

# Tutorato 5 AL310

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**Exercise 1.** Let  $\mathbb{F}_5[\alpha]$  the stem field with  $\alpha^2 = 2$ ; determine his number of elements and write the generators of  $\mathbb{F}_5[\alpha]^*$ .

**Exercise 2.** Find the zeros of the polynomial  $f(x) = x^{16} + x^{12} + 1$  in  $\mathbb{F}_2[\alpha]$  with  $\alpha^4 = \alpha + 1$ .

**Exercise 3.** After describing the elements of  $\text{Aut}(\mathbb{Q}(5^{\frac{1}{3}}, \sqrt{-3})/\mathbb{Q})$  determine the order of any of them.

**Exercise 4.** Describe the elements of the Galois group of the polynomial  $f(x) = x^4 - 14x^2 + 9 \in \mathbb{Q}[x]$  and determine to which group  $\text{Gal}(\mathbb{Q}_f/\mathbb{Q})$  is isomorph.

**Exercise 5.** Describe the elements of the Galois group of the polynomial  $f(x) = x^4 - x^2 - 6 \in \mathbb{Q}[x]$  and determine to which group  $\text{Gal}(\mathbb{Q}_f/\mathbb{Q})$  is isomorph.

**Exercise 6.** Describe the lattice of subfields of  $\mathbb{Q}(\zeta_5)$ .

**Exercise 7.** Describe  $G = \text{Aut}(\mathbb{Q}_f/\mathbb{Q})$ , where  $\mathbb{Q}_f$  is the splitting field of the polynomial  $f(x) = (x^2 + 1)(x^4 - 3)$ . Calculate the order of the elements of  $G$  and determine to which group it is isomorph.