QUADRICS CONTAINING A PRYM-CANONICAL CURVE

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Introduction

Let X denote a smooth projective nonsingular curve of genus g and let σ be a nontrivial line bundle on X such that $\sigma^2 = 0$. The line bundle $L = \omega\sigma$, where ω is the canonical line bundle on X, is called a *Prym-canonical line bundle*; if L is globally generated and birationally very ample the curve $\varphi_L(X) \subset \mathbf{P}^{g-2}$ is a *Prym-canonical curve* (where $\varphi_L : X \to \mathbf{P}^{g-2}$ denotes the morphism defined by the global sections of L). In this paper we study the linear system of quadrics containing a Prym-canonical curve, proving some results wich relate its base locus to the Clifford index of the curve.

We consider curves X of Clifford index 3 or more, in which case any Prym-canonical line bundle is very ample and normally generated. From results of Green and Lazarsfeld it follows immediately that if $Cliff(X) \geq 5$ then $\varphi_L(X)$ is the intersection of the quadrics which contain it, in symbols:

$$\bigcap_{Q \supset \varphi_L(X)} Q = \varphi_L(X)$$

If $3 \leq Cliff(X) \leq 4$ this does not happen any more in general. If $Cliff(X) = 4$ then $\varphi_L(X)$ can have at most finitely many trisecants and we have

$$\bigcap_{Q \supset \varphi_L(X)} Q = \varphi_L(X) \cup (trisecants)$$

This is again consequence of a result of Green-Lazarsfeld.

If $Cliff(X) = 3$ the situation can be more complicated. We study this case and we prove that if $g \geq 9$ then

$$\bigcap_{Q \supset \varphi_L(X)} Q \subset \varphi_L(X) \cup \Lambda_1 \cup \cdots \cup \Lambda_k$$

where $\Lambda_1, \ldots, \Lambda_k$ are proper linear subspaces of $\mathbf{P}^{g-2}$. We also give examples of Prym canonical curves of Clifford index 3 and 4 that have trisecants and therefore are not intersection of quadrics (remark (2.7)).

The motivation for this work came from an attempt to understand the Torelli problem for Prym varieties. According to one of the known strategies for the proof of the generic Torelli theorem (see [D] and [T]) one of the steps of the proof is to show that the Prym canonical model of a curve X, relative to $L = \omega\sigma$ where σ is the line bundle which defines a given unramified double cover, can be recovered from the quadrics containing the curve. Our analysis shows that this is indeed the case if $Cliff(X) \geq 3$ because $\bigcap_{Q \supset \varphi_L(X)} Q$ contains only one nondegenerate curve; in particular it contains only one Prym-canonical curve. We discuss this matter in section 3. In section 1 we recall the

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results of Green-Lazarsfeld that we need. Section 2 deals with Prym canonical curves and quadrics containing them.

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We work over the field $\mathbf{C}$ of complex numbers. For all the notation used and not explained we refer the reader to [ACGH].

1. The results of Green-Lazarsfeld

In this section we introduce the notions and the results we will need in section 2. In what follows we denote by X a smooth connected projective curve of genus g. The *Clifford index of a line bundle* $M \in \mathrm{Pic}(X)$ is defined as:

$$Cliff(M) := deg(M) - 2h^0(M) + 2$$

and the *Clifford index of* X as

$$Cliff(X) := min\{Cliff(M) : h^0(M) \geq 2, h^1(M) \geq 2\}$$

If $h^0(M) \geq 2$ and $h^1(M) \geq 2$ then one says that M *contributes to the Clifford index of* X. If moreover $Cliff(M) = Cliff(X)$ we say that M *computes the Clifford index of* X.

Note that $Cliff(M) = Cliff(\omega M^{-1})$ for every $M \in \mathrm{Pic}(X)$

It is well known that

$$0 \leq Cliff(X) \leq [\frac{g-1}{2}]$$

and the second inequality is an equality if X is a sufficiently general curve of genus g (see [La]).

Let $L \in \mathrm{Pic}(X)$ be a globally generated line bundle. We denote by $|L|$ the complete linear system defined by L, and by

$$\varphi_L : X \to \mathbf{P}(H^0(L))^{\check{}} = \mathbf{P}(H^1(\omega L^{-1}))$$

the morphism defined by L. Recall that L is said to be *normally generated* if it is very ample and $\varphi_L(X)$ is projectively normal, equivalently if the natural homomorphism:

$$\bigotimes^k H^0(L) \to H^0(L^k)$$

is surjective for each k.

