FORZE ELASTICHE

Legge di Hooke $\vec{F} = -k \Delta l \hat{k}$

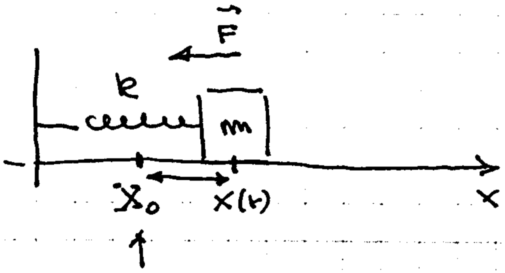
Forze elastiche : forze di richiamo centrali - conservative (posizionali)

→ $\boxed{\text{MOTI ARMONICI}}$: $\vec{F} = -kx\hat{\imath}$

$$dU = -F\,dx$$
$$U = \frac{1}{2}kx^2$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$a = -\frac{k}{m}x = -\omega^2 x$$



Posizione di equilibrio

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$ Equazione moto armonico

Soluzioni:

$$x = a \operatorname{sen} \omega t + b \cos \omega t$$

$$\begin{cases} x(t) = A \operatorname{sen}(\omega t + \varphi) \\ x(t) = B \cos(\omega t + \psi) \end{cases}$$

$$\begin{cases} a = A\cos\varphi \\ b = A \operatorname{sen}\varphi \end{cases}$$

$$\begin{cases} a = A \operatorname{sen}\psi \\ b = -B\cos\psi \end{cases}$$

φ = fase iniziale

A = elongazione massima attorno ad X_0

T = periodo

$$x(t) = x(t+T) \Rightarrow A\operatorname{sen}(\omega t + \varphi) = A\operatorname{sen}[\omega(t+T)+\varphi]$$

$$\Downarrow$$

$$\omega = 2\pi/T$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$\nu = \frac{1}{T}$ frequenza : indipendente dall' elongazione A!

Condizioni iniziali $t = 0$ $x = x_0$, $v = v_0$

$$x(t) = A\operatorname{sen}(\omega t + \varphi)$$

$$\begin{cases} x_0 = A\operatorname{sen}\varphi \\ v_0 = A\omega\cos\varphi \end{cases}$$

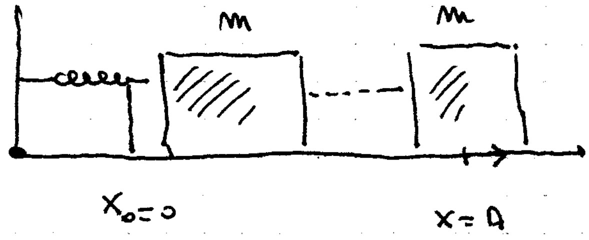
$x_0 \neq X_0$ in generale.

$$\varphi = \frac{2\pi}{T} \cdot \tau$$

τ = fase temporale : Δt tra passaggio per X_0 e x_0

<u>CASI PARTICOLARI</u> 1°) $x = X_0 = 0$

$x(t=0) = A$ m parte dalla posizione $x = A$



$t=0$ $x_0 = A$ $v_0 = 0$

$$\boxed{x(t) = A \operatorname{sen}(\omega t + \pi/2) = A \cos\omega t}$$

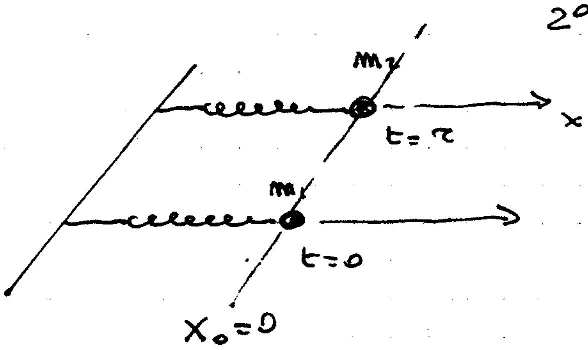
$\varphi = \pi/2$; $\tau = T/4$

$$\begin{cases} v(t) = -A\omega \operatorname{sen}\omega t \\ a(t) = -\omega^2 x = -\omega^2 A \cos\omega t \end{cases}$$

$$\begin{cases} t = 0 \;;\; v = v_0 \;;\; a = a_{MAX} = -\omega^2 A \\ t = T/4 \quad x = 0 = X_0 \quad v = v_{MAX} = -A\omega \;;\; a = 0 \end{cases}$$

$\longrightarrow$

2°)



$$\begin{cases} x_1(t) = A \operatorname{sen} \frac{2\pi}{T} t = A \operatorname{sen} \omega t \\ x_2(t) = A \operatorname{sen} \frac{2\pi}{T}(t+\tau) = A \operatorname{sen}(\omega t + \varphi) \end{cases}$$

$$\begin{cases} x_1 = A \quad \text{per} \quad t = T/4 \\ x_2(T/4) = A \cos \frac{2\pi}{T}\tau = A\cos\varphi < A \end{cases}$$

<u>Moti armonici sfasati</u> : non passano per gli stessi punti nello stesso istante.

a) $\tau = T/4$ $\varphi = \pi/2$

$$\begin{cases} x_1 = A \operatorname{sen}\omega t \\ x_2 = A \operatorname{sen}(\omega t + \pi/2) = A\cos\omega t \end{cases}$$

m_1 passa per A ; m_2 passa per X_0

Moti in quadratura di fase.

b) $\tau = T/2 \qquad \Phi = \pi$

$$\begin{cases} x_1 = A \operatorname{sen} \omega t \\ x_2 = A \operatorname{sen}(\omega t + \pi) = -A \operatorname{sen} \omega t \end{cases}$$

m_1 passa per A; $\quad m_2$ passa per $-A$

Moti in opposizione di fase.

