Tutorato 4 AL310

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Martedì 6 novembre 2018

Esercizio 1. *Verify that* $cos(\frac{\pi}{18})$ *and* $sen(\frac{\pi}{12})$ *are algebraics and calculate their minimal polynomials.*

Esercizio 2. *Calculate the number of irreducible and monic polynomials with degree* 7 *over* $\mathbb{F}_{11}$ *and with degree* 8 *over* $\mathbb{F}_2$.

Esercizio 3.

- *Verify that* $\frac{\mathbb{F}_3[x]}{x^2+1}$ *and* $\frac{\mathbb{F}_3[x]}{x^2-x-1}$ *are fields.*
- *Are these the only possibilities for a field with* 9 *elements?*
- *Find an isomorphism between* $\frac{\mathbb{F}_3[x]}{x^2+1}$ *and* $\frac{\mathbb{F}_3[x]}{x^2-x-1}$.

Esercizio 4. *Describe the* F*-homomorphisms of* E *over* $\mathbb{C}$ *in the following cases:*

- $E = \mathbb{Q}(\sqrt{2}, \sqrt{10})$ *and* $F = \mathbb{Q}(\sqrt{5})$;
- $E = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ *and* $F = \mathbb{Q}(\sqrt{6})$.

Esercizio 5. *Calculate the number of elements in the splitting field of the polynomial* $f(x) = (x^6 + x^2 + 3)(x^{32} + x^2)$ *over* $\mathbb{F}_2[x]$.

Esercizio 6. *Calculate the degrees of the splitting fields over* $\mathbb{Q}$, $\mathbb{F}_2$, $\mathbb{F}_3$ *and* $\mathbb{F}_5$ *of the polynomials* $f(x) = x^3 + 2$ *and* $g(x) = x^4 - 2$.

Esercizio 7. *Calculate the minimal polynomials of* $\frac{1}{\alpha}$ *and* $\frac{1}{\alpha-1}$ *over the field* $\mathbb{Q}(\alpha)$, *where* $\alpha^4 = \alpha + 1$.

Esercizio 8. *Calculate the zeros of the polynomial* $f(x) = x^3 + x + 1$ *in* $\mathbb{F}_2(\alpha)$, *where* $\alpha^3 = 1 + \alpha^2$.

Esercizio 9. *Let* $f \in \mathbb{Q}[x]$, *irreducible and with degree* 8. *Considerate the field* $\mathbb{Q}(\alpha)$, $f(\alpha) = 0$ *and verify that* $\mathbb{Q}(\alpha) = \mathbb{Q}(\alpha^3)$. *Give an example of* f *such this but with the property that* $\mathbb{Q}(\alpha) \supset \mathbb{Q}(\alpha^2) \supset \mathbb{Q}$.

Esercizio 10. *Let* d *an integer positive and odd. Verify that* $f_d = X^4 - 2X^2 - 2d \in \mathbb{Q}[x]$ *is irreducible and indicate* $F_d = \mathbb{Q}(\alpha)$, $\alpha^4 = 2\alpha^2 + 2d$.

- *Verify that* F_d *has a subfield which is isomorphic to* $\mathbb{Q}(\sqrt{1+2d})$,
- *Calculate the degree of the splitting field of* f_d *over* $\mathbb{Q}$.

Esercizio 11. *Determine the splitting field over* Q *of* $f(X) = x^{15} - x^8 - x^7 + 1 \in \mathbb{Q}[x]$ *and calculate his degree over* $\mathbb{Q}$.

Esercizio 12. *Verify that* $\mathbb{Q}(\sqrt{2}) \subset \mathbb{Q}(\zeta_8)$ *and describe the* $Q(\sqrt{2})$*–homomorphisms of the field* $\mathbb{Q}(\zeta_8)$ *over* $\mathbb{C}$.