We will need the following result of Green and Lazarsfeld.

(1.1) Theorem(Green-Lazarsfeld) *Let L be a very ample line bundle on X such that*

$$deg(L) \geq 2g + 1 - 2h^1(L) - Cliff(X)$$

Then L is normally generated.

Proof See [La], theorem 2.2.1.

We will need another result of Green-Lazarsfeld whose proof is outlined in [La], proposition 2.4.2. Since we will state it in a slightly different form from the one given there, we will give a complete proof of it. We start with a lemma.

(1.2) Lemma *Assume that L is a very ample line bundle on X and that the natural map*

$$H^0(L) \otimes H^0(L) \to H^0(L^2)$$

is surjective. Let $p \in \mathbf{P}(H^1(\omega L^{-1}))$. Then $p \in \bigcap_{Q \supset \varphi_L(X)} Q$ if and only if there is an extension $\eta \in Ext^1(L, \omega L^{-1})$ such that the corresponding coboundary map

$$\delta_\eta : H^0(L) \to H^1(\omega L^{-1})$$

has image equal to the 1-dimensional vector subspace that defines p.

Proof Let $\mathbf{P} = \mathbf{P}(H^0(L))\check{}$ be the ambient space of $\varphi_L(X)$. Consider the commutative diagram of linear maps:

$$\begin{array}{ccc} H^0(L) \otimes H^0(L) & & \\ s \downarrow & \searrow \mu & \\ \rho : S^2 H^0(L) & \to & H^0(L^2) \end{array}$$

A hyperplane W of $S^2H^0(L)$ represents a codimension 1 linear system of quadrics of $\mathbf{P}$. In particular the linear system of all quadrics through p is represented by the hyperplane $s(V_p \otimes H^0(L))$ of $S^2H^0(L)$, where $V_p \subset H^0(L)$ is the hyperplane corresponding to p.

To a nonzero linear form $\eta : H^0(L^2) \to \mathbf{C}$ corresponds the linear system $ker(\eta \cdot \rho)$, which contains all the quadrics containing $\varphi_L(X)$; this linear system coincides with $s(V_p \otimes H^0(L))$ if and only if $p \in \bigcap_{Q \supset \varphi_L(X)} Q$.

Under the identification

$$\mathrm{Ext}^1(L, \omega L^{-1}) = H^0(L^2)\check{}$$

a linear form η on $H^0(L^2)$ defines an extension

$$\eta : 0 \to \omega L^{-1} \to E \to L \to 0$$

The condition that the coboundary map

$$\delta_\eta : H^0(L) \to H^1(\omega L^{-1})$$

has 1-dimensional image equal to the subspace representing p is equivalent to the condition that

$$ker(\delta_\eta) = V_p$$

and this is in turn equivalent to the condition that

$$ker(\eta \cdot \mu) = V_p \otimes H^0(L)$$

This concludes the proof. *qed*

(1.3) Theorem (Green-Lazarsfeld) *Let L be a very ample line bundle on X such that*

$$deg(L) \geq 2g + 2 - 2h^1(L) - Cliff(X) \tag{1}$$

Then

$$\bigcap_{Q \supset \varphi_L(X)} Q = \varphi_L(X) \cup (trisecants)$$

Proof The inclusion $\supset$ is obvious. Conversely, let $p \in \bigcap_{Q \supset \varphi_L(X)} Q$, and assume that $p \notin X$. We will show that p belongs to a trisecant of X. Let

$$\eta : 0 \to \omega L^{-1} \to E \to L \to 0$$

be the extension associated to p as in lemma (1.2). Applying Segre's theorem to E (see [S] or [L] for a modern proof) we deduce the existence of an extension

$$0 \to A \to E \to \omega L^{-1} \to 0$$

such that $deg(A) \geq \frac{g-2}{2}$, or equivalently:

$$deg(A) \geq [\frac{g-1}{2}] \tag{2}$$

On the other hand from (1) one immediately computes that:

$$Cliff(X) \geq Cliff(\omega L^{-1}) + 2$$

which implies $h^1(L) \leq 1$ and moreover implies:

$$deg(\omega L^{-1}) \leq 2h^1(L) + Cliff(X) - 4 \leq [\frac{g-1}{2}] - 2$$

Therefore in the diagram:

$$\begin{tikzcd}
& & 0 \arrow[d] & & \\
& & A \arrow[d] \arrow[dr] & & \\
0 \arrow[r] & \omega L^{-1} \arrow[r] & E \arrow[r] \arrow[d] & L \arrow[r] & 0 \\
& & \omega A^{-1} \arrow[d] & & \\
& & 0 & &
\end{tikzcd}$$

the diagonal arrow $A \to L$ is not zero. It follows that $A = L(-D)$, with $D > 0$.

