

ALGEBRAIC NUMBER THEORY – FIELDS
& GALOIS THEORY
Tribhuvan University

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Solve the maximum number of the following exercises:

1. Choose one element d in the set $\{3, 6, 7, 10, 11, 14, 15, 19, 22\}$ and show that the group of the units $(\mathbf{Z}[\sqrt{d}])^*$ is infinite.

2. Consider the stem field $\mathbf{Q}[\alpha]$, $\alpha^3 = -2\alpha + 1$. Compute $a, b, c \in \mathbf{Q}$ such that

$$\frac{1}{\alpha^2 + 1} = a + b\alpha + c\alpha^2$$

Can you produce infinitely many units in the ring $\mathbf{Z}[\alpha]$?

3. Write the definition of stem-field, write a stem for the field $\mathbf{Q}[\sqrt{3} - \sqrt[3]{5}]$ and prove that $\mathbf{Q}[\sqrt{3}, \sqrt[3]{5}] = \mathbf{Q}[\sqrt{3} - \sqrt[3]{5}]$.

4. Prove that $\sqrt{-20} \in \mathbf{Q}[\sqrt{5} + i]$

5. Give the definition of Galois field.