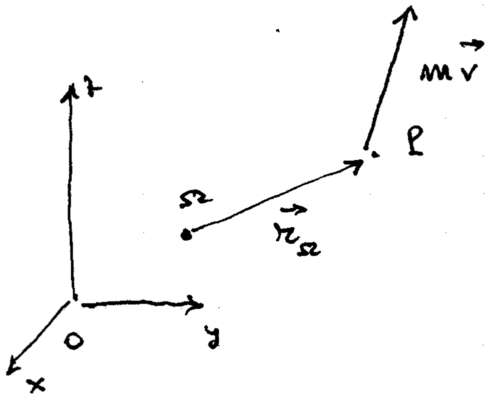
# FORZE CENTRALI

**Forza centrale** : diretta costantemente verso un punto O.

$$\vec F(\vec r) = f(r)\,\hat n$$

**Momento angolare** $\quad \vec l = \vec r_\Omega \wedge \vec Q = \vec r_\Omega \wedge m\vec v$




**Momento di una forza** $\quad \vec M = \vec r_\Omega \wedge \vec F$

$$\vec r_\Omega = \vec{OP} - \vec{O\Omega}$$

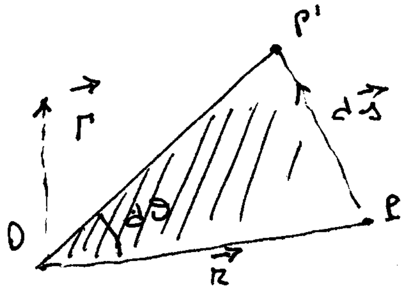
$$\boxed{\vec M = \frac{d\vec l}{dt} + \vec v_\Omega \wedge \vec Q}$$

FORZE CENTRALI $\vec M = 0$

$\vec L = \text{cost.}$

**Velocità areolare** $\quad \Gamma = \frac{1}{2}\,|\vec r \wedge \vec v|$ : aree spazzate nell'unità di tempo

$$\vec \Gamma = \frac{1}{2}\,\vec r \wedge \vec v$$



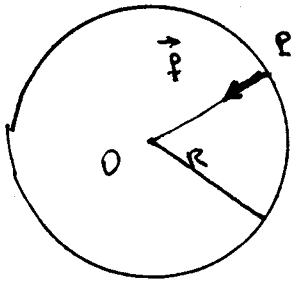

**Sistema di punti materiali** $\quad \frac{d\vec L}{dt} = \vec M^{(e)} + \vec M^{(i)} - \vec v_\Omega \wedge \vec Q$

$$\boxed{\frac{d\vec L}{dt} = \vec M^{(e)} - \vec v_\Omega \wedge \vec Q}$$

$$\vec M^{(i)} = \sum_{i=1}^{N} \vec M_i^{(i)} = 0$$

(Forze interne stessa retta applicazione)

# FORZE CENTRIPETE



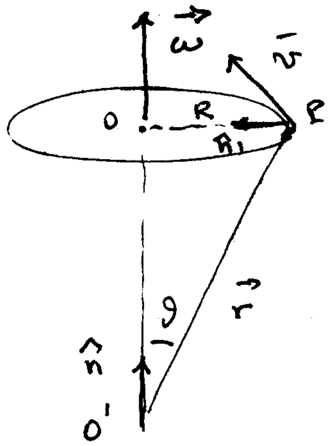

$$f(r) = mv^2/R \qquad \begin{cases} v = \text{cost} \\ R = \text{raggio} \end{cases}$$

$$a_c = v^2/R$$

$$\omega = \frac{d\vartheta}{dt} = \frac{1}{R}\frac{ds}{dt} = \frac{v}{R}$$ VELOCITÀ ANGOLARE

$$T = \frac{2\pi}{\omega}$$ PERIODO

$$\nu = \frac{1}{T}$$ FREQUENZA




$$\vec{\omega} = \frac{v}{R}\hat{n}$$ VETTORE VELOCITÀ ANGOLARE

$$\bar{v} = \bar{\omega} \wedge \bar{R}$$

$$\boxed{\frac{d\vec{r}}{dt} = \vec{\omega} \wedge \vec{r}}$$ RELAZIONE DI POISSON

$$\vec{a}_c = \bar{\omega} \wedge (\bar{\omega} \wedge \bar{R}) = \vec{\omega} \wedge \vec{v} = \frac{v^2}{R}\hat{n}'$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d\omega}{dt}\hat{n} + \underbrace{\omega\frac{d\hat{n}}{dt}}_{=0} = \frac{d\omega}{dt}\hat{n}$$