Note that the linear span $\langle D \rangle$ of D in $\mathbf{P}$ contains p: this follows immediately from the inclusion

$$H^0(L(-D)) \subset ker(\delta_\eta)$$

and from the fact that $ker(\delta_\eta)$ is the set of hyperplanes containing p. Computing the cohomology sequence of both extensions we get:

$$h^0(E) \begin{cases} = h^0(L) + h^0(\omega L^{-1}) - 1 & = g - Cliff(L) \\ \leq h^0(A) + h^0(\omega A^{-1}) & = g + 1 - Cliff(A) \end{cases}$$

which gives:

$$Cliff(A) \leq Cliff(L) + 1 \tag{3}$$

Writing:

$$Cliff(A) = deg(L) - deg(D) - 2h^0(L(-D)) + 2$$

$$Cliff(L) + 1 = deg(L) - 2h^0(L) + 3$$

we deduce from (3) the following inequality:

$$deg(D) \geq 2[h^0(L) - h^0(L(-D))] - 1 = 2\varepsilon - 1$$

where we have defined:

$$\varepsilon := h^0(L) - h^0(L(-D))$$

We have the following possibilities:

$\varepsilon = 0$: since L is globally generated this implies $D = 0$: hence η splits, a contradiction.

$\varepsilon = 1$: since L is very ample this implies that $deg(D) = 1$ and $p \in X$, a contradiction again.

$\varepsilon = 2$: the subspace $\langle D \rangle$ is a trisecant line containing p.

$\varepsilon \geq 3$: in this case we have:

$$deg(A) - 2h^0(A) + 2 = Cliff(A) \leq Cliff(L) + 1 <$$

$$< Cliff(X) \leq [\frac{g-1}{2}] \leq deg(A)$$

where the first inequality is (3), the second is implicit in (1), the last is (2). This implies $h^0(A) > 1$. On the other hand we also have:

$$h^1(A) = h^0(L) - deg(L) + g - 1 - \varepsilon + deg(D) =$$

$$= h^1(L) - \varepsilon + deg(D) \geq h^1(L) + \varepsilon - 1 \geq h^1(L) + 2 \geq 2$$

We deduce that A contributes to $Cliff(X)$. Therefore:

$$Cliff(X) \leq Cliff(A) \leq Cliff(L) + 1 < Cliff(X)$$

where the second inequality is (3) and the third descends from (1). We have a contradiction, hence the case $\varepsilon \geq 3$ is impossible. *qed*

2. Prym canonical curves

In this section we let $L = \omega\sigma$, where $\omega = \omega_X$ is the canonical line bundle on X and σ is a nontrivial line bundle such that $\sigma^2 = 0$. If L is globally generated then the image of $\varphi_L : X \to \mathbf{P}^{g-2}$ is a (possibly singular) curve of degree $2g-2$ called a *Prym canonical model* of X.

(2.1) Lemma *If $Cliff(X) \geq 3$ then L is normally generated.*

Proof Assume that there is $x \in X$ such that $h^0(L(-x)) = h^0(L)$. Then:

$$1 = h^1(L(-x)) = h^0(\sigma(x))$$

Therefore there is a point $y \in X$, $y \neq x$, such that $2y \sim 2x$. This means that X is hyperelliptic, a contradiction. Therefore L is globally generated.

Assume now that

$$h^0(L(-x-y)) = h^0(L) - 1$$

for some $x, y \in X$. Then:

$$1 = h^1(L(-x-y)) = h^0(\sigma(x+y))$$

This means that there exist points $w, z \in X$ such that

$$w + z \not\sim x + y,\ 2(w+z) \sim 2(x+y)$$

hence X has a g^1_d with $d \leq 4$, a contradiction. Therefore L is very ample.

