Geometry of interaction and non-determinism

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joint work with Mario Piazza, SNS Pisa

XXVIII Incontro di Logica AILA

Udine, 3-6 settembre 2024

Outline

- actions of streams

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- lambda calculus and choice as syntactic sugar

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STREAMS & MACHINES

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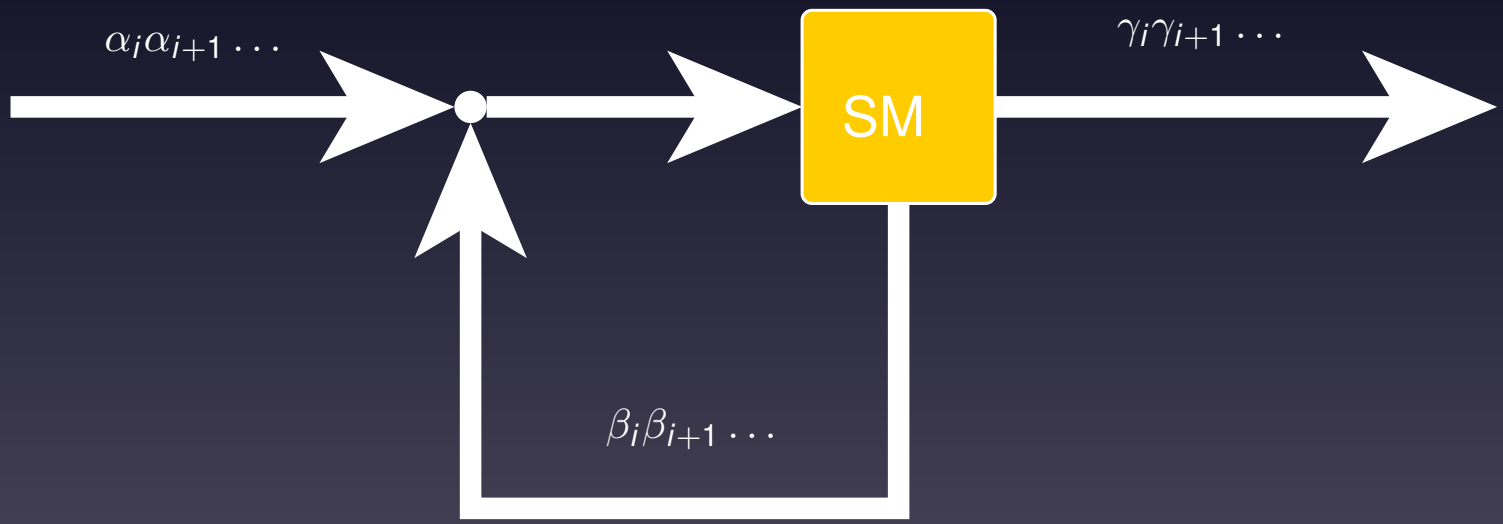
When an action is processed possibly many new actions are generated, feeding the stream.



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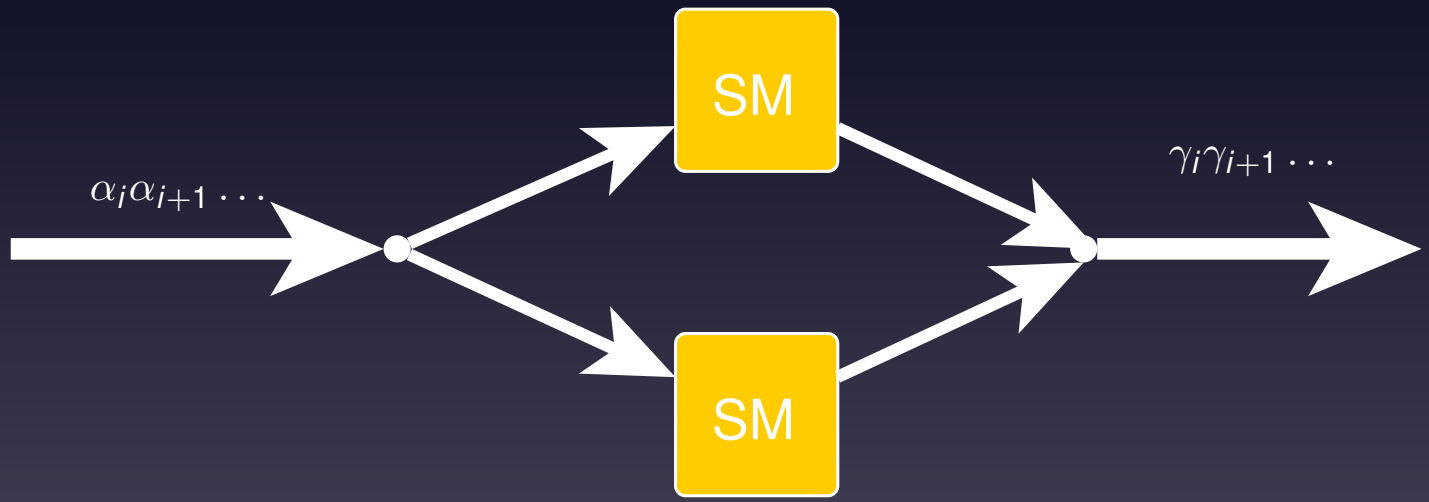

