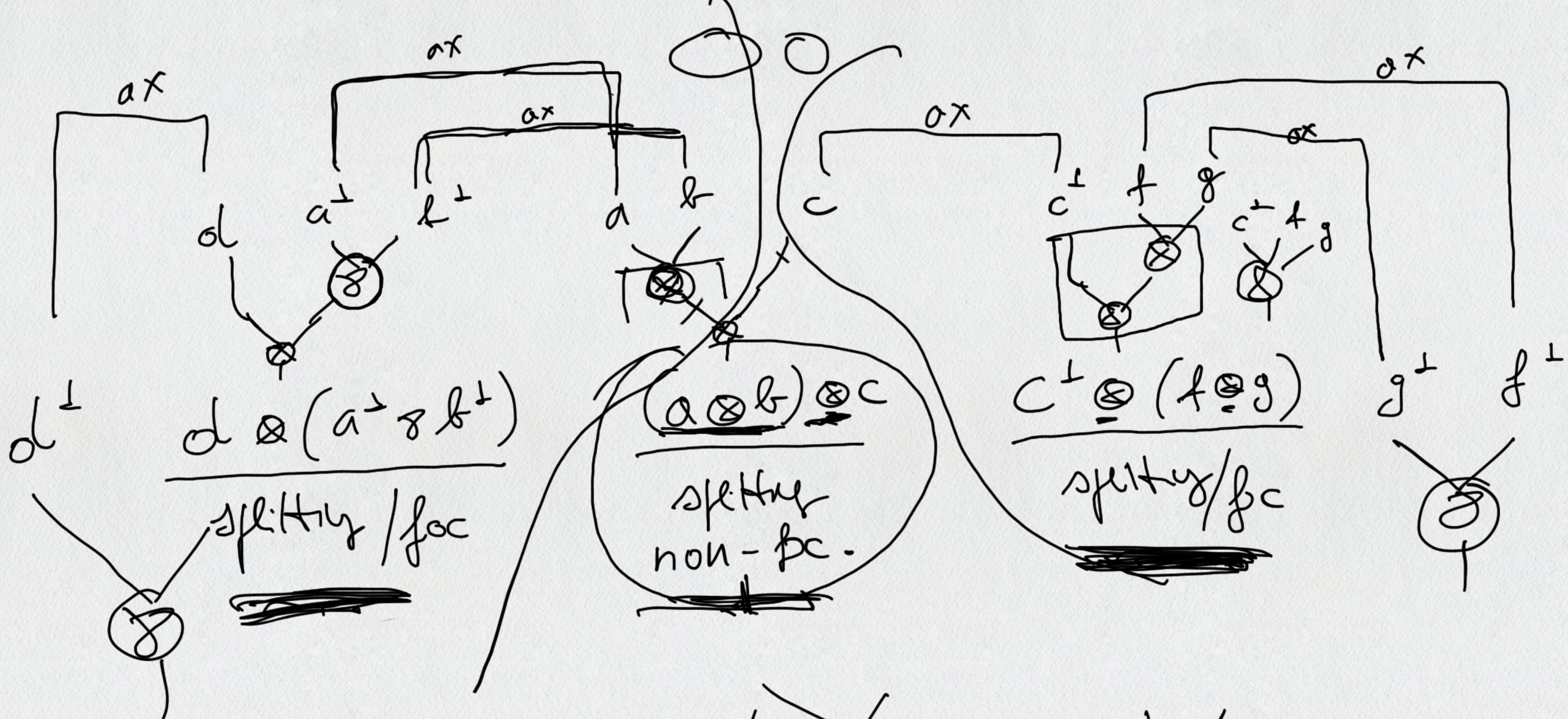
1) A è asynchrono $A_1 \parr A_2$ oppure è un letterale $a, a^\perp$
   oppure A è synchrono $A_1 \otimes A_2$ ed $A \in foc(\Pi_A)$

