Tutorial 1

Mohamed Anwar

October 15, 2018

1. Use the Division Algorithm to find the greatest common divisor, $d(x)$, of the following two polynomials
$$x^3 + x^2 + 2x + 2, \quad x^5 + x + 1 \quad \text{in} \quad \mathbb{F}_3[x], \mathbb{F}_5[x].$$
Moreover, find the polynomials $f(x)$ and $g(x)$ in $F_3[x]$ such that
$$d(x) = f(x)(x^5 + x + 1) + g(x)(x^3 + x^2 + 2x + 2).$$

2. Prove that $x^4 - 10x^2 + 1$ is irreducible over $\mathbb{Q}[x]$.

3. Prove or disprove that the following polynomials are irreducible in $\mathbb{Q}[x]$, if not find the irreducible factors:
$$x^4 + x^2 + 1, \quad x^4 + 1, \quad x^5 - 1, \quad x^4 + x^3 + x^2 + x + 1.$$

4. Determine the degree of the following field extensions:
$$[\mathbb{Q}(\sqrt[5]{2}, \zeta_5) : \mathbb{Q}], \quad [\mathbb{Q}(\sqrt[4]{2}) : \mathbb{Q}(\sqrt{2})], \quad [\mathbb{Q}(\zeta_8, \sqrt{2}) : \mathbb{Q}].$$

5. Find a polynomial $f(x) \in \mathbb{Z}[x]$ of degree n such that it is irreducible over $\mathbb{Q}$, where $n = 2, 3, 4, 79$.

6. Determine whether $f(x) = 2x^5 - 5x^3 - 4x^2 - 3x - 3$ is invertable or not in $\mathbb{Q}[x]/(g(x))$, where $g(x) = 2x^4 - 7x^2 - 4$.

7. Prove that: (a) If $[F : E] = p$, p is prime, then F is simple extension of E.
(b)Give an example of an algebraic extension of rationals of infinite degree.

8. Write the following symetric polynomial over $\mathbb{Z}[x, y, z]$ as a polynomial interms of the elementary symetric functions:
$$X^2Y + X^2Z + Y^2X + Y^2Z + Z^2X + Z^2Y.$$

9. Calculate the minimal polynomial of $\zeta_{12}, \zeta_6, \zeta_{14}$ over $\mathbb{Q}$.