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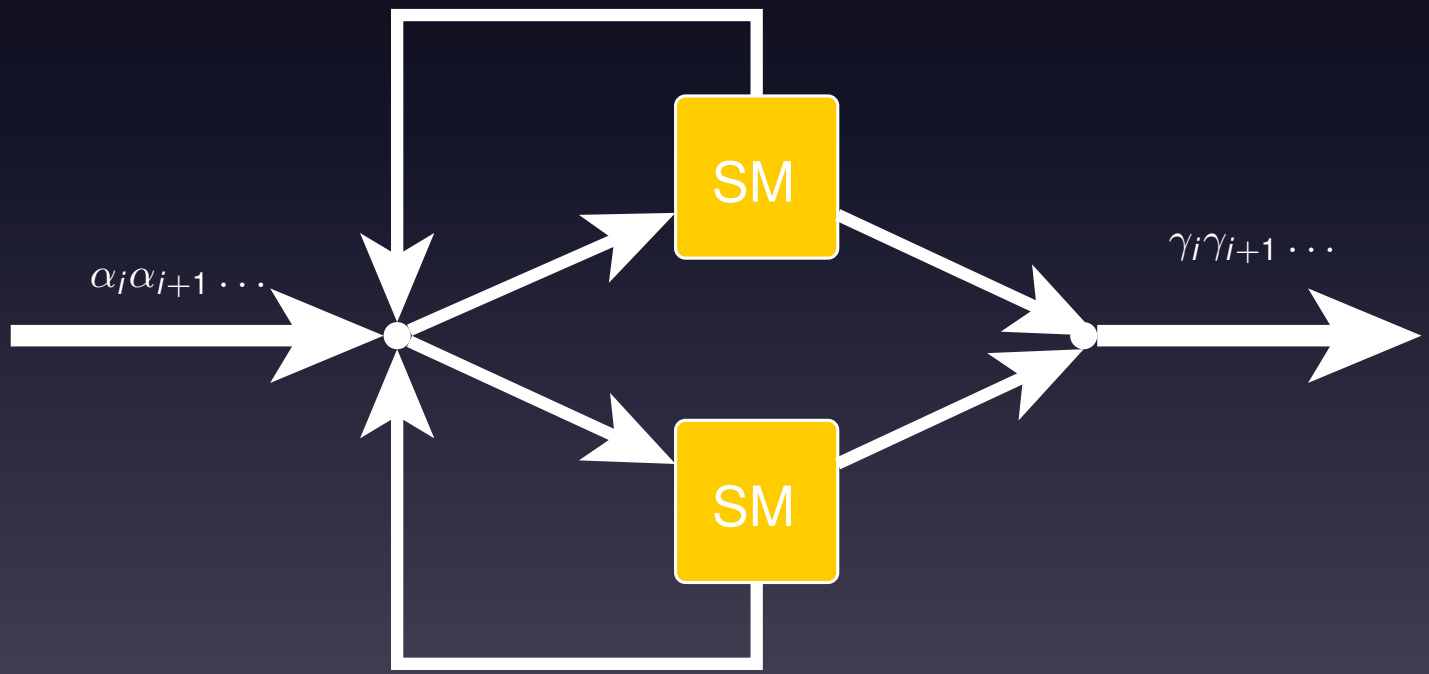

