MOMENTO DELLE FORZE : Sistema R - Polo c.m

$$\sum_{i=1}^{N} \vec{r}_i^* \wedge \vec{F}_i = \sum_{i=1}^{N} \vec{r}_i^* \wedge \left(\vec{F}_i^{(i)} + \vec{F}_i^{(e)}\right) = \vec{M}_c^{(e)}$$

Sistema R^* - Polo cm.

$$\sum_{i=1}^{N} \vec{r}_i^* \wedge \vec{F}_i^* = \sum_{i=1}^{N} \vec{r}_i^* \wedge \left(\vec{F}_i^{(i)} + \vec{F}_i^{(e)}\right) - \sum_i r_i m_i \wedge \vec{a}_{cm}$$

$$= \qquad '' \qquad - M \vec{r}^*_{cm} \wedge \vec{a}_{cm}$$

$\hookrightarrow = 0$

$$\boxed{\vec{M}_c^{(e)} = \vec{M}_c^{*(e)}}$$

MOMENTO ANGOLARE : Sistema R - Polo cm

$$\vec{L}_c = \sum \vec{r}_i^* \wedge m_i \vec{v}_i = \sum r_i^* \wedge m_i \left(\vec{v}_i^* + \vec{v}_{cm}\right) = \sum m_i \vec{r}_i^* \wedge \vec{v}_i^*$$

$$\vec{L}_c = \vec{L}_c^*$$

1° TEOREMA DI KONIG : $\vec{L}_o = \vec{L}_c^* + \vec{L}_o^{c.m} = \vec{L}_c + \vec{L}_o^{cm}$

$$\frac{d\vec{L}_c}{dt} = \frac{d\vec{L}_c^*}{dt} = \vec{M}_c^{(e)}$$

Polo O di R

$$\vec{L}_o = \sum m_i \vec{r}_i \wedge \vec{v}_i = \sum \left(\vec{r}_i^* + \vec{r}_{cm}\right) \wedge m_i \left(\vec{v}_i^* + \vec{v}_{cm}\right)$$

$$= \sum_{i=1}^{N} m_i r_i^* \wedge v_i^* + \underbrace{\left(\sum m_i \vec{r}_i^*\right)}_{=0} \wedge \vec{v}_{cm} + \vec{r}_{cm} \wedge \underbrace{\left(\sum m_i v_i^*\right)}_{=0} + \vec{r}_{cm} \wedge \vec{v}_{cm}$$

$$\vec{L}_o = \vec{L}_c^* + \vec{L}_o^{cm} = \vec{L}_c + \vec{L}_o^{cm}$$

2° TEOREMA DI KONIG : $T = T^* + T_{cm}$

$$T = \frac{1}{2} \sum m_i v_i^2 = \frac{1}{2} \sum m_i \left(v_i^* + v_{cm}\right)^2 = \frac{1}{2} \sum m_i v_i^{*2} + \left(\sum m_i \vec{v}_i^*\right) \times \vec{v}_{cm}$$

$$+ \frac{1}{2} \sum m_i v_{cm}^2$$