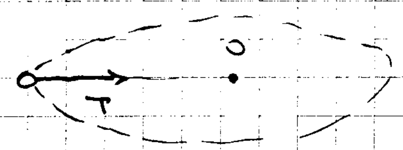
$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_t + \vec{a}_n$$

$$= \frac{d}{dt}(\vec{\omega} \wedge \vec{r}) = \frac{d\vec{\omega}}{dt} \wedge \vec{r} + \vec{\omega} \wedge \vec{v}$$

$$\begin{cases} \vec{a}_t = \vec{\alpha} \wedge \vec{r} & \text{accelerazione tangenziale} \\ \vec{a}_n = \vec{\omega} \wedge \vec{v} & \text{accelerazione centripete} \end{cases}$$

# ESEMPI FORZE CENTRIPETE

a) Palla legata ad un filo orizzontale:

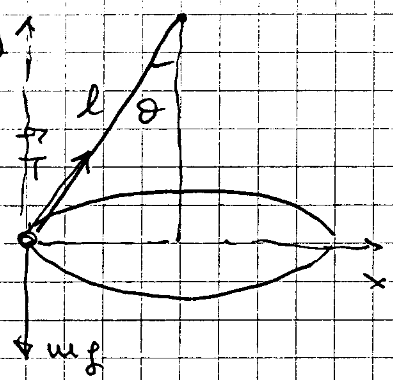



Tensione massima

$$T_0 = m v_0^2 / R$$

$$v_0 = \left( \frac{T_0 R}{m} \right)^{1/2} \qquad v > v_0 \quad \text{moto rettilineo uniforme}$$

b) Pendolo conico




$$\begin{cases} T \sin\theta = m v^2 / R \\ T \cos\theta = mg \end{cases} \qquad R = l \sin\theta$$

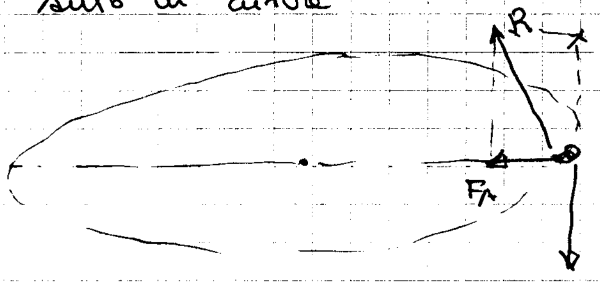
$$T = m v^2 / R \sin\theta \Rightarrow \frac{m v^2}{R \sin\theta} \cdot \cos\theta = mg$$

$$v^2 = mg \cdot \frac{R \sin\theta}{m \cos\theta} = \frac{l g \sin^2\theta}{\cos\theta} =$$

$$\omega = \frac{v}{R} = \frac{v}{l \sin\theta} = \frac{\sin\theta}{l \sin\theta} \left\{ l g \, tg\,\theta \right\}^{1/2} \rightarrow$$

$$\omega = \left\{ \frac{g}{l} \, tg\,\theta \right\}^{1/2}$$

c) Auto in curva




$$\begin{cases} F_A = m v^2 / R \\ N = mg \end{cases} \qquad F_A = \mu_s N = \mu_s mg$$

$$\mu_s g = m v^2 / R$$

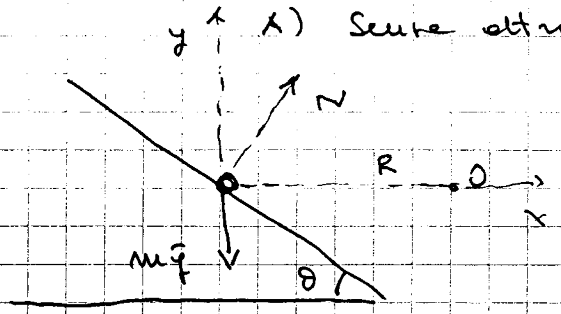
Attrito statico: assenza di sbandamento radiale