PELCR-VM

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idle cycle (if α is the zero action)

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Execution of an action α to be performed within a context

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generates new actions in pairs:

$$\alpha_i := \langle(\varepsilon_i, \varepsilon), (v_i, v_i'), a_i'\rangle \quad \text{and} \quad \beta_i' := \langle(\varepsilon_s, -\varepsilon), (v_s, v_i'), b_i'\rangle,$$

where ε is arbitrarily chosen in $\{+, -\}$ and v_i' is a newly generated node and if $b_i^* a \neq 0$, $a_i' b_i'^*$ is the stable form of $b_i^* a$.

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Essentially an action is a **polarised edge in a weighted graph**:

$$\langle(\varepsilon_t, \varepsilon_s), (v_t, v_s), w\rangle,$$

and tracks the logical structure of a path in the proof-net.

Parallel Evaluation

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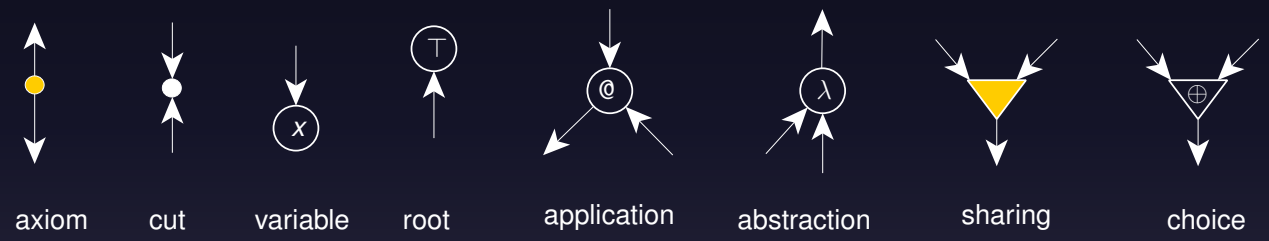