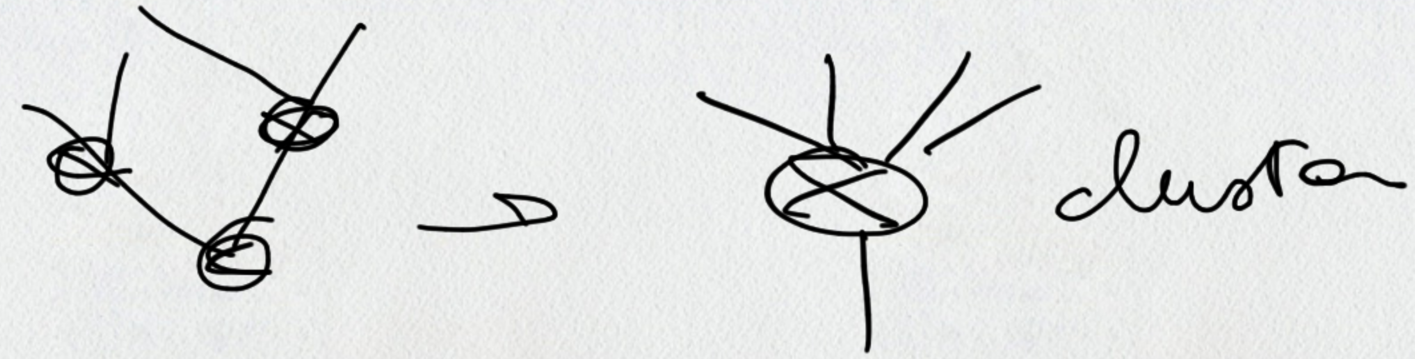
2) B è asynchrono $B_1 \parr B_2$ oppure è un letterale $b, b^\perp$
   oppure B è synchrono $B_1 \otimes B_2$ ed $B \in foc(\Pi_B)$

Theorem focusing: se Π è una ~~deriv~~ proof net di Γ e Γ è in splitting condition (non contiene conclusioni $\parr$ e contiene almeno una conclusione $\otimes$) allora $\exists F$ in Γ t.c. $F \in foc(\Pi)$.

Esempio:

