Family Name .... *Name* .... *Student ID(Matricola):* ....

Solve the problems adding to the replies short and essential explenations. *Please write the solutions in the designed areas. NO EXTRA SHEETS WILL BE ACCEPTED.* 1 Problem = 4 marks. Duration: 2 hours. No questions allowed in the first hour and in the last 20 minutes.

1	2	3a	3b	3c	3d	4	5	6	TOTAL

1. Calculate the continued fraction expansion of $\sqrt{39}$

2. An irrational number has continued fraction expansion $[3; \overline{1,4}]$. Compute it.

3. State some of the important facts of the Theory of continues fractions.

4. Show that $p \neq 3$ is a prime number such that $p = x^2 + 3y^2$ if and only if $p \equiv 1 \bmod 3$.

5. Show that if an integer m has the form $m = 2(7 + 8k)$, the m does not have the form $m = x^2 + y^2 + 2z^2$.

6. Prove that there are infinitely many integers that can be written in exactly 20 distict ways as the sum of 2 squares.

7. Let n and m be integers such that $5n \equiv 3 \mod 8m$ and $m \equiv 13 \mod 60$. Compute the Jacobi symbol $\left(\frac{m}{n}\right)$.

8. Compute the density of the integers that are odd and with last two digits in base 5 equal to 22.

9. Prove the following asymptotic formula:

$$\sum_{p \leq T} \frac{\log p}{p} = \log T + O(1).$$

hint: *first show that* $\sum_{m \leq T} \Lambda(m)m = \log T + O(1)$ *using a formula proven in class. Then estimate the contribution of prime powers.*