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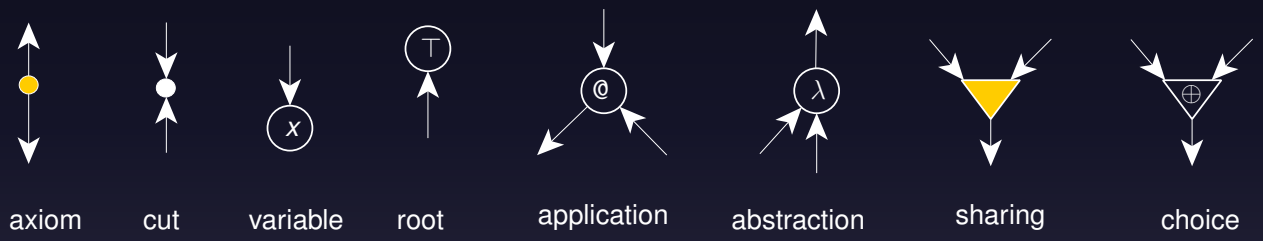
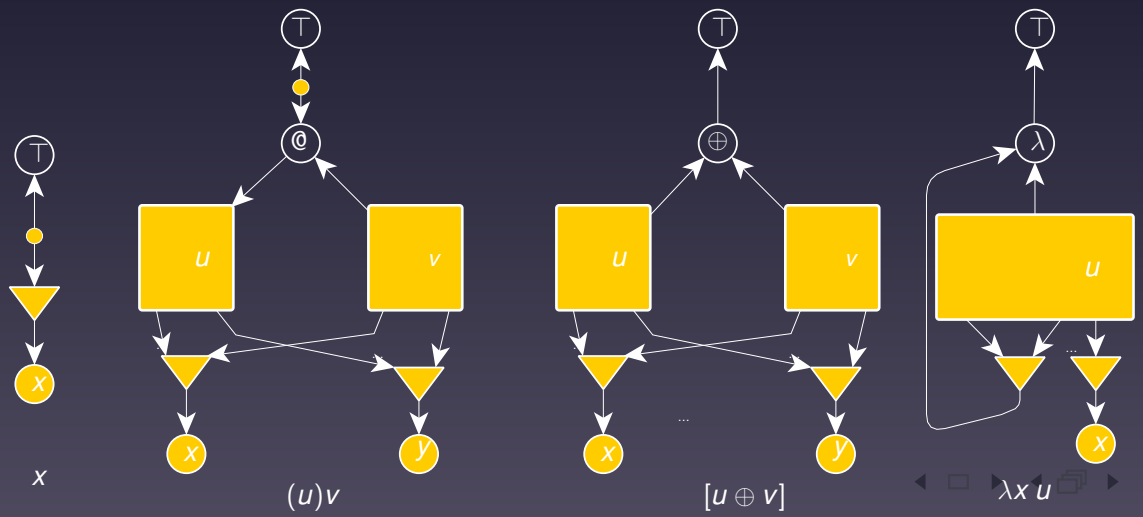
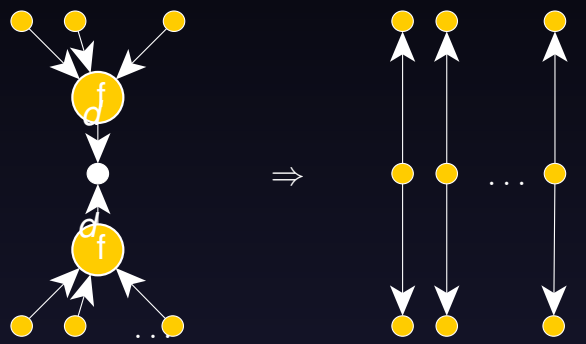
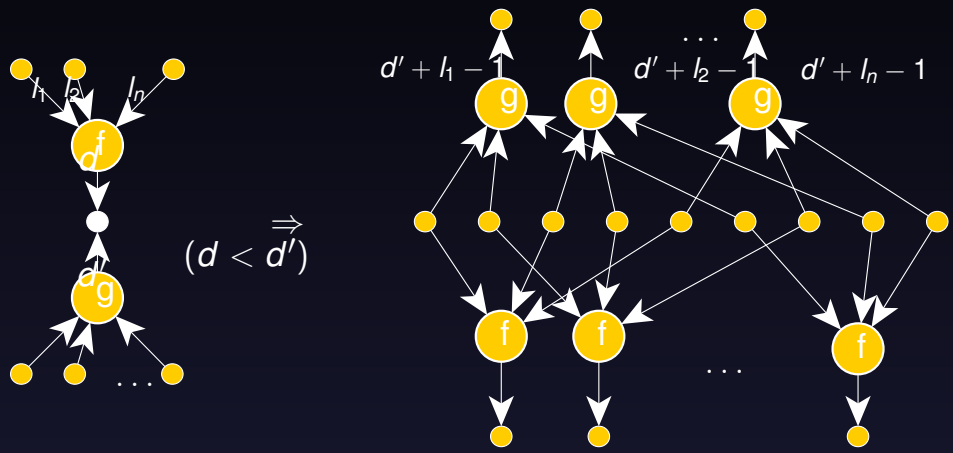