Π è corretta PN



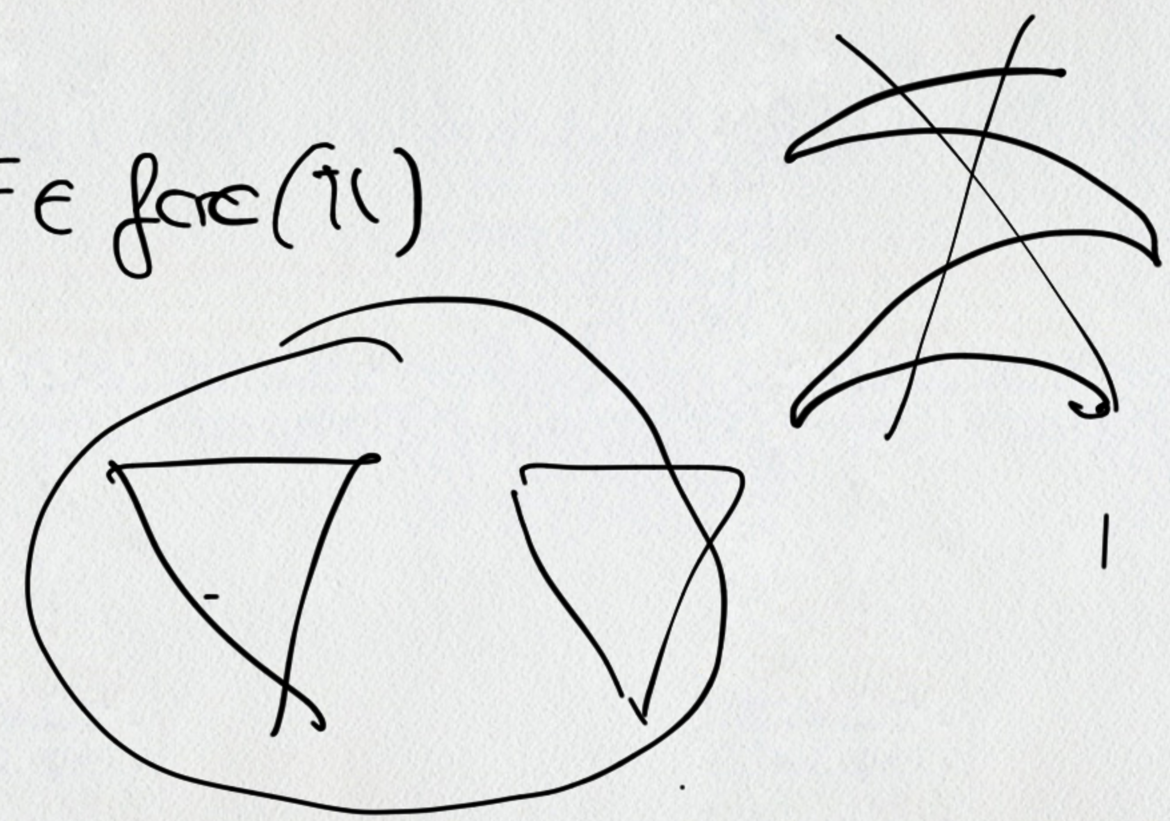


strategia fc

– prima unisci tutti i ⅋

– obp se è in splitting cond ⊥ $\Rightarrow$ ∃ un $F \in$ foc(Π)

selezionalo / cercalo (?)



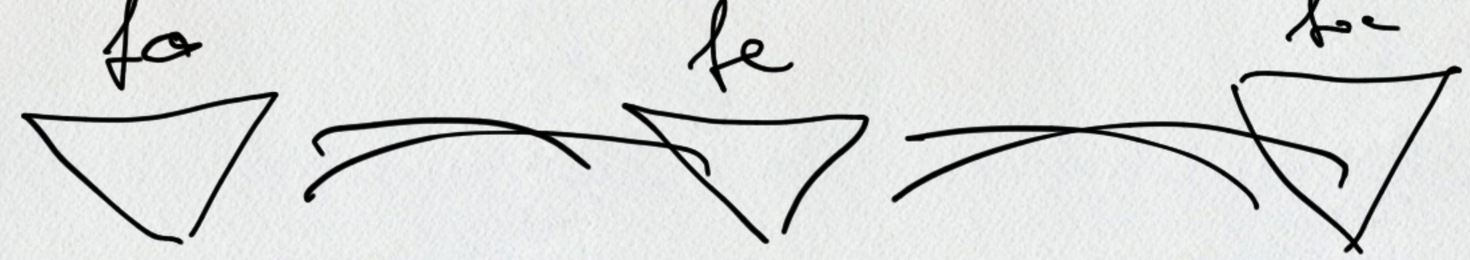
J.M. Andreoli: Calcolo di Sequenti

Andreoli – Maieli: focusing lemma.

foc $\begin{array}{l} \perp\perp cca \ - \\ \otimes\otimes co \ + \\ \perp\otimes cc \ - \end{array}$

$$\dfrac{\dfrac{\dfrac{\overline{a^\perp, a} \quad \overline{b^\perp, b} \quad \overline{c^\perp, c}}{a^\perp, b^\perp, (a\otimes b)\otimes c, c^\perp}\ foc^+}{\overline{\vdash d^\perp, d} \quad (a^\perp ⅋ b^\perp), (a\otimes b)\otimes c, c^\perp}\ unfoc\ ⅋^-}{d^\perp, d\otimes(a^\perp ⅋ b^\perp), (a\otimes b)\otimes c, c^\perp}\ foc^+ \qquad \overline{g, g^\perp} \quad \overline{h, h^\perp}$$