The normal generation of L follows from theorem (1.1). *qed*

(2.2) Proposition *Suppose that L is very ample and that $\varphi_L(X)$ has a trisecant. Then $Cliff(X) \leq 4$*

Proof From the hypothesis it follows that there is an effective divisor $x+y+z$ on X such that $h^0(L(-x-y-z)) = h^0(L) - 2$. Then

$$1 = h^1(L(-x-y-z)) = h^0(\sigma(x+y+z)).$$

Hence there is an effective divisor $u+v+w$ such that $u+v+w \not\sim x+y+z$ and $2(x+y+z) \sim 2(u+v+w)$. This implies that X has a g^1_d with $d \leq 6$. *qed*

(2.3) Corollary *If $Cliff(X) \geq 5$ then*

$$X = \bigcap_{Q \supset \varphi_L(X)} Q$$

Proof Since L is very ample by proposition (2.1), we are in the hypothesis of theorem (1.3): then the conclusion follows from proposition (2.2). *qed*

(2.4) Proposition *If* $3 \leq Cliff(X \leq 4$ *and* $g \geq 9$ *then* $\varphi_L(X)$ *has at most finitely many trisecants.*

Proof From the proof of Proposition (2.2) it follows that each pair of trisecants ℓ, ℓ' defines linear equivalences

$$u + v + w - (x + y + z) \sim \sigma \sim u' + v' + w' - (x' + y' + z') \tag{4}$$

From (4) it follows that the divisor $x + y + z + u' + v' + w'$ defines a g^1_d with $d \leq 6$, and we deduce that X has a 2-dimensional family of g^1_6's. From Keem's theorem (see [ACGH], page 200) and its extension by Coppens to the cases $g = 9, 10$ ([C]), it follows that $Cliff(X) \leq 2$, a contradiction. *qed*

(2.5) Corollary *If* $Cliff(X) = 4$ *then*

$$\bigcap_{Q \supset \varphi_L(X)} Q = \varphi_L(X) \cup \ell_1 \cup \ldots \cup \ell_k$$

where $\ell_1, \ldots, \ell_k$ *are the trisecants of* $\varphi_L(X)$.

Proof By theorem (1.3) we have

$$\bigcap_{Q \supset \varphi_L(X)} Q = \varphi_L(X) \cup (trisecants)$$

The conclusion follows from proposition (2.4). *qed*

We will next consider the case $Cliff(X) = 3$.

(2.6) Proposition *Assume that* $Cliff(X) = 3$ *and* $g \geq 9$. *Then*

$$\bigcap_{Q \supset \varphi_L(X)} Q \subset \varphi_L(X) \cup \Lambda_1 \cup \cdots \cup \Lambda_k$$

where $\Lambda_1, \ldots, \Lambda_k$ *are proper linear subspaces of* $\mathbf{P}^{g-2}$.

Proof Let $p \in \bigcap_{Q \supset \varphi_L(X)} Q$. By lemma (2.1) the hypotheses of lemma (1.2) are satisfied and therefore, as in the proof of theorem (1.3), we can find extensions:

$$\begin{tikzcd}
 & & 0 \arrow[d] & & \\
 & & A \arrow[d] \arrow[dr] & & \\
0 \arrow[r] & \omega L^{-1} \arrow[r] & E \arrow[r] \arrow[d] & L \arrow[r] & 0 \\
 & & \omega A^{-1} \arrow[d] & & \\
 & & 0 & &
\end{tikzcd}$$

with A such that inequality (2) is satisfied. We may choose A as a subbundle of E of maximal degree. Since $deg(\omega L^{-1}) = 0$ the diagonal arrow $A \to L$ is not zero. Therefore $A = L(-D)$ with $D > 0$ and $p \in \langle D \rangle$. By [LN], lemma 2.1, the line bundle A determines the divisor D and thus the linear span $\langle D \rangle$ uniquely. As in the proof of theorem (1.3) one shows that inequality (3) is satisfied, which in this case takes the form:

$$Cliff(A) \leq 3 \tag{5}$$

Therefore we obtain the following chain of inequalities:

$$deg(A) - 2h^0(A) + 2 = Cliff(A) \leq 3 =$$

$$= Cliff(X) \leq [\frac{g-1}{2}] \leq deg(A) \tag{6}$$

From (6) we immediately deduce that $h^0(A) \geq 1$ and if $h^0(A) = 1$ then $7 \leq g \leq 8$ which contradicts our assumptions. Therefore we may assume $h^0(A) \geq 2$.