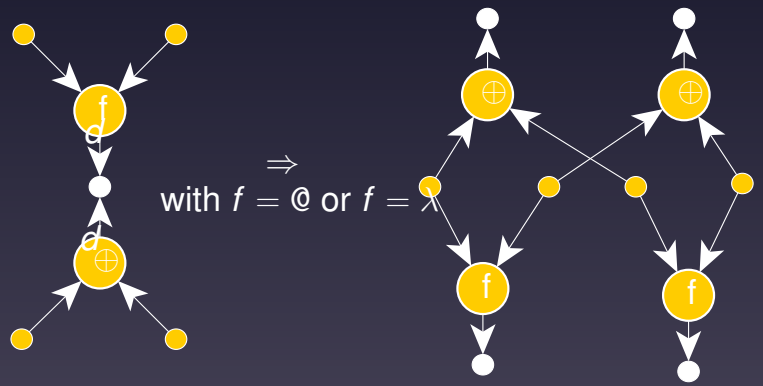
Reduction Rules



(a) annihilation

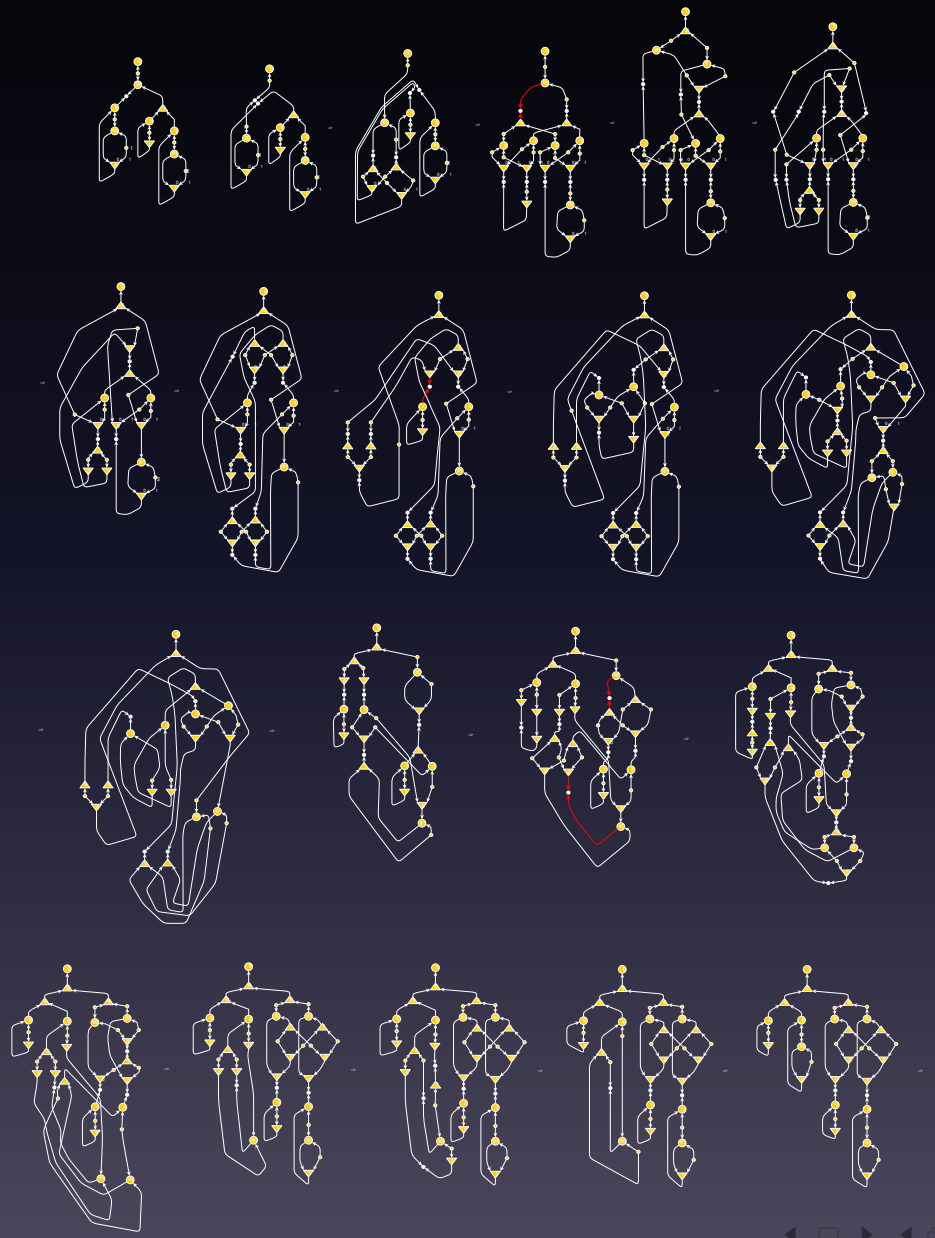


(b) commutation



(c) commutation of choice

Example of Reduction



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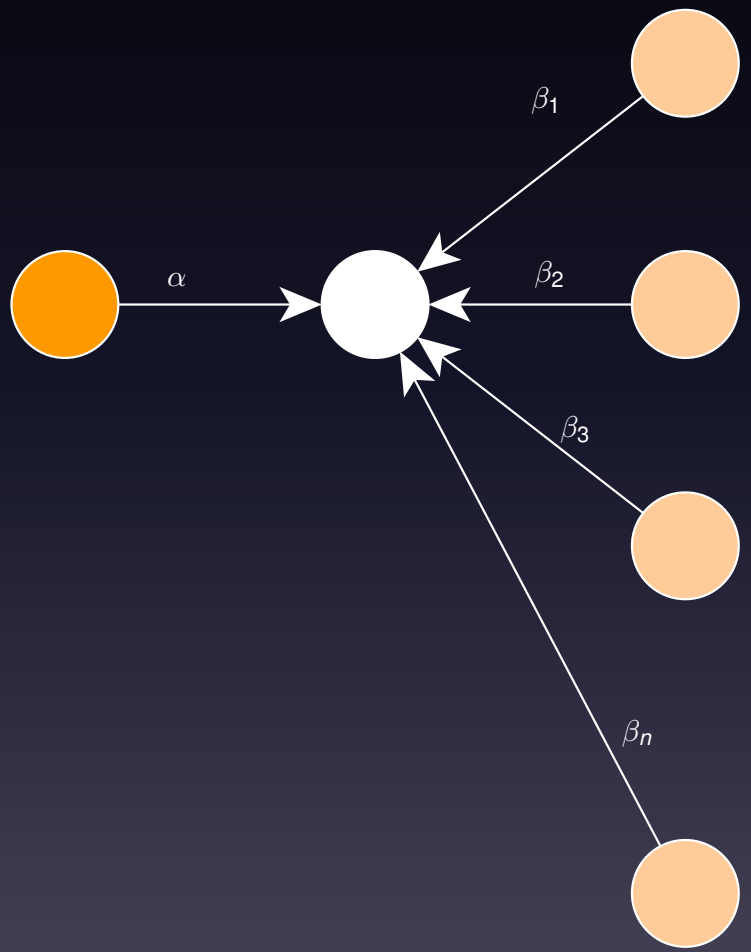

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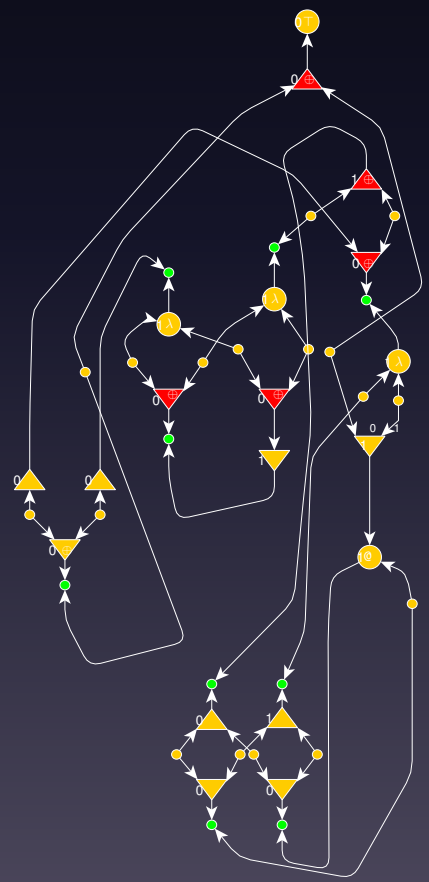

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MOLECULES

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How to compute them ?

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5. **Time to Next Reaction:** Calculate the time to the next reaction $\tau = \frac{1}{a_0} \ln \frac{1}{r_1}$.
6. **Determine Reaction Rule:** Find reaction j such that $\sum_{k=1}^{j-1} a_k < r_2 a_0 \leq \sum_{k=1}^{j} a_k$.

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3. **Total Propensity:** Compute the total propensity $a_0 = \sum_j a_j$.
4. **Generate Random Numbers:** Generate two random numbers $r_1, r_2 \in [0, 1]$.
5. **Time to Next Reaction:** Calculate the time to the next reaction $\tau = \frac{1}{a_0} \ln \frac{1}{r_1}$.
6. **Determine Reaction Rule:** Find reaction j such that $\sum_{k=1}^{j-1} a_k < r_2 a_0 \leq \sum_{k=1}^{j} a_k$.
7. **Update:** Update the system state by executing reaction j. Update time $t = t + \tau$.

