Family Name .... *Name* .... *Student ID(Matricola):* ....

Solve the problems adding to the replies short and essential explenations. *Please write the solutions in the designed areas. NO EXTRA SHEETS WILL BE ACCEPTED.* 1 Problem = 4 marks. Duration: 2 hours. No questions allowed in the first hour and in the last 20 minutes.

1	2	3a	3b	3c	3d	4	5	6	TOTAL

1. Compute the 13–adic valuation $v_{13}(200!)$.

2. Approximate $\sqrt{2}+\sqrt{3}$ by a rational using continued fractions. You may stop after three divisions.

3. Let $\omega(n)$ be the number of prime divisors of $n \in \mathbf{N}$ and let $\alpha \in \mathbf{C}$. We define $f_\alpha(n) := \alpha^{\omega(n)}$.

a. Show that $f_\alpha(n)$ is multiplicative function. For which values of α is f_α totally multiplicative?

b. Prove that $f_2(n) = d(n)$ if and only if n is square free.

c. Compute $f_{\sqrt{5}}(300)$.

d. Show that $f_2 * \mu = \mu^2$.

4. Prove that for any prime p, there exists a primitive root modulo p.

5. Prove that if $n \equiv 10 \bmod 16$, the n is not of the form $x^2 + 2y^2 + 3z^2$ with $x, y, z \in \mathbf{Z}$.

6. Let $m \in \mathbf{N}$. Prove that if $x^2 \equiv 2 \bmod n$ has solution, then the Jacobi symbol $\left(\frac{2}{m})\right)_J = 1$ but there are infinitely many integers m such that $\left(\frac{2}{m})\right) = 1$ but 2 is NOT a square modulo m.