$\otimes^2$

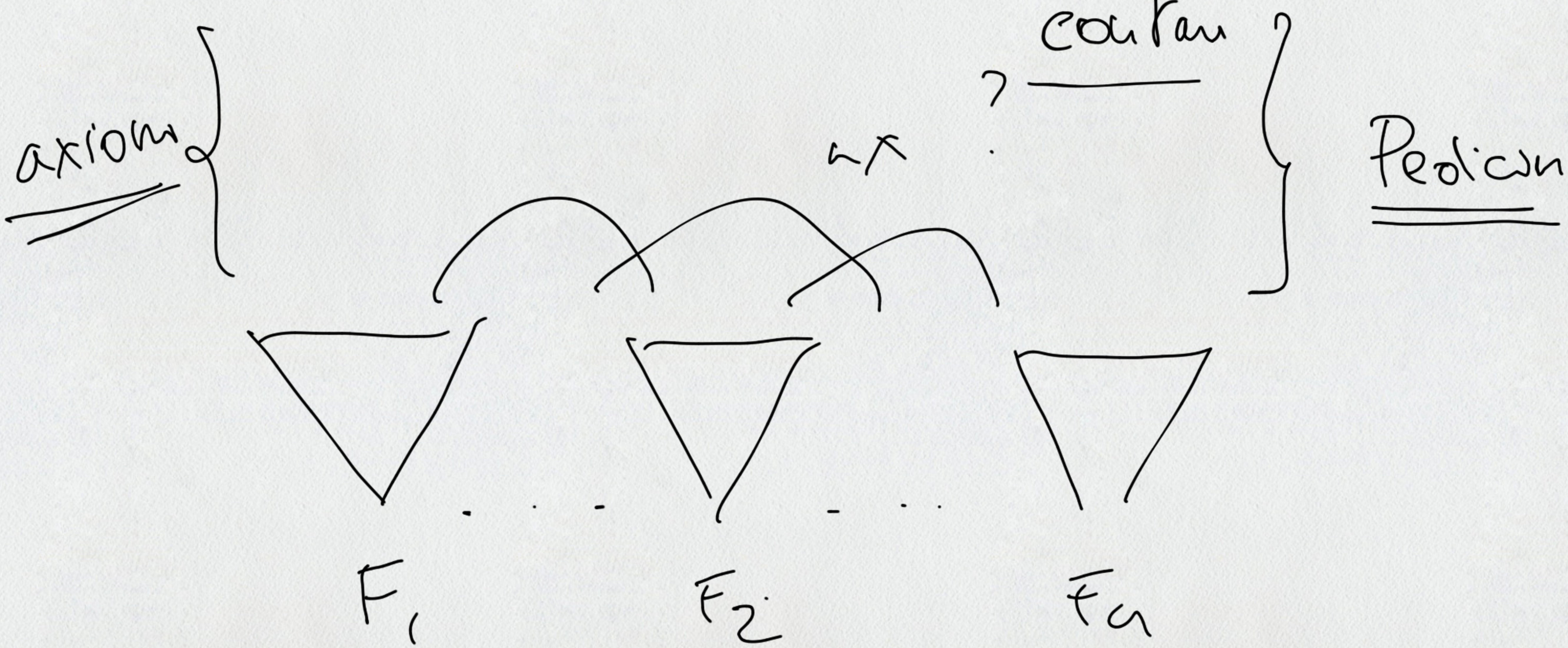


$\exists F foc$ $\qquad \vdash d^\perp, d\otimes(a^\perp ⅋ b^\perp), (a\otimes b)\otimes c, c^\perp\otimes(f\otimes g), g^\perp, f^\perp$

fase negativa $\qquad \overline{\qquad\qquad}\ ⅋_1 \ \cdots\ ⅋_2 \ \overline{\qquad\qquad}$

$$\vdash d^\perp ⅋_1 (d\otimes(a^\perp ⅋ b^\perp)), (a\otimes b)\otimes c, \underbrace{c^\perp\otimes(f\otimes g)}_{foc}, g^\perp ⅋_2 f^\perp$$

Proof net construction



foc.

- MLL problemi ⅋, ⊗

ultime settimane

MALL { somme / prodotti — &, ⊕ / ⊗, ⅋

MELL (λ-calcolo)

δ - !, ?, ⊗, ⅋

Giovedì : domande/risposte esercizi.