<u>Energia di un moto armonico</u>

$$E = T + U \qquad T = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \varphi)$$

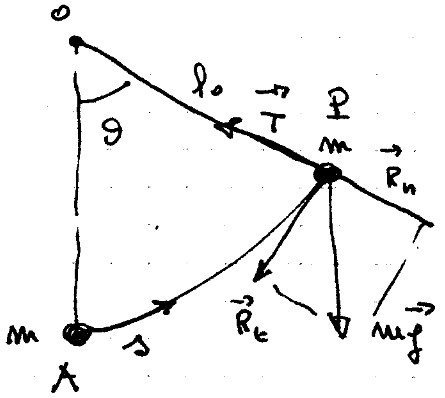
$$-\frac{dU}{dx} = -kx \Rightarrow U = \frac{1}{2} k x^2 =$$

$$= \frac{1}{2} k A^2 \operatorname{sen}^2(\omega t + \varphi)$$

$$E = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \varphi) + \frac{1}{2} k A^2 \operatorname{sen}^2(\omega t + \varphi)$$

$$\boxed{E = \frac{1}{2} k A^2 = \text{cost}}$$

→ PENDOLO SEMPLICE



Equazione del moto

$$\vec{T} + m\vec{g} = m\vec{a}$$

$$\begin{cases} R_t = -mg \operatorname{sen} \vartheta = m a_t = m \dfrac{d^2 s}{dt^2} = m l_0 \dfrac{d^2 \vartheta}{dt^2} \\ T - mg \cos \vartheta = m a_n = m v^2 / l_0 \end{cases}$$

$$\boxed{\frac{d^2 \vartheta}{dt^2} + \frac{g}{l_0} \operatorname{sen} \vartheta = 0}$$

Piccole oscillazioni: $\quad \operatorname{sen} \vartheta \sim \vartheta \qquad \vartheta = 30^\circ = 0.52 \text{ rad}$

$$\boxed{\frac{d^2 \vartheta}{dt^2} + \frac{g}{l} \vartheta = 0}$$ Equazione del pendolo

$$\vartheta = \vartheta_0 \operatorname{sen}(\omega t + \varphi) \qquad \omega = \sqrt{\frac{g}{l}}$$

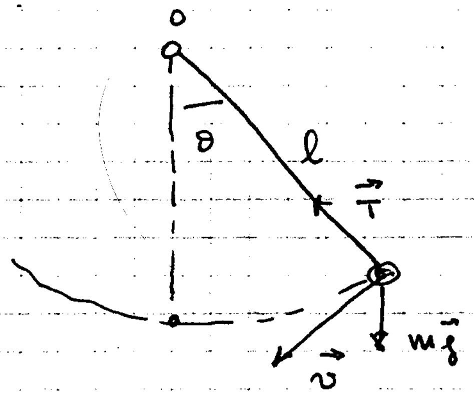
oscillazioni <u>isocrone</u>: $\quad T = 2\pi \sqrt{\frac{l}{g}}$

$$\begin{cases} s = l_0 \vartheta = l_0 \vartheta_0 \,\text{sen}\,(\omega t + \varphi) = s_0 \,\text{sen}\,(\omega t + \varphi) \\ v = v_0 \cos(\omega t + \varphi) \end{cases}$$

$$T = m\left[g \cos\vartheta + v^2/l_0 \right]$$

$$T_{max} = m\left[g + v_0^2/l_0 \right] \qquad \text{per } \vartheta = 0$$

— PIANO DI OSCILLAZIONE COSTANTE



Momento delle forze rispetto ad O

$$\vec{M} = \vec{\ell} \wedge \vec{F} + \vec{\ell} \wedge \vec{T} = \vec{\ell} \wedge \vec{F} = \vec{\ell} \wedge m\vec{g}$$

ha componenti solo lungo l'asse z:

$M_x = 0$, $M_y = 0$ $\quad M_z = mg\ell \,\text{sen}\,\vartheta$

quindi moto solo nel piano xy: $\vec{M} \,//\, \vec{L}$

$$\vec{M} = \frac{d\vec{L}}{dt} \Rightarrow mg\ell \,\text{sen}\,\vartheta = -\ell m \frac{dv}{dt}$$

$$\Downarrow$$

$$g \,\text{sen}\,\vartheta = -\frac{dv}{dt} \qquad \frac{dv}{dt} = \frac{d^2 s}{dt^2} = \ell \frac{d^2\vartheta}{dt^2}$$

$$\frac{d^2\vartheta}{dt^2} + \frac{g}{\ell}\vartheta = 0 \qquad \text{sen}\,\vartheta \simeq \vartheta$$