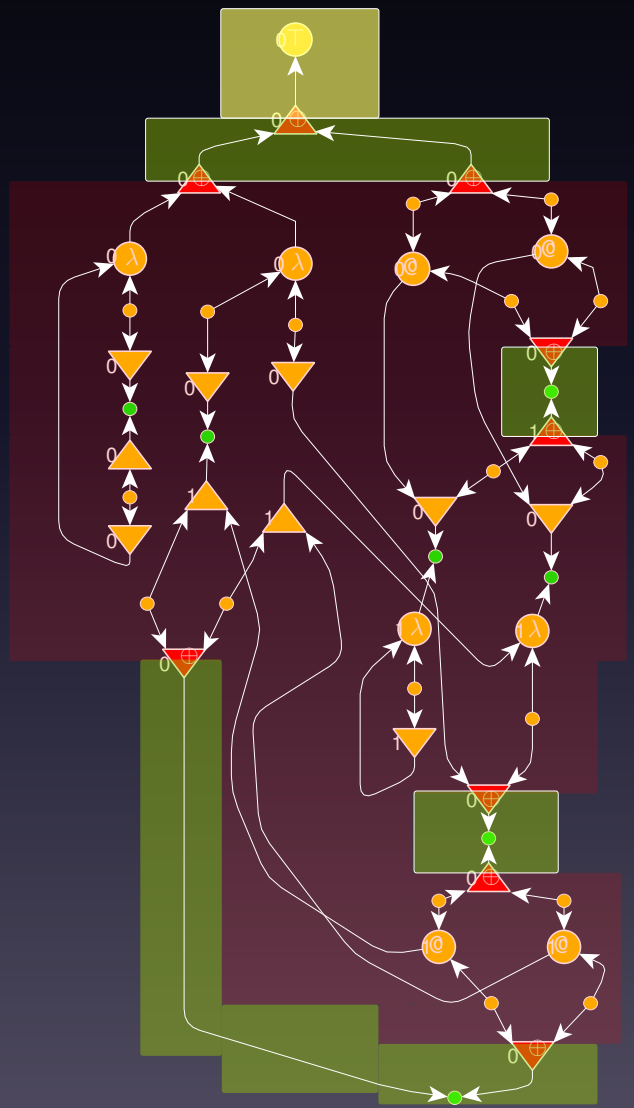
Gillespie's Algorithm

The Gillespie algorithm is a stochastic simulation method used to simulate the time evolution of a set of coupled chemical reactions within a reacting system.

1. **Initialization:** Set the initial number of molecules for each species, time $t = 0$.
2. **Calculate Propensities:** For each reaction j, calculate the propensity function $a_j = c_j \cdot h_j(X)$ where c_j is the rate constant and $h_j(X)$ is the number of possible combinations of reactant molecules.
3. **Total Propensity:** Compute the total propensity $a_0 = \sum_j a_j$.
4. **Generate Random Numbers:** Generate two random numbers $r_1, r_2 \in [0, 1]$.
5. **Time to Next Reaction:** Calculate the time to the next reaction $\tau = \frac{1}{a_0} \ln \frac{1}{r_1}$.
6. **Determine Reaction Rule:** Find reaction j such that $\sum_{k=1}^{j-1} a_k < r_2 a_0 \leq \sum_{k=1}^{j} a_k$.
7. **Update:** Update the system state by executing reaction j. Update time $t = t + \tau$.
8. **Repeat:** Repeat steps 2-7 until the desired simulation time is reached.

Probabilistic Geometry of Interaction

Open Question any action has a choice nesting depth (in the sharing graph).



- we give an interpretation of the choice nesting depth d as the rate $c_d = 2^{-d}$,
- we count the total number of active configurations at any depth d: $h(d)$
- then we apply Gillespie algorithm to schedule the current action $a_d = c_d h(d)$ and normalise against the total propensity $a = \sum_i a_i$.

Concluding remarks

- We introduced **sharing graphs** as a novel representation for the **non-deterministic lambda calculus**.
- The sharing graphs were interpreted within the framework of the **geometry of interactions** (GoI), providing a new perspective on the computational processes involved.
- We demonstrated that this interpretation can be effectively **evaluated using the PELCR-VM**, a virtual machine designed for efficient computation in this setting.
- This work opens new avenues for **further investigations** into the **operational nature of non-determinism** in computational systems.
- Future research could explore optimizing the evaluation strategies and understanding the implications of non-deterministic behaviors in broader computational contexts.

THANKS!

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