From (5) we deduce the inequality:

$$deg(D) \geq 2[g - 1 - h^0(L(-D))] - 1 = 2\varepsilon - 1 \tag{7}$$

where we have defined

$$\varepsilon := g - 1 - h^0(L(-D)) = dim(\langle D \rangle) + 1$$

.

The cases $\varepsilon = 0, 1$ are impossible as above because L is very ample. If $\varepsilon = 2$ then $\langle D \rangle$ is a trisecant line containing p: by proposition (2.4) we know that there are at most finitely many trisecants.

Assume now that $\varepsilon \geq 3$. Then we obtain:

$$h^1(A) = h^0(A) - deg(A) + g - 1 = h^0(L) - \varepsilon - [deg(L) - deg(D)] + g - 1 =$$

$$= deg(D) - \varepsilon \geq \varepsilon - 1 \geq 2 \tag{8}$$

It follows that A contributes to $Cliff(X)$ and actually computes it. Therefore

$$Cliff(A) = Cliff(X) = 3$$

and we have the following equalities, deduced from (7) and (8):

$$deg(D) = 2\varepsilon - 1$$

$$h^1(A) = deg(D) - \varepsilon = \varepsilon - 1 = dim(\langle D \rangle)$$

and:

$$3 = deg(A) - 2h^0(A) + 2$$

which follows from (6).

Assume $deg(A) \leq g-1$. Then by [CM], theorem C, we have $deg(A) \leq 10$ and it follows that one of the following cases occurs:

$\lvert A\rvert$	$dim(\langle D\rangle)$
g^1_5	$g-4$
g^2_7	$g-5$
g^3_9	$g-6$

From the hypothesis $Cliff(X)=3$ it follows that in all cases $|A|$ is base point free and simple. By [C] there are finitely many $|A|$'s as above on X unless X is a plane sextic, which is impossible because $Cliff(X)=3$. Therefore only finitely many such spaces $\langle D\rangle$ can occur in this case.

Assume finally that $deg(A) \geq g$. Then $deg(\omega A^{-1}) \leq g-1$ and theorem C of [CM] applies again to ωA^{-1}. We deduce that $deg(\omega A^{-1}) \leq 10$ and it follows that one of the following cases occurs:

$\lvert\omega A^{-1}\rvert$	$dim(\langle D\rangle)$
g^1_5	2
g^2_7	3
g^3_9	4

As already shown, there are finitely many such linear series on X and it follows that also in this case finitely many such spaces $\langle D\rangle$ can occur.

Summarizing we see that, since the point p is contained in one of the spaces $\langle D\rangle$ and since there are only finitely many such spaces, all of dimension strictly smaller than $g-2$, the conclusion follows. *qed*

(2.7) Remark One might ask whether for $Cliff(X)=3,4$ the curve $\varphi_L(X)$ actually admits trisecants. The following examples show that this is the case. Consider a smooth plane curve Y of degree 7 (and genus 15) having two distinct tritangent lines λ_1, λ_2 concurring at a point $p \in Y$. Assuming for a moment the existence of this curve, let x, y, z (resp. u, v, w) be the points of contact of Y with λ_1 (resp. with λ_2). Then we have the linear equivalence

$$p+2x+2y+2z \sim p+2u+2v+2w$$

from which we deduce that the line bundle $\sigma := \mathcal{O}(x+y+z-u-v-w)$ satisfies $\sigma^2=0$. The curve $\varphi_L(Y) \subset \mathbf{P}^{13}$, where $L=\omega\sigma$, has the two trisecants $\langle x+y+z\rangle$ and $\langle u+v+w\rangle$. We have $Cliff(Y)=3$.

Similarly consider a plane irreducible curve Z of degree 8 and genus 20 having an ordinary double point p and two tritangent lines λ_1, λ_2 concurring at p. Assuming the existence of Z, let x, y, z (resp. u, v, w) be the points of contact of Z with λ_1 (resp. with λ_2). On the normalization $\hat{Z}$ of Z consider the line bundle $\sigma := \mathcal{O}(x+y+z-u-v-w)$. Then $\sigma^2=0$, and the curve $\varphi_L(\hat{Z})$ of $\mathbf{P}^{18}$, where $L=\omega\sigma$, has the two trisecants $\langle x+y+z\rangle$ and $\langle u+v+w\rangle$. We have $Cliff(\hat{Z})=4$.