Noto $\mu_s$ : velocità massima

$$v_M = (\mu_s g R)^{1/2}$$

d) Sopraelevate :

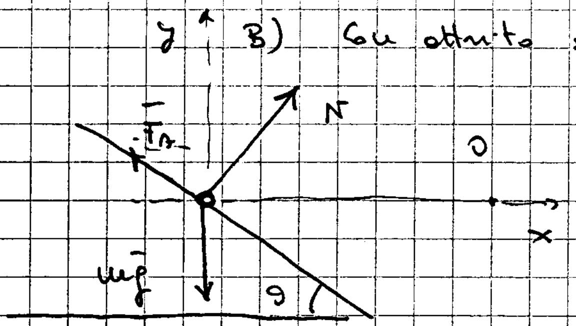
A) Senza attrito con velocità $v_0$ : calcolo R




$$\begin{cases} N\cos\theta = mg \\ N\sin\theta = m v_0^2/R \end{cases}$$

$$R = \frac{v_0^2}{g\,\mathrm{tg}\,\theta}$$

B) Con attrito : calcolo $\mu_s$ ; $v < v_0$




$$\begin{cases} N\cos\theta + F_A\sin\theta = mg \\ N\sin\theta - \bar{F}_A\cos\theta = m v^2/R \end{cases}$$

$$\begin{cases} N\cos^2\theta + \bar{F}_A\cos\theta\sin\theta = mg\cos\theta \\ N\sin^2\theta - F_A\cos\theta\sin\theta = \frac{mv^2}{R}\sin\theta \end{cases}$$

Sommando

$$N = m\left(g\cos\theta + \frac{v^2}{R}\sin\theta\right)$$

Sottraendo

$$\mu_s = \frac{\bar{F}_A}{N} = \frac{1}{\cos\theta}\left\{\sin\theta - \frac{mv^2/R}{N}\right\} =$$

$$= \frac{1}{\cos\theta}\left\{\frac{mg\sin\theta\cos\theta + mv^2/R\,\sin^2\theta - m v^2/R}{mg\cos\theta + \frac{mv^2}{R}\sin\theta}\right\} =$$

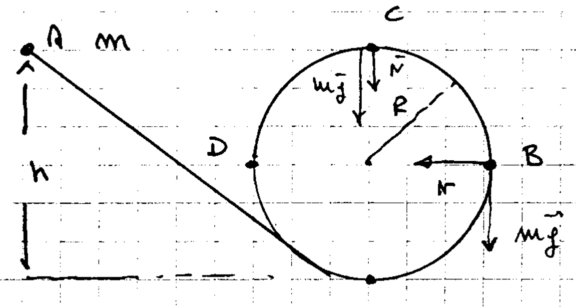
$v < v_0$ (sbandata verso il basso)

$$\mu_s = \frac{g\sin\theta - v^2/R\cdot\cos\theta}{g\cos\theta + \frac{v^2}{R}\sin\theta}$$

$v > v_0$ (sbandata verso l'alto)

$$\mu_s = \frac{g\sin\theta + \frac{v^2}{R}\sin\theta}{g\cos\theta + \frac{v^2}{R}\sin\theta}$$

d) Guida circolare : calcolare h minimo perché m non si stacchi
" forza in B ed in D.




Nel punto C si ha

$$mg + N = m v_c^2/R$$

Il valore minimo di $v_c^2$ prima del distacco lo si ha per
$N=0$

$$mg = m v_c^2/R \quad \Rightarrow \quad v_c^2 = gR$$

Conservazione dell'energia

in C) $\quad mgh = 2R\cdot mg + \frac{1}{2} m v_c^2 \quad \Rightarrow gh = 2Rg + \frac{1}{2} gR$

$$h_0 = \frac{5}{2} R$$

in B) $\quad N = m v_B^2/R$

Conservazione dell'energia

$$mgh_0 = mgR + \frac{1}{2} m v_B^2 \quad \Rightarrow \quad \frac{5}{2} gR = gR + \frac{1}{2} v_B^2$$

