Family Name .... *Given Name* .... *ROLL NO.* ....

Solve the maximum number of problems providing, in each case, clear and synthetic explanations. *Insert your answers in the appropriate slots. Please NO SEPARATE SHEETS.* 1 Exercise = 4 points. Total time: 2 hours.

SIGNATURE	1	2	3	4	5	6	7	8	TOT.
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1. Answer to the following questions providing a justification of 1 line:

   a. Is it true that there exists a unique field, up to isomorphisms, with 26 elements?

   ....

   b. Is it true that if $f, g \in \mathbf{F}_q[X]$ and $(f - g)$ is divisible by $X^q - X$, then, as functions from $\mathbf{F}_q$ to $\mathbf{F}_q$, f and g are the same?

   ....

   c. Give an example of an irreducible polynomial of degree 2 in $\mathbf{F}_5$.

   ....

   d. When does a cyclotomic polynomial split in $\mathbf{F}_q[X]$?.

   ....

2. After having given the definition of permutation polynomial in $\mathbf{F}_q[X]$, show that all linear polynomials are permutation polynomials.

3. Describe all permutation polynomials in $\mathbf{F}_3[X]$ and $\mathbf{F}_4[X]$.

4. Let $N(a, q)$ be the number of irreducible polynomials of degree a with coefficients in $\mathbf{F}_q$. Prove that if a is prime, then $N(a, q) = \frac{q^a - q}{a}$.

5. Let $\mathbf{F} = \mathbf{F}_2[\alpha], \alpha^3 = \alpha + 1$. After having proven that $\mathbf{F}$ is a field, having computed its characteristic and its cardinality, compute the multiplicative order of $\alpha + 1$.

6. Consider the equation $2x^3 + 3x^2y + 5zt^2 - 10 = 0$. Prove that it has at least 7 solutions $(x_0, y_0, z_0, t_0) \in \mathbf{F}_7^4$. *hint: This is an application of the Chevalley's Theorem.*

7. State a Theorem that you judge of great importance about finite fields.

8. What is the numer of solutions of the equation $x^2 + 2y^2 + 3z^2 = 3$ over $\mathbf{F}_7$?