Tutorato 3 AL310

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Exercise 1. *Let* $\mathbb{Q}(\alpha)$ *be the stem field with* $\alpha^3 = \alpha + 1$. *Calculate* $\frac{1}{\alpha}$, $\frac{1}{\alpha+4}$, α^5 *and* $\frac{1}{\alpha^2}$.

Exercise 2. *Calculate the degrees of the following field extensions:*

- $[\mathbb{Q}(2^{\frac{1}{5}}, \zeta_5) : \mathbb{Q}]$
- $[\mathbb{Q}(2^{\frac{1}{4}}) : \mathbb{Q}(\sqrt{2})]$
- $[\mathbb{Q}(\zeta_3, \sqrt{2}) : \mathbb{Q}]$

Exercise 3. *Determine the splitting field of the following polynomials and calculate their degree on* $\mathbb{Q}$:

- $f(x) = x^4 - x^3 + 2x^2 - x + 1$
- $g(x) = (x^4 - 2)(x^2 + 1)((x - 3)^2 + 6)$

Exercise 4. *Verify that* $cos(\frac{2\pi}{9})$ *and* $cos(\frac{2\pi}{5})$ *are algebraics and calculate their minimal polynomials.*

Exercise 5.

- *Describe the* $\mathbb{Q}(\sqrt{-1})$*-homomorphisms of* $\mathbb{Q}(\zeta_{16})$ *over* $\mathbb{C}$.
- *Describe the* $\mathbb{Q}(\sqrt{-1})$*-homomorphisms of* $\mathbb{Q}(\sqrt{-3}, \sqrt{3})$ *over* $\mathbb{C}$.

Exercise 6. *Find a* $\mathbb{Q}$*-basis of the splitting field of the polynomial* $f(x) = (x^2 - 2)(x^2 - 3)$ *over* $\mathbb{Q}[x]$.

Exercise 7. *Find the number of irreducible factors of the polynomial* $f(x) = x^{255} - 1$ *over* $\mathbb{Q}[x]$ *and* $\mathbb{F}_2[x]$.

Exercise 8. *Calculate the number of elements in the splitting field of the polynomial* $(x^{2^8} - x)(x^8 + x^4 + 1)(X^{12} + x^4 + 1)(x^5 + x)$ *over* $\mathbb{F}_2[x]$.