$$v_B^2 = 3gR$$

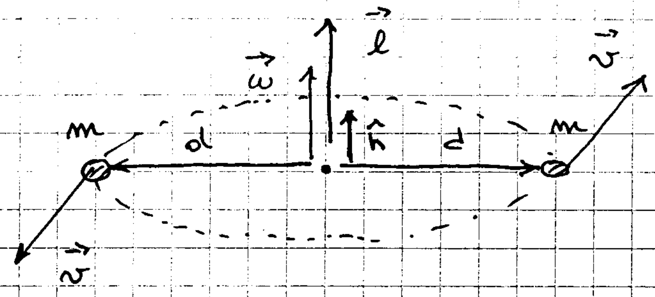
$$\Downarrow$$

$$N = 3mg$$

Forza in B = Forza in D

$$F = \sqrt{N^2 + m^2g^2} = mg\sqrt{10}$$

# MOMENTO ANGOLARE - ROTAZIONI - PRECESSIONE




$\vec l = \vec l_1 + \vec l_2 = 2\vec d \wedge m\vec v =$

$= 2md^2\omega\,\hat n = k\omega\,\hat n$

$$\vec M = \frac{d\vec l}{dt} = k\frac{d\omega}{dt}\hat n + k\omega\frac{d\hat n}{dt} = \vec M_\parallel + \vec M_\perp$$

a) Variazione modulo $\omega$ ; $\vec M \parallel \vec l$ : $\vec M_\parallel = k\frac{d\omega}{dt}\hat n$

momento motore

$$\frac{d^2\vartheta}{dt^2} = \alpha \Rightarrow \begin{cases} \omega = \omega_0 + \alpha t \\ \vartheta = \frac{1}{2}\alpha t^2 + \omega_0 t + \vartheta_0 \end{cases}$$

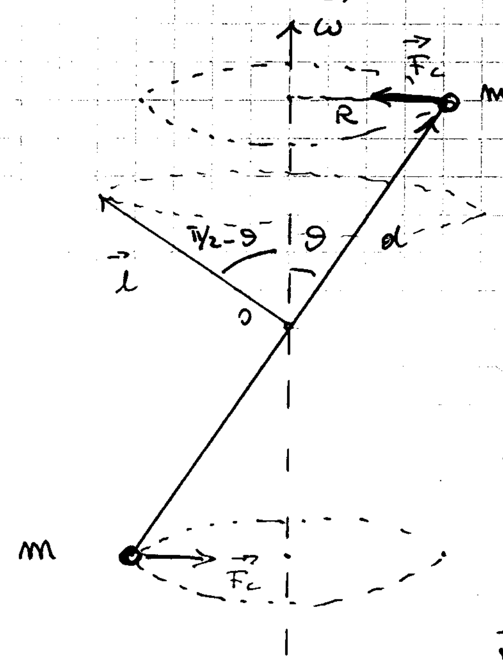
$$\begin{cases} a_t = \frac{dv}{dt} = \frac{d\omega}{dt}\cdot d = \alpha d \rightarrow v = \alpha t \cdot d \quad (\omega d) \\ a_n = \frac{v^2}{d} = \alpha^2 t^2 d \end{cases}$$

$$\Downarrow$$

$$a = (a_t^2 + a_n^2)^{1/2} = \alpha d\,(1+\alpha^2 t^4)^{1/2}$$

b) Variazione direzione di $\vec\omega$ ; $\vec M \perp \vec l$ : $\vec M_\perp = k\omega\frac{d\hat n}{dt}$

MOMENTO FORZA : RESPONSABILE ROTAZIONI




$\vec l = \vec l_1 + \vec l_2 = 2d^2 m\omega \sin\vartheta = 2dRm\omega$

$\omega$ costante $\Rightarrow$ $l$ costante ma non $\vec l$

$\vec M = \frac{d\vec l}{dt} = \vec\omega \wedge \vec l$ ($\vec M$ ruota $\perp$ piano $\vec\omega, \vec l$)

MOTO DI PRECESSIONE

$M = \omega l \cos\vartheta = 2dmR\omega^2\cos\vartheta$

Momento Forze centripete

$\bar M = 2\bar d \wedge \bar F_c$ $\quad$ $M = 2dF_c\cos\vartheta = 2dmR\omega^2\cos\vartheta$

Responsabile moto precessione, momento delle forze centripete.

Non si compie lavoro!