The existence of the curve Y is shown as follows. Start with three distinct lines $\lambda_1, \lambda_2, \lambda$ in $\mathbf{P}^2$ with a common point p. Then consider irreducible conics C_1, C_2, C_3 tangent to λ_1 and to λ_2, and sufficiently general with this property, and a general cubic curve E. Now let

$$Y' = C_1 + C_2 + C_3 + \lambda,\ Y'' = 2\lambda_1 + 2\lambda_2 + E$$

Then a general curve Y belonging to the pencil of Y' and Y'' has the required properties. Infact all the base points of the pencil are smooth for either Y' or Y'': therefore Y is smooth. Moreover Y is tangent λ_i at the three points $C_1 \cap \lambda_i$, $C_2 \cap \lambda_i$ and $C_3 \cap \lambda_i$ because Y' and Y'' are.

The existence of Z is proved similarly, replacing E with a general quartic F, and λ with two general lines λ', λ'' through p.

3. Relation with the Torelli problem for Prym varieties

Let $\mathcal{R}_g$ denote the moduli space of connected etale double coverings $f : \hat{X} \to X$ with X of genus g, and by $\mathcal{A}_{g-1}$ the moduli space of principally polarized abelian varieties of dimension $g-1$. Associating to a double cover $f : \hat{X} \to X$ its *Prym variety* one defines a morphism

$$p_g : \mathcal{R}_g \to \mathcal{A}_{g-1}$$

called the *Prym map* (see [LB]). The Torelli problem is the following:

(3.1) Question *At which points of $\mathcal{R}_g$ is p_g injective ?*

A complete answer to this question is not known. The following generic result is known:

(3.2) Generic Torelli theorem for Prym varieties: *For $g \geq 7$ the Prym map p_g is generically injective.*

Proofs of theorem (3.2) have been given by Friedman-Smith in 1982 ([FS]),by Kanev for $g \geq 9$ in 1983 ([K]), by Welters in 1987 ([W]), and by Debarre in 1989 ([D]).

On the negative side R. Donagi showed in [Do] that injectivity of p_g fails at the double covers of curves having a base point free g^1_4. He also conjectured that this is the only case where injectivity of p_g fails. Subsequently Verra showed in [V] that p_g is not injective also at the double covers of plane nonsingular sextics, thus giving a counterexample to Donagi's conjecture. The results of Donagi and Verra are summarized in the following statement:

(3.3) *p_g is not injective at double covers of curves X such that $Cliff(X) = 2$.*

It is natural to ask whether the following modified form of Donagi's conjecture is true:

(3.4) Modified Donagi's Conjecture: *p_g is injective at double covers of curves X such that $Cliff(X) \geq 3$.*

One of the strategies of proof of theorem (3.2), proposed in [T] (but not completely carried out), and in [D], and analogous to the proof of a generic version of the classical theorem of Torelli given by Green in [G], consists of two main steps.

The first step is to show that for a given unramified double cover $f : \hat{X} \to X$ defined by a nontrivial line bundle σ such that $\sigma^2 = 0$, the projectivized tangent cones at the double points of the theta divisor of the Prym variety generate the linear system of quadrics containing the Prym canonical curve $\varphi_L(X)$, where $L = \omega\sigma$. This step has been carried out by Debarre for *generic* double covers using a degeneration argument; it parallels the result proved by M. Green in [G] for the tangent cones of the theta divisor of any non hyperelliptic jacobians.

The second step consists in proving that $\varphi_L(X)$ can be recovered from $\bigcap_{Q \supset \varphi_L(X)} Q$. This has also been shown by Debarre for generic Prym canonical curves.

Our results of section 2 show that the second step can be carried out for the unramified double covers of every curve X having Clifford index 3 or more. Infact our analysis shows that $\varphi_L(X)$ is the only nondegenerate curve contained in $\bigcap_{Q \supset \varphi_L(X)} Q$ and therefore can be recovered from the quadrics containing it. This implies that conjecture (3.4) can fail only if the first step just described fails for some curve of Clifford index 3 or more. Therefore our results give some new evidence for conjecture (3.4).

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